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BY

ERNEST L. THURSTON
SUPERINTENDENT OF PUBLIC SCHOOLS, DISTRICT OF COLUMBIA

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PREFACE.

The use and applications of number vary with the changes and development of our industrial, commercial and social relations. The arithmetic of to-day is not the arithmetic of ten years ago. The fundamental principles and processes do not alter, but new applications constantly develop and older applications lessen in usefulness. In this sense, arithmetic is a living subject which deals with living phenomena.

In these days arithmetic is commonly used not only for the necessary routine computations incident to our private, business, or scientific affairs, but its processes are called upon to study facts through number and to *interpret* these facts—to put number *into* things rather than to take number from them. Moreover, the modern use of number often takes the form of a language, as in the construction or reading of numerical illustrations, and statements or statistics which are designed to impress upon others certain facts or conditions. Thus modern arithmetic may be said to have a use as a tool, i. e., for routine computation, an interpretive use, and a language use.

But accuracy and facility in the use of most working instruments presuppose a knowledge of the fundamental parts of the instrument, of the interrelation of these parts, of their working principles, and of the classes of material or work to which they may be applied. Thus, facility in the three "uses" of the arithmetic instrument is founded, primarily, on a logical working knowledge of the fundamental operations and principles. This is the more necessary because the arithmetic problems of actual life do not have a text-book form of statement. If formulated at all, they may offer no direct clue to a particular subdivision of the subject. Often the problem must first be formulated, at least mentally, the power of

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selection being exercised to choose from a group of number facts those essential to the computation desired.

This book is an attempt to construct the arithmetic instrument logically, in its simplest form, as a branch of mathematics; then to apply this tool to general computation, and to the study, preparation and interpretation of varied number material. Involved in this is the purpose to develop arithmetic as a *language of business*, or as a means of interpretation and study of business, and economic conditions; to acquire the capacity properly to present numerical facts by tabulations or graphs; to cultivate clearness of thought and expression, and an appreciation of the value of order and system in applied number work.

In the preparation of the subject matter of this book, no attempt has been made to draw from other text-books, although much may be found here that is in harmony with them. Material has been gathered only from living and reliable sources. The material has been selected with the purpose of emphasizing how arithmetic is applied for a definite object. The problem exercises are not intended, as is so often the case, as a means of conveying to the mind indigestible information on most known subjects. The author believes that the book will be found none the less rich in content because it aims at the digestion of facts, rather than at surfeiting with them.

Much thought has been given to the problem work and, as one result, considerable variety will be noticed in form of statement. Thus the old time "problem in question form" is used, also the incomplete statement, the statistical table, the business form, the written article, the memorandum, etc. Place is also given to series or related problems, composite problems, and "central" problems, to be viewed from different standpoints. Exercises in the preparation of original problems are included for the purpose of cultivating powers of selection

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and statement, and of distinguishing between essentials and non-essentials. Mental development, and growth in power of analysis, must necessarily follow the doing of the work that these exercises require.

A method of development has been followed, in condensed and simplified form, and rules have been reduced to an absolute minimum. It has been assumed, however, that the pupil has had considerable experience in grade arithmetic, before undertaking the study of this book. For this reason illustrative examples are frequently given only in outline. To give facility in the fundamental operations and main processes, a considerable quantity of abstract work has been introduced. In this line, special emphasis has been laid on the fundamental processes as applied to whole numbers, decimals and fractions. Short methods have been emphasized for paper work, and methods of checking have been introduced or suggested. It is felt that every important computation should be checked.

Oral exercises, planned with the utmost care to serve as models, have been given throughout the book, but they should be supplemented by further work along the lines that they suggest. The exercises should vary in form, being, at different times, sight exercises, exercises by dictation, and exercises based upon some fundamental expressions that are placed upon the board, and around which individual examples are built. Many types of exercises will be found illustrated in the following pages.

In all oral work, it should be remembered that facility will come (1) from constantly varying the simple number combinations, or "ringing the changes" on fundamentals, and (2) from fitting the examples to the individual pupil, giving him something that he *can* do and that will develop him in the doing. Attention to these requirements will bring a rich return in the increased capacity of the pupils.

The author wishes to acknowledge his indebtedness to Mr.

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Charles Hart, head of department of business practice, Business High School, Washington, D. C., who read the first proof, and whose wise counsel and careful criticism have been helpful.

E. L. T.

WASHINGTON, D. C.

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EDITOR'S INTRODUCTION.

No subject of the curriculum has received recently more discussion and criticism than has arithmetic. In that most suggestive book "School and Society" John Dewey dwells on the facts that there should be established a natural connection of the everyday life of the child with "the business environment of the world around him," and "it is the affair of the school to clarify and liberalize this connection." Observation confirms the dictum of John Stuart Mill that the child is by nature logical—he seeks the reasons for things. The teaching of mathematics as an abstract science based on phenomena outside of the child's experience serves to dull his natural logical habit of mind.

The three-fold aim of teaching arithmetic is now generally conceded, and a new text can scarcely fail to take these into account. First, it should give the ability to compute correctly and with some rapidity. Second, it should sustain the interest of the learner, that is, arithmetic should present a body of knowledge that is fresh and attractive. Third, the teaching of arithmetic should develop the power of applying number concepts to the activities of everyday life.

The statement of these ideals is easy: the difficulties come in their realization. Arithmetic can be made an instrument of education, both for what has commonly been termed culture and for practical affairs. No school study offers better opportunities for teaching pupils to think or for directing their thinking to the sort of phenomena with which they will deal when school is at an end. The author and the editor of this book recognized these ideals when the book was planned, eight years ago. In preparing the book certain fixed prin-

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ciples have been borne in mind. One is Herbert Spencer's—that "so far as possible the child should make his own investigations and draw his own conclusions." Constantly in this book the pupil is asked to investigate for himself, to formulate or arrange his data, and to deduce his own conclusions.

Secondly, this book is built on the assumption that through the use of concrete problems, it is possible to do three things at once, namely, to give drill, to keep up interest, and to train in the facility of practical application. In the selection of material little or no regard has been given to text-books heretofore existing. Problems have been drawn almost entirely from home, store, farm, shop, the engineer's office, and other like sources.

We have progressed too far educationally to spend time in arguing that it is saner to teach pupils to think on the data that they will later use, than to try to teach them from the obsolete data of the books. Not only does the first method arouse more interest; it gives the only assurance that the pupils will be trained for practical affairs.

The author would seem to have had an almost ideal preparation for the writing of this book. In the first place he was broadly trained in mathematics, and for several years had experience as a teacher in an institution of higher learning. In addition to this he had some years experience as a teacher of arithmetic in the Business High School at Washington, and more recently has served as a supervisor in methods of teaching in elementary schools. Best of all he is a trained and careful investigator, who has been willing to forsake the traditions of arithmetic to work out by practical investigation a book along new lines.

This book gives but limited attention to the fundamental operations, and the attention that is given is only to emphasize the relative value of the fundamental processes. As with the other book of the series, decimals are made to follow im-

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mediately after whole numbers, thus combining these closely related parts of our notation.

The difficulty with much arithmetic teaching is that the pupil considers arithmetic entirely a matter of the book, having nothing to do with practical affairs. The pupil who works through this book should have an intelligent notion of the modern organization and operation of business, a grasp of himself and arithmetical processes, and such facility in the application of these processes as will enable him to go out and make himself useful in the work-a-day world.

C. A. H.

GIRARD COLLEGE,
January 1, 1913.

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CHAPTER I.

UNITS AND NUMBERS.

In applied arithmetic, constant use must be made of units of value and of measure. *Unity means one-ness*, but a unit is not necessarily "one," or a single thing, for it may be *any quantity definitely measured or stated.* One dollar is a unit, but the dollar is one hundred cents, or a certain number of unit grains of gold and alloy, that is, a collection of other units. In practice, units vary with the things with which they are associated. A retailer sells goods by the can—the can is his ***selling*** unit; but he buys by the case, say of 24 cans, and the case may be termed his ***buying*** unit. The idea of a unit as a definite quantity is emphasized in many occupations, as, for example, in engineering, where a *unit distance* is often taken as 100 or 1000 feet.

It is evident that *all units are divisible.* If the unit is *one* or a quantity considered as one, it is termed an *integral unit.* If an integral unit is divided into tenths, hundredths, etc., one of the parts so obtained is termed a *decimal unit.* If the integral unit is divided into any number of equal parts, it is termed a *fractional unit.*

A number is a unit or a collection of units. It is therefore integral, decimal or fractional according to the character of units of which it is composed. If the number is entirely unaffected by association with any object or expression it is an

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abstract number. (Examples: 87, ¼.) If the number is associated with some idea it is *concrete.* (Example, 8 foxes.) If the associated expressions are those of standard measures, the number is *denominate.* (Example, 12 yards.)

Numbers are *like or unlike,* according as they have the same or different unit values. Thus "five months" and "eight months" are like numbers, while "five months" and "eight pounds" are unlike.

Numbers are *simple* if they contain units of the same kind; they are *compound* if they contain different forms of units which are reducible to a single form of unit. Thus "4 lb." is simple, but "4 lb. 8 oz." is a compound number which may be reduced to the simple number "72 oz."

EXERCISE.

1. Name six units of measure.
2. Name five units employed in different trades or professions.
3. Classify these numbers and name the unit in each case:
   (*a*) 976. (*d*) $3.85. (*g*) 5½ gal.
   (*b*) .08. (*e*) 5 yd. 2 ft. (*h*) ¼ lb.
   (*c*) 45 lb. (*f*) 276 payments. (*i*) 4 horses, 2 mules.
4. Name and classify the numbers and units in these expressions:
   (*a*) "The cost of this cloth is 55 cents per yard."
   (*b*) "The goods are packed in cases of 48 boxes each."
   (*c*) "The temperature reached 96° to-day."
   (*d*) "It will take at least half-an-hour and probably forty minutes."
   (*e*) 240 is ½ of 480.

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CHAPTER II.

NOTATION AND NUMERATION.

INTRODUCTORY EXERCISE.

1. How do the expressions "3 yd." and ".03 yd." differ in meaning?

2. Add ciphers to the numbers "40" and ".04" in such a way as not to alter their values.

3. Where does the insertion of a cipher cause a change of value?

4. In expressing numbers orally, what are the values of the syllables "teen" and "ty"?

ARABIC NOTATION.

The Arabic or Hindu notation is the system universally used in civilized countries. The significant figures or digits, 1 to 9, were in use in India 2,000 years ago, but calculations were performed with extreme difficulty until the invention of the tenth symbol, the cipher or zero, 800 years later. The zero made modern computation possible. In the seventeenth century, the invention of the decimal point made possible the present varied use and application of number. The present tendency is toward a continually broadening use of decimals.

The "significant" figures of the system are those that have a *name* or face value, and a *position* value. The zero serves only to alter the position value of other figures.

PRINCIPLES OF USE.

1. The *name* value of each significant figure is constant.

2. The *position* value increases in ten-fold ratio from right to left.

NOTE. Hence called the "decimal system," from the Latin *decem* meaning "ten."

3. A *decimal point*, or period, separates units and tenths of units.

NOTE. The portion of the number to the left of the decimal point is termed the *whole number* or *integer;* the portion to the right is the decimal.

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4. The zero increases the position of significant figures to its left ten fold; it decreases the value of significant figures in a decimal, to its right, to one-tenth their previous value.

5. In any number, where a significant intermediate figure is lacking, a zero is written in its position.

6. Place values are named under the French or English system. Under the French system, used in this country, the figures of large numbers are divided into groups or periods of three figures each, as follows:

Place	13	12	11	10	9	8	7	6	5	4	3	2	1	.	1	2	3	4	5	6
Name	Trillions	Hundred billions	Ten billions	Billions	Hundred millions	Ten millions	Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Units	Decimal point	Tenths	Hundredths	Thousandths	Ten-thousandths	Hundred-thousandths	Millionths
Period	Billions				Millions			Thousands			Hundreds				Thousandths			Millionths		

ILLUSTRATION OF NUMERATION. The numbers should be read in the simplest way, the word "and," or "decimal," being used for the decimal point, thus:

14,264.216 is read: Fourteen thousand, two hundred, sixty-four *and* two hundred sixteen thousandths.

2,132. is read: Twenty-one hundred, thirty-two.

3.02 is read: Three *and* two hundredths.

30,000.003 is read: Thirty thousand *and* three thousandths.

NOTE. In English tables, the periods contain six places each, called units, millions, etc. Under this system the number 36,000,000,000 would be read: Thirty-six thousand millions.

EXERCISE.

1. Which is the larger, 3 yd. or .030 yd.?
2. How does the decimal point determine the size of the part taken?
3. Read:
   52,608
   1,292,723
   329,624,006
   50,007,064,329
4. Read:
   2.43
   17.061
   228.0406
   5,060.38001
5. Read:
   30.0009
   2.0703125
   .627724
   2.390
6. Read by both English and French systems:
   472,693,684,721,125.001.

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7. Insert a cipher in the number 17,246 in five different positions in turn, and read the results. In which position has the cipher the least effect? The greatest?

8. Insert a cipher, in turn, between each two significant figures in the number 86.043201, and read the resulting numbers.

9. Read .4 as hundredths; .65 as ten-thousandths; .025 as hundredths; .002 as millionths.

10. What is the decimal of lowest value, that can be written with the figures 3 and 7 and four ciphers?

NOTE. In writing large numbers, separate periods by commas, or by spacing.

11. Write in column: Seven billion, two hundred forty-eight million, seventy thousand, eight hundred twenty-eight; two hundred twenty-six million, four hundred thousand, thirty; eighteen hundred thousand, four; six ten-thousandths; five and four millionths; sixty-three thousand and six hundred thirty hundred-thousandths; four hundred fifty-nine and eight hundred eleven ten-thousandths; fifty-four tenths; fifty-two hundred-thousandths.

12. Write, substituting numbers for numerical words: (*a*) The circumference of a circle is three *and* fourteen thousand, one hundred fifty-nine hundred-thousandths times the diameter. (*b*) A nautical mile is one and one hundred fifty-two thousandths of a statute mile. (*c*) The total elongation of a bar on a steel bridge, under a maximum load, was two hundred fifty-six thousandths inches.

ROMAN NOTATION.

The Roman system employs as number symbols seven letters of the Roman alphabet. These characters have a name value and a variable position value.

I	V	X	L	C	D	M
1	5	10	50	100	500	1000

PRINCIPLES OF USE.

1. Combinations of symbols are written from left to right in order of value. The total value is the sum of the constituent values.

ILLUSTRATIONS. CX = 100 + 10 = 110. MDCC = 1000 + 500 + 100 + 100 = 1700.

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2. To avoid the consecutive use of symbols four times, a letter of less value may be written before one of greater value, the combination being equal to the difference of the name values.

ILLUSTRATIONS. L = 50, but XL = 50 − 10 = 40. IX = 10 − 1 = 9.

3. The mark "—" over a symbol multiplies its value by 1000.

ILLUSTRATION. XIV = 14, but $\overline{\text{XIV}}$ = 14,000.

EXERCISE.

1. Write the Roman numerals from 1 to 100.

2. Spell the Roman numerals from 1854 to 1913.

3. The corner stone of a public building is marked MDCCCLXVIII. It was laid in what year?.

4. Write in Arabic notation: XIV; XCVI; MDVIII; MCDIX; $\overline{\text{XXIII}}$; CX; DCXII.

5. Name three common uses of the Roman notation.

6. Name objections to the general use of the system.

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CHAPTER III.

ADDITION.

The fundamental operations on which the working methods of arithmetic are based are: (1) addition; (2) subtraction; (3) multiplication; and (4) division. These processes are applied to whole numbers, decimal fractions and common fractions. They will be considered, first, in their relation to whole numbers and decimals.

Addition is the process of combining two or more numbers into a single equivalent number, called the *sum*, the *amount*, or the *total*. It is the most universally employed fundamental process, and should be performed with absolute accuracy and reasonable speed. From a business standpoint, this accuracy and speed depend on: (1) careful formation of figures; (2) alignment of figures; (3) capacity to give and take number dictation; (4) capacity to copy figures accurately; (5) working knowledge of "grouping"; and (6) continued, systematic practice.

Careful Writing. Figures should be uniform in size, legible, simple and distinct in form, evenly aligned vertically and horizontally, and closely spaced but not crowded.

EXERCISE.

1. Give reasons for the requirements just stated.
2. How do carelessly formed figures reduce speed in addition?
3. Write an original column of 6-place figures. (Exchange papers and criticise for form.)
4. The money column on page 331 is "unit ruled." What is unit ruling? Of what value is it in addition?

Number Dictation is frequently incidental and preparatory to addition. One clerk may dictate values to another, who

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writes them in column, or "checks off" a list he holds. In dictation, numbers must be clearly enunciated in an even tone, without unnecessary words, and at uniform speed.

SUGGESTION FOR TEACHERS.

1. Dictate a column of figures. Have pupils check accuracy by "dictating back."

2. Let pupils dictate original columns to each other. Later, check by addition.

Copying is more common than dictating. Office clerks are continually transferring figures from one record to another. Thus a bank clerk lists a depositor's checks, or a book-keeper posts from a Cash Book to a Ledger.

SUGGESTION FOR TEACHERS.

1. Have the class copy a column of blackboard figures. Check by dictation.

2. Exchange original columns for copying.

General Addition.

The mental process of adding consists in grouping digits of the same order. The thought should be placed on the result. In adding 5, 8, 9 and 7, think of each sub-total (13, 22, 29). Facility in addition is gained by reading *instantly* any two or three digits as a sum. Thus, "8 + 9" should be read "17."

DRILL TABLE.

The forty-five two-figure combinations: Name sums at sight.

7	4	2	4	1	3	4	3	3	1	4	2	2	1	1	8	9	8	5	6	4	5	5
7	6	5	3	7	6	2	3	2	5	1	2	1	3	1	9	9	8	5	1	4	3	4
-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
7	1	5	6	6	8	9	8	7	7	4	9	7	6	7	5	3	2	4	5	7	6	
2	8	6	6	9	6	1	2	3	5	8	3	8	7	9	9	8	9	9	8	4	2	
-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	

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EXERCISE.

1. Name the totals. $4 + 8$, $6 + 9$, $3 + 1$, $4 + 7$, $2 + 8$, $1 + 7$, $3 + 9$, $4 + 5$, $4 + 4$, $1 + 2 + 3$, $2 + 4 + 3$, $5 + 2 + 7$, $6 + 7 + 3$, $8 + 2 + 4$, $1 + 1 + 3 + 2$, $1 + 8 + 8$.

2. List all possible groups of two or three digits equaling ten. Learn them.

3. Develop systematically a list of the 165 possible combinations of three digits.

ILLUSTRATION. $1 + 1 + 1$, $1 + 1 + 2$, $1 + 1 + 3$, etc.

4. Give, orally, the consecutive totals obtained by adding: 3s to 58 (58, 61, 64, etc.), 4s to 127, 9s to 156, 8s to 69, 7s to 32, alternate 3s and 8s to 14, alternate 2s and 4s to 52. (Limit 300)

Grouping. In adding columns of figures, it is advisable to group digits into sub-totals, and to combine these sub-totals. The grouping may be *consecutive, irregular* or by *tens and twenties.*

ILLUSTRATIONS.

I.	II.	III.
3	4	5
6	9	9
9	3	4
2	8	1
4	7	6
6	2	8
3	9	3
5	6	2
2	1	7
1	49	5
7		50
48		

(I.) Read instantly the joined numbers as single numbers. The consecutive totals, downward, are: 9, 20, 30, 38, 48.

(II.) Add upward, grouping simple numbers, even if non-consecutive. Totals: 7, 16, 25, 33, 40, 49.

(III.) Since "tens" are easier to add than irregular combinations, group "tens" as shown. Totals upward: 5, 15, 25, 35, 45, 50.

It is frequently necessary to combine groups of two-place numbers. Some accountants regularly add double columns in place of single columns. Until totals are possible by inspection, add the tens and then the units, thus: $46 + 57 = 40 + 50 + 6 + 7$.

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ORAL EXERCISE.

Name the sums of the following:

36	21	84	73	27	62	81	94	78	55	43	16	21	62	43	67
43	69	72	69	41	25	73	18	63	57	54	11	21	29	13	58
—	—	—	—	—	—	—	—	—	—	—	14	10	71	2	11

Complements. The complement of a number is the difference between it and the lowest number of the next higher order. Thus the complement of 8 is 10 − 8, or 2; the complement of 87 is 100 − 87, or 13. Complements are an aid to rapid addition.

ILLUSTRATION. 376 + 96 = 376 + 100 − 4.

ORAL EXERCISE.

1. Name the complements of the following: 7, 21, 36, 54, 77, 129, 973, 9959.

2. When is it advisable to use the complement in mental addition?

3. Add by complements: 64 + 97, 88 + 93, 173 + 98, 77 + 84, 568 + 92.

EXERCISE.

Add these columns without copying figures, grouping simple numbers:

1.	2.	3.	4.	5.
524	3164	529	234,567	1 234 567
689	43	61324	123,456	2 456 673
306	721	399	912,345	5 928 431
1247	2903	2401	891,234	4 241 326
1201	8127	88	789,123	5 261 252
3824	655	1120	678,912	4 683 392
6735	432	21205	567,891	9 221 243
3289	288	4630	456,789	4 322 881
6476	179	880	345,678	3 333 333

6. Why is column 1 easier to add than column 3?

7. What aids to quick addition are found in columns 4 and 5?

8. Write two original columns, one of 20 six-place figures, the other of 60 two-place figures. Add. Which example is more difficult? Why?

9. Name ten simple applications of addition, in everyday life.

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Checking. All addition should be "checked," or tested by one of the following methods.

(*a*) Reverse direction. Add down after adding up, etc. Why?

(*b*) After adding by consecutive numbers or groups, group by tens. Why?

(*c*) Divide the column at one or more points by horizontal lines. Determine the total of each partial column. Set out these totals and add.

(*d*) Add the unit column, writing the total at one side; add the "tens" column independently, offsetting the total as shown. Continue the process. The total of the sub-totals should equal the column total. This method is also used for direct addition by one who is likely to be interrupted frequently. One may stop adding at any point, and continue again when opportunity offers.

3216	18
2479	15
3821	20
3652	11
13168	13168

(*e*) Casting or counting out nines.

To cast out nines from a number, add its digits and whenever the sum equals or exceeds 9 discard 9, adding the remainder to the succeeding digits, etc.

To check addition, discard the nines from the numbers added and also from their sum. Discard the nines from the remainders of the numbers added. If the final remainder equals the remainder of the sum, the sum may be presumed to be correct.

ILLUSTRATION.

	Final Remainder.
426	3
824	5
936	0
1542	3
3728	2, check remainder.

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WRITTEN EXERCISE.

Below is shown a portion of the circulation statement of a newspaper, for a recent year. Find and check the monthly totals, using the methods just outlined.

Date	January	February	March	April	May	June
1	45578	59471	60105	60150	58408	59635
2	50542	59723	59886	48302	59392	56880
3	56551	59116	60036	59937	59029	56746
4	57318	59400	61096	59887	58713	46006
5	57273	49249	49512	61030	58395	57192
7	57942	59522	59469	60105	47000	57745
8	46937	59400	60107	60400	58410	57579
9	57771	59281	60484	47861	58910	57163
10	57909	59613	59960	60415	58614	56662
11	58330	59395	59837	60634	58164	46349
12	58533	59035	48821	60485	58084	57155
13	58106	59372	60149	60332	57955	56571
14	58428	59306	59575	59953	47006	57875
15	48047	59536	59680	59939	57965	57957
16	57797	59469	59839	47505	58102	57859
17	57945	59470	60663	58577	57654	56226
18	58443	59477	59950	60825	57335	46547
19	58696	49410	48875	60134	57349	57287
20	58393	59526	60143	61332	56774	57684
21	58697	59394	60329	60874	46346	57107
22	48676	53016	60260	60504	57296	57208
23	58396	60116	59825	48023	57197	56767
24	59334	59823	60366	59989	57927	56350
25	59015	59934	60060	59751	57884	46069
26	59453	49187	48110	59658	57362	56293
27	59544	59981	60250	59170	57281	56989
28	59075	59732	60023	59147	46522	57203
29	49327		60569	58858	57068	56927
30	59580		60316	47412	49684	56919
31	59381		60064		57070	
Monthly total						

Horizontal Addition. It is often necessary to find the sum of several numbers written in a horizontal row, without re-writing them in vertical columns. Such addition is common in billing, accounting and statistics.

ILLUSTRATION.

				Total Col.
2472	3269	3014	5207	13962

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In adding, follow the usual rule. Add the units (22) writing the unit value of the total and carrying '2'; add the tens, etc. Be careful of positions after passing the second place.

EXERCISE.

Sales recapitulation sheets are used in department stores, factories, etc., to sum up, for fixed periods, the total sales of each department, of each class of output, of each salesman, etc. Their totaling often involves both horizontal and vertical addition, as in the following:

RECAPITULATION OF DEPARTMENT SALES.

February, 19—.

Day	Dept. A	Dept. B	Dept. C	Dept. D	Dept. E	Dept. F	Totals
1	$460.77	$344.00	$450.55	$305.50	$355.06	$455.56	?
2	544.04	550.00	405.60	366.67	460.60	758.24	
3	477.07	637.89	400.33	536.56	367.89	457.65	
5	477.08	460.70	512.45	435.56	404.40	460.70	
6	546.87	315.85	428.67	345.67	546.82	467.19	
7	478.89	211.81	543.78	377.69	512.87	601.03	
8	213.56	110.87	232.16	215.46	127.83	105.68	
9	136.46	98.67	205.38	122.69	200.80	149.03	
10	568.67	511.84	690.57	477.89	624.35	589.98	
12	477.21	647.29	489.39	436.29	527.69	672.39	
13	567.85	437.27	512.11	532.56	455.21	532.24	
14	547.54	512.37	544.65	522.22	499.05	542.26	
15	600.25	590.01	504.71	488.34	473.56	532.56	
16	580.35	543.26	379.67	476.58	532.23	433.59	
17	555.88	496.36	665.00	299.99	548.84	477.25	
19	788.27	564.88	675.50	590.54	693.47	657.76	
20	433.19	459.23	435.67	467.89	590.12	534.56	
21	512.25	511.11	561.00	500.15	509.90	503.32	
22	485.20	500.44	503.67	483.74	468.49	589.70	
23	522.16	460.69	544.36	467.78	488.29	512.61	
24	501.44	577.10	600.00	503.92	605.58	505.45	
26	473.36	546.46	516.17	563.10	718.14	503.25	
27	494.58	536.25	500.26	468.87	600.21	580.41	
28	456.67	478.75	601.37	503.22	496.59	502.45	
Totals							

(*a*) Total the vertical columns. What do they show?

(*b*) Total the horizontal columns. What do they show?

(*c*) Suggest a "check" for the vertical and horizontal totals.

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(*d*) Indicate the possible effects of weather, season, special sales, and other causes, on values appearing in departmental columns. Show what values in the above table might be specially accounted for.

NOTE. Most tabular addition in business offices, and considerable computation requiring other fundamental operations, is now done by machinery. As early as A.D. 968, Gilbert, Archibishop of Rheims, perfected some ingenious computing machines, which, however, sometimes failed to compute correctly. Pascal (1642–45) invented a machine correct in theory, which, while not practically useful, formed the basis for some modern machines of great capacity.

The modern machines generally perform the fundamental operations. They are either "listing" or "non-listing" machines. One standard machine has eight or more "banks" of keys, each containing nine digits and having its own place value. The keys to represent a certain number are pressed down and the movement of a lever, or the pressing of an electric button causes the printing of the number on a continuous roll of paper. The next number is printed below the first, the changing total appearing in the machine. A special movement prints the final total on the paper, and "clears the machine" for the next operation. Some machines list abstract numbers; others list money values, fractions, etc. Some are attached to typewriters so that bills of merchandise may be written and automatically totalled.

TABULATIONS AND STATISTICS.

EXERCISE.

These two news items have appeared in print:

(Tabulated.)

The Department of Agriculture estimates the minimum annual damage done to crops by our most destructive insects, as follows:

Chinch-bug	$60,000,000
Grasshopper	50,000,000
Hessian fly	40,000,000
Corn-root worm	20,000,000
Corn-ear worm	20,000,000
Cotton-boll weevil	20,000,000
Codling moth (Apple)	20,000,000
Army worm	15,000,000
Cotton-boll worm	12,000,000
Grain weevil	10,000,000
San Jose scale	10,000,000
Cotton-leaf worm	8,000,000
Potato-bug	8,000,000
Cabbage worm	5,000,000
Total	?

(Untabulated.)

The losses caused by insects to the products of the country, during an average year, are estimated by the Department of Agriculture, as follows: Cereals, $200,000,000; animal products, $175,000,000; forests and forest products, $111,000,000; products in storage, $100,000,000; hay and forage, $53,000,000; truck crops, $53,000,000; cotton, $50,000,000; tobacco, $5,300,000; fruits, $27,000,000; sugars, $5,000,000; miscellaneous crops, $5,800,000. Total, $——?——.

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(*a*) Find the missing totals. In which case is it easier? Why?

(*b*) Which statement is read more easily? Which is more noticeable?

(*c*) From which statement is it easier to select, or reclassify values? Why?

(*d*) Compare capitalization in the two cases. Compare use of "$."

(*e*) Tabulate the second illustration, using a brief title in place of the introductory clause.

The illustrations given above show the value to the statistician of the "column" and "total" in conveying numerical information. The column tabulation attracts attention, and brings each numerical value in sharp contrast with other values, while allowing comparison with the total. In tabulating, names attached to values should be brief, but definite. Each tabulation should have a brief descriptive title.

EXERCISE.

1. Tabulate and find the lowest cost of construction of a building for which the following bids were submitted:

For masonry, iron work, carpentry, glazing, tinning, plastering, etc.—A. B. Ryan, $49,845; J. D. Norton, $41,900; C. A. Olhaus, $42,344; Robert Adams, $48,275; Brown & Mann, $35,691; Wm. Harris, $37,573; W. B. Norton, $44,678; A. S. Keene, $43,536; Northern Cons. Co., $40,547; Jas. Davis, $35,386.

For materials and labor necessary to furnish, install and complete a low pressure, steam-heating apparatus:—Robert Davis, $4,646; Morris Heating Cons. Co., $3,674; Evanson & Son, $3,940; E. T. Roberts Co., $3,970; Barton Mfg. Co., $4,180; Geo. A. Sanderson, $4,900; Brown & Mann, $4,950; Wm. Harris, $3,398.

For materials, labor, etc., for plumbing work:—D. F. Kean, $3,325; Robert Harris Co., $5,132; Wm. Harris, $4,736; A. B. Ryan, $4,728; Harper & Son, $4,198; Brown & Mann, $5,763.

From the tabulation just made, find: (*a*) what bidder will complete the work for the lowest sum; (*b*) the lowest sum for which the work may be done by giving sub-contracts to different bidders.

Decimal Addition.

INTRODUCTORY EXERCISE.

1. Annex a cipher to .6, and read result.

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2. Reduce 4 to tenths.

3. Reduce 5.3 to hundredths.

4. Add .3 and .6.

5. Add .4, .6, .9, .8.

6. Add .04, .83, .07, 001.

7. Add .047 and .07. (.07 = how many thousandths?)

8. How does reduction to a common order aid in decimal addition?

There is practically no difference between integral and decimal addition. Integral or decimal units of the same order are added. In writing in column, be sure that the decimal points are in vertical alignment, as this will ensure proper "order" position, and also the position of the decimal point in the total.

ILLUSTRATION.

$$\begin{array}{r} 46.205 \\ 392.0872 \\ 1245.01232 \\ 254.67005 \\ \hline 1937.97457 \end{array}$$

ORAL EXERCISE.

1. How does the number of decimal places in the total compare with the number of places in the constituent numbers?

2. May the total contain a less number of decimal places than any significant number?

Add:

3. .4 and .85.

4. .072 and 0096.

5. .12, .065 and .363.

6. .013, .21, and .220.

7. .01, 2.006 and 15.0126.

8. 2.33, 1.062 and 2.0382.

9. 3.25, 1.0006 and 8.0129.

10. .46, .05, .8 and .002.

11. 6.3205 and .00061.

12. .001, 2.01 and .0352.

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EXERCISE.

Total these columns:

(1)	(2)	(3)
23.075	123.456729	4.37
06.5082	54.08296	83.070096
16.09327	5.4729825	5.9276
5463.382473	.007369	329.832
1.29634	527.8308896	4.00892
50069.8002173	5.2100009	.0006732
542.06345276	67.31438	1.01
12.47234	5729.61	15.2729
5067.30821	80.8	.888005

NOTE. A large proportion of decimal addition deals with money values:

(4)	(5)	(6)
$ 326.925	£ 456.29	45.625 fr.
48.065	39.625	129.8
723.8	1233.40	46239.03
9654.47	56.1235	596.008
830.425	5.60	63.9
65.72	1283.55	1321.48
12756.23	14756.125	32635.835
52.08	677.0875	4.006
926.00	1239.05	5282.56
7259.12	36.075	43.45
5.69	5296.5125	6.735
627.33	83.0125	529.62
46.09	962.825	88.9

7. Find the totals, in units of a thousand feet, of these orders for lumber: 6246 ft., 15,389 ft., 7500 ft., 4382 ft., 965 ft., 45 ft., 12,925 ft., 6750 ft.

8. Total these engineering measurements, and express total in units of 1000 ft.: 564′, 296.5′, 4521′, 885.2′, 750′, 1325′, 632.8′, 983.7′, 66.8′.

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CHAPTER IV.

SUBTRACTION.

INTRODUCTORY EXERCISE.

1. Name the difference between 2 and 9, 16 and 21.
2. From 129 subtract 4s.
3. From 138 subtract alternate 3s and 5s.
4. Give several illustrations of the use of subtraction in everyday life.

Subtraction is one phase of addition. It is the process of finding the number to be added to the lesser of two given numbers in order that the sum may equal the greater.

ILLUSTRATION. Find the difference between 774 and 1969.

1969 minuend.
774 subtrahend.
1195 difference.

Process. Add to the subtrahend, digit by digit, until the minuend is obtained, thus: 4 (sub.) + 5 = 9 (min.). Write 5, as unit difference. 7 (sub.) + 9 = 16 (min.). Write 9 in ten's place of difference. 7 (sub.) + 1 = 8 (9 − 1, min.). 0 (sub.) + 1 = 1 (min.).

NOTE. In formal work, it is often inconvenient to write the lesser number under the greater. Position, however, makes no difference in method. The above example might be written:

(*a*) 774 sub.
1969 min.
1195 diff.

(*b*) 1969 − 774 = 1195.

EXERCISE.

1. Show that these subtraction "checks" are true.
   (*a*) The sum of subtrahend and difference equals minuend.
   (*b*) The minuend less the difference equals the subtrahend.

Solve by inspection:

2. Find by addition the complements of 36, 29, 64, 83, 91, 896.
3. Solve by addition:

54 − 13	76 − 23	148 − 93	168 − 72
59 − 37	43 − 29	124 − 58	249 − 68

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NOTE. Oral work is sometimes simplified by subtracting the orders of the subtrahend independently, beginning with the highest. This is termed subtraction from the left. Thus 578 − 364 = 578 − 300 (278) − 60 (218) − 4 = 214.

4. Solve, by subtraction from the left:

249 − 67	328 − 243	1527 − 892	968 − 871
1116 − 452	1793 − 896	529 − 381	4473 − 269

NOTE. Irregular numbers may be subtracted mentally by subtracting an even number of the next higher order, correcting the difference of the numbers by adding the difference of the subtrahends. Thus 384 − 47 = 384 − 50 (334) + 3 = 337.

5. Solve, explaining the process:

362 − 98	473 − 57	734 − 96	263 − 99
654 − 319	328 − 246	899 − 473	346 − 228

NOTE. Often, in practical work, the subtrahend must first be obtained by addition.

6. 624 − (32 + 16 + 11) = ?

7. 1754 − (200 + 311 + 40) = ?

8. Sales of 72, 14 and 11 pounds from a stock of 360 lb. leave how many pounds on hand?

EXERCISE.

1. 59,768,003 − 407,019 = ?

2.
$$\begin{array}{r} \$3,876,547 \\ 108,976 \\ \hline ? \end{array}$$

3.
$$\begin{array}{r} \$567,839 \\ 203,678 \\ \hline ? \end{array}$$

4.
$$\begin{array}{r} 987,654,321 \\ 123,456,789 \\ \hline ? \end{array}$$

In the following, subtract the last number from the sum of the balance of the column:

5.
$$\begin{array}{r} 7239 \\ 2146 \\ 1299 \\ 881 \\ 4731 \\ \hline \end{array}$$

6.
$$\begin{array}{r} 1209 \\ 9660 \\ 5138 \\ 1234 \\ 8108 \\ \hline \end{array}$$

7.
$$\begin{array}{r} 213 \\ 32 \\ 921 \\ 4238 \\ 1119 \\ \hline \end{array}$$

8.
$$\begin{array}{r} 32139 \\ 4781 \\ 61020 \\ 3121 \\ 5288 \\ \hline \end{array}$$

9–12. Repeat the last four examples, subtracting the first number from the sum of the balance of the column.

NOTE. Subtraction is often combined with direct addition, especially in business forms and statistical tables.

13. This form is one of many used in business to keep account of stock on hand.

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STOCK RECORD OF	LIMIT	WHERE KEPT
Office Desks, No. 34672	*40*	*Warehouse A—21*

STOCK RECEIVED			STOCK SOLD			ON HAND
DATE	FROM WHOM	QUAN.	DATE	TO WHOM	QUAN.	QUAN.
19—			*19—*			
3-3	*Inventory*	*172*	*3-5*	**#3762*	*9*	*163*
			3-8	*3964*	*27*	*?*
			3-9	*4287*	*15*	*?*
			3-12	*4561*	*42*	*?*
3-21	*Sanborn Co.*	*50*	*3-21*	*5265*	*18*	*?*
			3-24	*5290*	*54*	*?*
			3-26	*5380*	*21*	*?*
			3-29	*5620*	*20*	*?*

(*a*) Extend the form.
(*b*) Explain the writing of the date.
(*c*) How are entries in the "On hand" column obtained?
(*d*) Explain the solution for Mar. 21.
(*e*) How many desks are on hand on Mar. 11?
(*f*) How may the "Received" and "Sold" columns be checked?

14. Many private families, living on a "cash" basis, keep simple record, showing in detail household income and expenditures. (Page 21.)
(*a*) Explain the first items in the "Total paid out" and "Balance" columns.
(*b*) Find the remaining values for these columns.
(*c*) Total all the vertical columns except the "Balance" column.
(*d*) Why is not the "Balance" column totaled?
(*e*) Find a check for the "Total paid out" total.
(*f*) Suggest a check for the final balance.
(*g*) Suggest how amounts in the different columns might vary with the season of the year.

The statistician who is tabulating crops, wealth, railway mileage, bank clearings, exports, etc., frequently gives figures for earlier periods and shows by a *difference* column the amount of change, the differences often lending special significance to the gross values. The business man, also, gives special atten-

* Order number identifies purchaser.

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Household Expenses from June 1 to June 15.

DATE		ITEMS	RECEIVED	TOTAL PAID OUT	BALANCE	RENT, TAXES, REPAIRS	LIGHT AND FUEL	INSURANCE AND INTEREST	FURNITURE AND SUPPLIES	CLOTHING	AMUSEMENTS, LUXURIES, TRAVEL	MISCELLANEOUS
June	*1*	*Salary*	*120 —*	*36 56*	*83 44*	*30 —*		*6 —*			*20*	*36*
	2	*Cash*		*?*	*?*		*5 75*		*6 42*			*38*
	3	*"*		*?*	*?*			*3 50*				*1 23*
	4	*"*		*?*	*?*		*1 62*		*1 40*	*13 65*		*47*
	5	*"*		*?*	*?*						*5 80*	*62*
	6	*"*		*?*	*?*				*12 92*			
	8	*"*		*?*	*?*			*25 40*				*81*
	9	*Comm.*	*40 —*	*?*	*?*		*2 60*		*3 17*			*2 17*
	10	*Cash*		*?*	*?*				*2 20*			*1 19*
	11	*"*		*?*	*?*				*60*			*46*
	11	*"*		*?*	*?*				*4 32*		*1 35*	*53*
	12	*"*		*?*	*?*					*5 21*		*07*
	13	*"*		*?*	*?*						*65*	*61*
	15	*"*		*?*	*?*		*1 25*		*1 35*	*2 10*		*16*
			?	*?*		*?*	*?*	*?*	*?*	*?*	*?*	*?*

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tion to the increases or decreases, as well as to the total of his sales, profits or expenses. In the exercise that follows, note how the differences add weight to the presentation of facts.

EXERCISE.

1. These figures from a financial article show certain facts concerning the business of the country for two consecutive years. Find the differences, placing decreases in the "increase" column, but marking them with a minus sign.

TWO YEARS' RECORD.

	1910	1911	Increase
Bank clearings	$158,450,000,000	$162,914,100,000	?
Railway earnings	2,395,800,000	2,423,400,000	?
Exports	1,867,605,000	1,637,256,000	?
Imports	1,392,500,000	1,426,200,000	?
Grain crops	2,702,458,000	2,489,000,000	?

EXERCISE.

1. Bring to class four illustrations of "difference" columns in statistical tables.

2. Sum up the different methods of checking addition and subtraction used or suggested in this chapter.

SUBTRACTION OF DECIMALS.

INTRODUCTORY EXERCISE.

1. From 36 hundredths subtract 12 hundredths.

2. $5.46 - 2.39 = ?$

3. $.52 = ?$ thousandths. $.52 - .412 = ?$

The only difference in process between whole number and decimal subtraction consists in the necessity for a reduction, in the latter, of both subtrahend and minuend to decimals of the same order.

ILLUSTRATION. Find the difference between 5.029 and 3.0014.

Solution. 5.029 is the same as 5.0290.

$$\begin{array}{r} 5.0290 \\ 3.0014 \\ \hline 2.0276 \end{array}$$

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EXERCISE.

By inspection find the difference between:

1. .51 and .402.
2. 3.6 and 1.32.
3. .9 and .09.
4. 1. and .0321.
5. 2.0004 and 1.3.
6. .01024 and .08.
7. .000004 and .0004.
8. 10.003 and .230.
9. From 3.75 subtract .05s.
10. From .324 subtract .004s.
11. From one and one-hundredth, subtract nine ten-thousandths.

EXERCISE.

1. 4.0739625 − 2.00100376 = ?
2. 5506.38 − 296.54325 = ?
3. 100,109.0001003 − 1202.12345 = ?
4. From 46 subtract the first of the following decimals; from the remainder subtract the second, and so on: 5.0621, 2.010309, 1.0042, .98396, 25.1013096, 1.24965. How may the accuracy of the final answer be checked?

Decimal subtraction is common in higher mathematics and in scientific and estimating problems. Two-place decimal subtraction is common in money computations. Usually subtraction is incidental to some other process.

EXERCISE.

1. A farmer's poultry account for January is as follows: Value of fowls, Jan. 1, $1800; sales of fowls during the month, $247.53; value of fowls at close, $1750; cost of food and care, $112.50; sales of eggs, $105.82; miscellaneous expenses, $54.27. Make a neat statement showing the net profit for the month.
2. A metal bar 16 inches long, is contracted under pressure .8583765 inches. What is its length under pressure?
3. A yard measure is equal to 0.914,402 meters. A meter exceeds a yard by what decimal of itself?
4. A certain cloth shrinks .1525 of a foot per yard in dyeing. A yard of the undyed cloth produces how many yards of dyed material?
5. The needle on a recording machine travels from 3.256 ft. on the scale to 4.105 ft., a distance of ? ft.

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6. In one city, having 5c car fares, the cost of carrying each passenger is 3.9246c. In another city, having a 4c fare, the cost is 3.1024c. Find the profit per passenger in each case, and the difference in cost per city.

7. Complete this memorandum of costs and profits.

THE NOVELTY MANUFACTURING COMPANY.

Memorandum of Costs and Profits per Article.

Article.	Cost to Make, ¢.	Net Sell. Pr. ¢.	Net Profit.
Butterfly cards	3.0245	7.5	?
Dime savings banks	14.308	16.5	?
Remembrance cards	.0965	2.15	?
Surprise box	4.85	6.1125	?
Three of a kind	13.0625	15.	?

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CHAPTER V.

MULTIPLICATION.

Multiplication, as a number process, is second in importance to addition, and has extensive and varied applications. It is really an addition "short cut," for expressions of multiplication, such as "5×381" may be written in the form "$381 + 381 + 381 + 381 + 381$." As an independent process, also, it lends itself to so many short methods as to make it worth a special study.

ILLUSTRATION. Multiply 3264 by 529.

3264	(multiplicand)
529	(multiplier)
29376	(units)
6528	(tens)
16320	(hundreds)
1726656	(product)

In the product there are 9×3264 units; 2×3264 tens and 5×3264 hundreds.

NOTE. The multiplicand and multiplier are called *factors* of the product.

While the multiplier is invariably an abstract number, the multiplicand and product may be either abstract or concrete, but they are always *like* numbers. As a result, the product is not altered by changing the order of the factors *considered as abstract numbers.*

ILLUSTRATION. The weight of 8724 bx. of 65 lb. each is either (8724×65) lb., or (65×8724) lb.

FOR DISCUSSION.

1. What is the objection to solving the examples illustrated above by addition?

2. At what point, in regular multiplication, does addition occur?

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3. In finding the product of 873,926 by 1001, which factor should be used as multiplier? Why?

4. Why is this statement incorrect:

"\$16 × 15 bbl. = \$180"?

ORAL EXERCISE.

1. Find the products of: 3×14, 7×18, $3 \times 3 \times 6$, $4 \times 2 \times 22$, 8×40 bu., 5×81c, $2 \times 3 \times 8 \times 4$, 2×24 ft.

2. Multiply each integer under 21, by itself and by each smaller integral number. These are the most used factors.

3. A camping party of three men, planning a sixty day trip, order rations in the proportions given below. Determine the required quantity of each article.

SUPPLIES PER MAN PER THIRTY DAYS.

Flour.....24 lb.	Tea, cocoa...1 lb.	Butter.... 2 lb.
Oatmeal..... 7 "	Coffee.....2 "	Bak. pow.. 1 "
Beans..... 6 "	Rice.....4 "	Bacon....11 "
Sugar.....12 "	Salt.....1 "	Pepper... 1 " (6 men)
Cond. milk.... 3 cn.	Prunes.....2 "	

4. A certain contractor found constant need for the products of the factors 80 to 100 by the factors 20 to 40. He prepared the form outlined below, the product of each two factors being written at the intersection of the proper horizontal and vertical columns. The product of 84 by 22 is shown. Compute the other products mentally. Show how to use an addition method for the work. Suggest a method of checking the computations.

	80	81	82	83	84	85	Etc.
20							
21							
22					1848		
Etc.							

5. Construct a similar multiplication table for the factors 1 to 25 inclusive.

EXERCISE.

Find the products of the following and check accuracy by reversing factors:

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1. $726{,}943 \times 120{,}301$.
2. $3{,}629{,}007 \times 20{,}905$.
3. $16{,}730{,}021 \times 802{,}001$.
4. $373{,}259 \times 129$.
5. $12{,}345 \times 67{,}890$.
6. $20{,}103 \times 1{,}387{,}654$.

7. A railway company sends out eight engineering parties of 19 men each, on reconnoisance and track location work. Rations are required for 200 days, based on a U. S. Government ration list. Prepare a tabulation showing the quantity of supplies to be provided.

RATION LIST FOR ONE MAN FOR ONE HUNDRED DAYS.

(Take first named article.)

100 lb. fresh meat, including fish and poultry.
50 " cured meat, canned meat, or cheese.
15 " lard.
80 " flour, bread, or crackers.
15 " corn meal, cereals, macaroni, sage, or corn starch.
5 " baking powder, or yeast cakes.
40 " sugar.
1 gal. molasses.
12 lb. coffee.
2 " tea or cocoa.
10 cn. condensed milk, or 50 qt. fresh milk.
10 lb. butter.
20 " dried fruit, or 100 lb. fresh fruit.
20 " rice or beans.
100 " potatoes or other fresh vegetables.
30 " vegetables or fruits.
4 oz. spices.
4 " flavoring extracts.
8 " pepper, or mustard.
3 qt. pickles.
1 " vinegar.
4 lb. salt.

8. As a first result of the survey, the Company decides to build 19 miles of single and 38 miles of double track. Compute the total quantity of the following supplies which are to be used in the construction: Cross-ties, at rate of 2640 per mile of single track; steel rail, at 90 pounds per yard; spikes, at the rate of 11,380 per mile of single track; 68 angle irons for special construction, 34 feet long, weighing 16 pounds per foot; 240 I-beams, 28 feet long, weighing 65 pounds per foot. Give quantities of iron and steel by weight.

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Short Methods.

Most fundamental processes of number may be abbreviated —often with marked increase in rapidity of solution and accuracy of result. The short methods used, however, should be such as one naturally develops for himself, or those that instinctively appeal to one as being simpler and more direct than the regular methods. Memorized rules for short methods are dangerous aids, as an inexact memory may cause an incorrect application of the rule at some critical moment. Short methods are commonly used to do away with computation on paper, or to simplify such computation, or to check the accuracy of computations performed by other methods.

Short Methods in Multiplication.

Some typical cases follow. Suggest other methods. In each case, show wherein the value of the method consists.

Multiplying by 10, 100, etc. $726 \times 10 =$ ——; $1428 \times 100 =$ ——; $345 \times 1000 =$ ——. Compare the significant figures of the product with those of the multiplicand. How may the product of such factors be written at sight? What effect has the annexing of a cipher? Of two ciphers?

ORAL EXERCISE.

1. $167 \times 10 = ?$
2. $100 \times 3{,}802 = ?$
3. $100 \times 50{,}201 = ?$
4. 100×26 lb. $= ?$
5. $10{,}000 \times 131$ ft. $= ?$
6. $10 \times 7{,}384$ yd. $= ?$
7. $1{,}000 \times 80{,}201 = ?$
8. $70{,}073 \times 100{,}000 = ?$

Find the weight of:

9. 246 bbl., averaging 100 lb. ea.
10. 1,246 loads, averaging 1000 lb.

Multiplying by 11, 21, 31, etc., and by multiples of 11. Outline a short method for these multipliers, suggested by the following illustrations:

(1) $11 \times 64 = 10 \times 64 + 64 = 640 + 64 = 704.$
(2) $31 \times 126 = 3\,(10 \times 126) + 126 = 3780 + 126 = 3906.$
(3) $101 \times 48 = 100 \times 48 + 48 = 4800 + 48 = 4848.$

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ORAL EXERCISE.

Find the product of:

1. 11×16.	**3.** 11×242.	**5.** 21×360.	**7.** 21×19.
2. 21×28.	**4.** 31×18.	**6.** 41×22.	**8.** 1001×287.

Find the weight of:

9. 240 ft. iron at 11 lb. per. ft.

10. 165 kegs, 31 lb. each.

Find the cost of:

11. 12 lb. butter, @ 31c.

12. 142 sq. yd. tiling, @ 21c.

Multiplying by 9, 19, 29, etc. After studying the illustrations, outline the short method and compare with the previous case.

ILLUSTRATIONS.

(1) $9 \times 148 = 10 \times 148 - 148 = 1480 - 148 = 1332$.

(2) $19 \times 320 = 2\,(10 \times 320) - 320 = 6400 - 320 = 6080$.

Used in paper work, also.

(3) Example. Multiply 32,689 by 19.

Long method.

$$\begin{array}{r} 32689 \\ 19 \\ \hline 294201 \\ 32689 \\ \hline 621091 \end{array}$$

Short method.

Factors not re-written.

653780 ($2 \times 10 \times 32689$, written at sight)

32689 (1×32689)

621091 (by subtraction)

NOTE. The short method substituted the multiplication by 2 for the more difficult multiplication by 9.

ORAL EXERCISE.

Find the product of:

1. 9×87.	**3.** 29×16.	**5.** 9×269.	**7.** 29×17.
2. 19×42.	**4.** 109×120.	**6.** 9×424.	**8.** 59×11.

9. Find the weight of 720 pc. castings, weighing 9 lb. each.

EXERCISE.

Using short methods, find the product of:

1. $19 \times 32{,}967$.	**4.** $29 \times 6{,}376{,}951$.	**7.** $99 \times 8{,}703{,}129$.
2. $9 \times 1{,}213{,}027$.	**5.** $19 \times 12{,}587{,}328$.	**8.** $1{,}009 \times 784{,}562$.
3. $39 \times 47{,}201$.	**6.** $109 \times 630{,}851$.	

Multiplying by 5, 15, etc.

ILLUSTRATIONS.

(1) $5 \times 620 =$ One-half of $10 \times 620 = \frac{6200}{2} = 3100$.

(2) $15 \times 48 = 10 \times 48 + \frac{1}{2}$ of $10 \times 48 = 480 + 240 = 720$.

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ORAL EXERCISE.

1. In multiplying a number, such as 7,268,316, by 5, why may it be simpler to use the short method?

2. Multiply by 5: 16, 21, 32, 54, 123, 1309, 677, 1893.

3. Multiply by 15: 24, 68, 152, 19, 137, 240, 62, 192, 1012.

4. Find the costs of: 124 cn. at 5c; 16 lb. nails at 5c; 68 yd. dress goods at 15c.

EXERCISE.

Using short methods, and omitting the re-writing of factors, find the products of:

1. 5×7236.
2. $15 \times 83{,}963$ lb.
3. 15×724 yd.
4. $5 \times 12{,}463$ qt.
5. 15×8039 qt.
6. 15×7204 oz.

Multiplying by factors of multiplier.

ILLUSTRATION: $36 \times 120 = 6 \times 6 \times 120 = 6\ (6 \times 120) = 6 \times 720 = 4320$.

ORAL EXERCISE.

1. Compare the short and the direct methods.

2. Is the short method of real advantage in oral or in written work?

3. How do the factors themselves limit the use of the method?

4. Reduce to two or more simple factors: 65, 24, 49, 18, 72, 96, 48, 45, 54, 84, 110, 39, 92, 120.

5. Determine these products by factoring multiplier, and check by other short methods:

25×160	122×12	99×48	16×32
240×18	12×118	15×72	20×155

Multiplying by parts (multiplication from the left).

ILLUSTRATION. (1) Find the weight of 722 bbl. of 112 lb. each.
$112 \times 722 = 100 \times 722 + 10 \times 722 + 2 \times 722 = 72200 + 7220 + 1444 = 80864$

(2) Written work. Multiply 738,421 by 321.

Regular method.	Method by parts.
738,421	Factors not re-written.
321	221,526,300 (3 × 100 × number)
738421	14,768,420 (2 × 10 × number)
1476842	738,421 (1 × number)
2215263	237,033,141
237033141	

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NOTE. The advantage of this method for oral work, lies in adding continuously decreasing sub-totals to a growing total, each new product being added as found.

ORAL EXERCISE.

Find the product of:

1. 115×36. 3. 23×42. 5. 62×312. 7. 103×428.
2. 111×249. 4. 22×124. 6. 210×122. 8. 51×122.

9. What is the output in 16 hours of a machine making 240 doz. cans per hour?

EXERCISE.

Find the product of:

1. $212 \times 324{,}567$. 2. $1325 \times 88{,}459{,}876$. 3. $1245 \times 87{,}960{,}345$.

SPECIAL EXERCISE.

1. Write a brief illustrative paper on "Short methods in Multiplication."

2. Write a paragraph on the dangers and sources of error in the use of short methods.

3. Write a brief paper on "Methods of Checking Multiplication."

DECIMAL MULTIPLICATION.

INTRODUCTORY EXERCISE.

1. One-tenth of 6 = ? One-hundredth of 6 = ?
2. Ten times .6 = ? One hundred times .6 = ?
3. Compare these numbers as to value: .06, .6, 6, 6000.
4. How is the value of a significant figure affected by moving the decimal point to the right.
5. Suggest a short method for multiplication of decimals by 10, 100, 1000, etc. Illustrate, using .006 and 4.2 as multiplicands.
6. One-tenth of 68 = 68 ÷ ?; or 68 × ?
7. Using 72.6 show the effect on its value of moving the decimal point to the left.
8. Suggest short methods of multiplication by .1, 01, 001, etc.

The process of multiplication of decimal fractions differs in no essential from the multiplication of integers, except in the determination of the decimal point of the product. Noticing that $.1 \times .01 = 1/10 \times 1/100 = 1/1000$ or .001, it is

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evident that the number of decimal places in the product equals the sum of those in the factors. In multiplication, therefore, decimals may be treated as integers, the proper number of places being pointed off after the numerical product is obtained.

ILLUSTRATIONS. (1) Multiply 5268.39 by 1.6.

Solution and analysis. 5268.39 contains 526,839 hundredths.

1.6 contains 16 tenths

526,839 hundredths × 16 tenths = 8,429,424 thousandths, or 8429.424

5268.39
1.6
3161034
526839
8429.424

(2) Find the product of 7.26 multiplied by 3600.

Solution. $3600 \times 7.26 = 36 \times 100 \times 7.26 = 36 \times 726.$

726
36
4356
2178
26136

NOTE. Notice how the decimal is cancelled before the multiplication of significant figures is begun. This is often possible.

ORAL EXERCISE.

1. How many decimal places in the products of:
   (*a*) $3.6 \times .004$ (*c*) $58 \times .020$ (*e*) $.00008 \times 4.00007$
   (*b*) $50.29 \times .00609$ (*d*) $1573.829 \times .032$ (*f*) 9.001×1.0000401
2. Find the products of:
   (*a*) $1.2 \times .12$ (*d*) $4003 \times .0002$ (*g*) $1.002 \times .021$
   (*b*) $.003 \times .0004$ (*e*) $6.01 \times .005$ (*h*) $.06 \times 6000$
   (*c*) $2.01 \times .00002$ (*f*) $.000003 \times .3$ (*i*) $7.25 \times .0002$
3. Compute the cost of:
   (*a*) 700 lb. @ 32c. (*b*) 2000 bu. @ 45c. (*c*) 2960 lb. @ $1.10.

EXERCISE.

(Check all products.)

1. Multiply 7206.021 by 3.001029.
2. Multiply 739.65482 by 549.03296.
3. Multiply 7.2098 by .02604, and the product by 100.09.

ORAL EXERCISE.

(Use the short methods given for integral numbers.)

1. 100×2 6 yd. = ?
2. $380 \times .2 \times .01 = ?$
3. $11 \times 3.6 = ?$
4. $1000 \times .005$ yd. = ?
5. $21 \times .4 = ?$
6. $101 \times .08 = ?$
7. $9 \times .8 = ?$
8. $19 \times 1.2 = ?$

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GENERAL EXERCISE.

NOTE. In the computation of costs and quantities by multiplication, decimals are usually involved. The following are typical illustrations.

1. For accounting purposes, Martin & Co. take an inventory, or valuation of their stock of skins, pricing them at the lowest market quotation, except for the gray fox which they price at the highest rate. Using the following quotations, make a neat tabulation showing the valuation of the entire stock.

WOOL AND HIDES.—Quotations for furs on No. 1 articles only. Wool, washed, free of burrs, per lb., 38a40; wool unwashed, per lb., 30a33; hides, green, per lb., 10; dry, per lb., 15a17; sheepskins, green, each, 1.00a 1.25; dry, each, 25a75; calf-skins, green, each, 1.00a1.50; muskrat, each, 12a18; skunk, each 25a75; mink, each, 3.00a4.00; rabbit skins, each, 10; opossum, each, 25a27; raccoon, each, 25a90; fox, red, each, 2.00a2.50; gray, each, 75a90.

The stock of Martin & Co. consists of: 2480 lb. wool, washed; 1600 lb. wool, unwashed; 750 lb. hides, green; 2375 lb. hides, dry; 128 calf skins; 46 muskrat skins; 256 rabbit skins; 13 gray fox skins; 129 opossum skins; 382 raccoon skins.

2. Tabulate individual and gross weights of the following order:

200 16′ I-beams, weighing 85.2 lb. per ft.
162 19′ I-beams, weighing 23.5 lb. per ft.
200′ trough plates, weighing 16.32 lb. per ft.
1150 18′ channel irons, weighing 33 lb. per ft.
450′ deck beams, weighing 27.23 lb. per ft.
620 14′ angle irons, weighing 17.2 lb. per ft.
145 Z-bars, 12′, weighing 29.8 lb. per ft.

3. This is an itemized tabulation of the cost of a masonry wall containing 12,642 cu. yd. of masonry. Compute the total cost.

	Per cu. yd.	Total cost.
Quarrying stone	$0.415	$?
Loading and hauling stone	.912	
Hoisting stone	.605	
Excavating	.300	
Loading and hauling sand	.355	
Cement	.345	
Mixing and delivering mortar	.205	
Masons and helpers	.915	
Carpentry	.885	
Blacksmithing	.190	
Tools and general supplies	.175	
Superintendence, foremen, etc.	.725	
	$?	$?

(How check?)

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4. Bids *a*, *b*, *c*, *d* and *e* have been submitted by different parties, to cover the cost of a granite block pavement. Find and tabulate the total cost of each item and of each bid. Determine what combination of different bids will give the lowest cost.

	a	*b*	*c*	*d*	*e*
400 cu. yd. concrete	$4.00	$3.00	$3.90	$2.55	$3.98
135 lin. ft. granite crossings	1.20	1.13	1.25	1.00	1.24
1178 lin. ft. 5″ granite curb	1.275	1.30	1.30	1.29	1.30
162 lin. ft. circular curb	2.00	2.10	1.95	2.00	2.00
65 lin. ft. curb, reset, etc.	.21	.15	.20	.32	.18
48250 cu. yd. grading	.43	.425	.41	.45	.50
2476 sq. yd. granite paving	3.25	3.18	3.25	3.00	3.30
2476 sq. yd. sub grade	.06	.06	.06	.06	.06
268 lin. ft. 24″ drain pipe	2.78	1.75	2.00	2.10	1.95
73 cu. yd. brick masonry	12.00	11.25	12.50	10.90	11.75
21 cu. yd. rubble masonry	5.75	6.00	4.25	5.00	4.80
12 pc. Palmer inlets	40.00	40.00	40.00	40.00	40.00
Totals					

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CHAPTER VI.

DIVISION.

INTRODUCTORY EXERCISE.

1. Add by 4s from 0 to 48. There are how many 4s in 48.
2. Subtract by 9s from 189 to 0. There are how many 9s in 189.
3. Compare division with addition and subtraction.
4. I agree to pay $40 cash and $6 per week for a $100 typewriter. Name the amount due after each payment. Determine the number of payments (1) by addition, (2) by subtraction, (3) by division. Lowering the weekly payment to $4 extends the time of settlement ? weeks.
5. $172 \div 4 = ?$ $? \times 4 = 172$.
6. Compare multiplication and division.

Division may be termed a short method in addition and subtraction, and the reverse of multiplication. It is the process of determining the unknown factor of a product from the product and a known factor. The known factor is the *divisor*, the product is the *dividend*, and the factor to be determined is the *quotient*. The *remainder* is the portion of the dividend that is left when division is not exact.

ORAL EXERCISE.

If the dividend is 64 and the divisor 8, show that:

1. The dividend divided by the divisor equals the quotient.
2. The quotient times the divisor equals the dividend.
3. The dividend divided by the quotient equals the divisor.

FACILITY EXERCISE.

(Solve by inspection.)

1. Divide by 2: 16, 38, 57, 96, 248, 1749, 62,482.
2. Divide by 3: 12, 21, 48, 69, 125, 683, 952, 667.
3. Divide by 2 and the quotient by 3: 36, 124, 88, 962.
4. Divide by 4: 76, 320, 888, 96, 154, 723.

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5. Divide by 5: 25, 65, 83, 195, 217, 1259.

6. Divide by 6: 42, 80, 172, 620, 38, 415.

7. Divide by 7: 49, 35, 735, 1420, 203, 533.

8. Divide by 8: 72, 30, 960, 1240, 884, 1720.

9. Divide by 9: 30, 75, 810, 640, 1850, 27630.

10. Multiply 24 and successive products by 2 (6 times). Divide by successive 4s.

Short Division. In short division the divisor and dividend are written, and the figures of the quotient, as obtained, but intermediate remainders and products are omitted.

ILLUSTRATION. Divide 7248 by 8.

Solution.

8)7248

906

Analysis. 72 hundreds ÷ 8 = 9 hundreds; 4 tens are not divisible by 8; 4 tens + 8 units = 48 units. 48 units ÷ 8 = 6 units.

NOTE. At times, when the divisor is reducible to very simple factors, it is possible to employ continued short division by the factors in turn. In such cases, of course, each factor must be an exact divisor until the last.

EXERCISE.

Divide:

1. 239,872,452 by 4.

2. 8,703,265 by 5.

3. 364,285,968 by 6.

4. 123,456,789 by 7.

5. 6888 by 56 (continued division).

6. 24,496 by 16.

7. 193,625 by 25.

8. 4,829,672 by 32.

ILLUSTRATIVE EXERCISE.

1. Solve: (*a*) 36 ÷ 4 = ?
(*b*) 36 ft. ÷ 4 = ?
(*c*) In 36 bbl. there are ? lots of 4 bbl. each.

2. What is "common" to these problems? What changes?

It is essential, in division, to determine whether the quotient and the remainder are abstract or concrete. Division by an abstract divisor is called *partition*, and by a concrete divisor, *measuring*. Thus, in 1 (*b*) in the preceding exercise "36 ft." is parted into 4 parts, and in 1 (*c*) "4 bbl." is used as a measure to determine the number of "4 barrel" lots in 36 bbl. It

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should be noted that if divisor and dividend are concrete, the quotient is abstract; while if the dividend is concrete and the divisor abstract, the quotient is a concrete number of the same order as the dividend.

ILLUSTRATIVE EXERCISE.

1. Find the remainders in the following:
   (*a*) 725 ÷ 4 = ?
   (*b*) 420 yd ÷ 8 = ?
   (*c*) For 88c I can buy ? yd. of cotton at 8c per yd.
2. When is the remainder abstract? When concrete?

Long Division. In long division, intermediate products and remainders are shown.

ILLUSTRATIONS. (1) Divide 76,945 by 162.

Solution.

474 (Quotient)

(Divisor) 162)76945 (Dividend)

(400 × 162) 648

1214

(70 × 162) 1134

805

(4 × 162) 648

157 (Remainder)

Check. 474 × 162 + 157 = 76945.

(2) Table of Multiples of 162.

By	
1	162
2	324
3	486
4	648
5	810
6	972
7	1134
8	1296
9	1458

If, as often happens in business and statistics, the same divisor is constantly employed, a table of its multiples (see (2) from 1 to 9, is prepared, and used for reference. This avoids repeated multiplication, and serves to show, without trial, each successive figure in the quotient.

EXERCISE.

Divide, expressing remainder separately:

1. 146,732 by 1396.
2. $372,486 by 2013.
3. 9,820,605 by 89,243.
4. 620,548 lb. by 63 lb.
5. In 468,246 lb. of grain there are ? bu. of 56 lb. each.

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6. Divide the following by 1237, using a table of multiples:
(*a*) 673,892. (*b*) 960,478. (*c*) 1,214,734.

7. Divide the following by 864, using a table of multiples:
(*a*) $4,327,432. (*b*) 8,921,065. (*c*) 1,234,056,789.

8. Find the number of tons of coal (2240 lb.) in each of these shipments, ignoring fractions of a ton.

Car No.	Wt. of Loaded Car.	Empty Car.	Net Weight.	Tons.
32,064	134,200	26,420	?	?
38,296	129,100	27,020		
43,129	126,000	24,320	?	?

Short Methods.

Division does not lend itself to so many easily applied short methods as does multiplication. The following are the most common:

Division by 10, 100, etc.

ORAL EXERCISE.

1. What effect has the decimal point on the comparative values of these numbers: 6800, 680 0, 68.00, 6.800?

2. Suggest a short method of division by powers of 10. Find the quotient of:

3. 720,000 ÷ 10.
4. 726,000 ÷ 1000.
5. $725,900 ÷ $100.
6. 30,000,000 lb. ÷ 10,000.
7. 368,000 ÷ 100.
8. $850 ÷ 10.

Division by multiples of 10. When a power of ten is a factor of both dividend and divisor, the example may be simplified by the principle of continued division.

Illustration. $4720 \div 80 = (4720 \div 10) \div 8 = 472 \div 8 = 59$.

EXERCISE.

Find the quotient by inspection.

1. 840 ÷ 70.
2. 725,000 ÷ 600.
3. 870 ÷ 30.
4. 320 lb. ÷ 20.
5. $7,600 ÷ 40.
6. 800,000 lb. ÷ 40.
7. $650 ÷ 130.
8. $987,500 ÷ 500.

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DECIMAL DIVISION.

INTRODUCTORY EXERCISE.

1. 4.2 is equivalent to ? tenths.

2. 4.2 ÷ 6 may be expressed: ? tenths ÷ 6. The quotient is —.

3. Divide by 6: $480, 48 lb, 4.8 T., .048 cu. yd., .00048.

4. Divide these same values by .6. What remains unchanged in the quotient? What changes?

It is evident that the location of the decimal point in the quotient is the new feature of the division process met with in passing from whole numbers to decimals.

ILLUSTRATIONS. (1) Divide 49.776 by 16.

Solution		*Condensed solution.*
	3 111	3.111
	16)49.776	16)49.776
(3 × 16)	48	48
	1 776 remaining.	17
(.1 × 16)	1 6	16
	.176 remaining.	17
(.01 × 16)	.16	16
	.016 remaining.	16
(.001 × 16)	.016	16

NOTE. If the divisor is integral, the position value of each figure in the quotient is the lowest position value in the partial dividend used in obtaining it. Thus "49" is the partial dividend used in obtaining "3" of the quotient, and its lowest position value is "unity." The advantage of writing quotient over dividend is evident, since the "placing" of the first figure of the quotient practically determines the place value of the rest.

(2) Divide 7.6845 by .15.

Solution. Multiply each term by 100, by short method.

7.6845 ÷ .15 = 768.45 ÷ 15.

51.23

15)768.45

NOTE. In case the divisor is decimal in form, it may be cleared of decimals, by moving the decimal point to the right in both dividend and divisor. The example then comes under the class shown in the previous illustration.

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EXERCISE.

Using short methods, if convenient, find by inspection the quotient of:

1. $3.6 \div 6$. **4.** $.0004 \div 2$. **7.** $3.824 \div 8$. **10.** 52.08 T. $\div 100$.

2. $.427 \div 7$. **5.** $2.64 \div 3$. **8.** $16.860 \div 12$. **11.** $3.62 \div 10$.

3. $.084 \div 16$. **6.** $1.6 \div 4$. **9.** $.00129 \div 30$. **12.** $29000.01 \div 1000$.

In the following, clear the divisor of decimals:

13. $.84 \div 1.2$. **14.** $63.08 \div .002$. **15.** $.0003 \div 006$. **16.** $2.4060 \div 1.2$.

Divide, by inspection:

17. 1 by .01, .001, .0001, 10000.

18. .01 by 10, 100, .01 and .00001.

19. .0001 by .005, .05, .02, 100.

20. 4.8 by 1.2, .012, .0006, .3.

21. 60 by .002, .3, .0005, .12.

22. 200, 4000, .08 and .0006 by 2.

23. .00012 by 6, 600, .3, .03, .006.

24. 4.24 by .0006 and by 6.

25. 63 by .007, .09, .7, .0003.

26. 660.15 by .015.

Exact and Approximate Results. Decimal ciphers may be added to the dividend indefinitely, to secure an accurate quotient to any number of decimal places, the final remainder being expressed fractionally. Commonly, however, results are required *exact to some stated number of places*, the remainder being unstated. Thus a business man may compute costs "to the nearest cent," or an engineer make measurements to "hundredths of an inch." Many working tables, also, are computed to four, five or six decimal places, remainders being ignored because, as a rule, they are too small to affect appreciably results computed by them. Often, if the remainder is one-half or more of the preceding decimal figure, that figure is increased by 1. Thus a quotient of 7.3086 may be written "to three decimal places" as 7.309.

When merely an approximate quotient is desired, the approximate method is sometimes used. This consists usually in direct division until the integral portion of the quotient is obtained. From that point, no further figure of the dividend is brought down but, at each step, the right hand figure of the previous divisor is cancelled.

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ILLUSTRATION. Divide 684.394683 by 56.138.

Contracted Solution.

```
                12.1914
          ------------------
56.138)684.3946839
       561 38
       ------
       123 014
       112 276
       -------
        10 738
(Div. 56.13)  5 613
              -----
              5 125
(Div. 56.1)   5 049
              -----
                 76
(Div. 56)        56
                 --
                 20
(Div. 5)         20
                 --
```

EXERCISE.

Divide:

1. $726 48 by 12.
2. 114,269 lb. by 11. (2 places.)
3. 4629.2 cu. yd. by .24.
4. .0297 by .31. (2 places)
5. 12.873 by 151.3. (1 place.)
6. 842,097 by .0036. (Approx.)
7. 23.0567385 by .07525. (3 places approximate.)
8. $7296.47 by 123. (3 places.)
9. 91.38 by 1.02545. (3 places, approximate.)
10. A speed of 592 knots in 24 hours is a speed of ? knots per hour. (2 places.)

EXERCISE.

NOTE. The weight of any material, as compared with an equal bulk of pure water, is termed its *specific gravity*. Thus a brick having a specific gravity of 1.602 weighs 1.602 times as much as the same volume of water.

Assuming the weight of a cubic foot of water as 62.355 lb., find the specific gravity of the following building materials:

Material.	Pounds per Cu. Ft.	Specific Gravity.
Asphaltum	87	———
Hard brick	125	———
Clay	120 to 145	—— to ——
Limestone	170 to 210	—— to ——
Plaster of paris	75 to 80	—— to ——
Slate	165 to 180	—— to ——
Tile	106 to 122	—— to ——
Sand stone	140 to 155	—— to ——

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Selected Individual Assignments.

1. Write a brief on "Methods of Checking Arithmetical Computations," giving illustrative examples.

2. Write a brief on "Short Methods in Computation." Illustrate short methods in common use. If possible, report on business men's attitude towards the use of short methods.

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CHAPTER VII.

ARITHMETICAL AVERAGING.

INTRODUCTORY EXERCISE.

1. What six equal numbers have a total equal to the sum of 12, 14, 30, 42, 18 and 10?

2. Five men give to a public cause, respectively, $40, $35, $20, $60, $25. The same sum would have been secured by their contributing equally, $—.

3. The total profit of a business for three years was $19,654.21. Assuming that there was no variation from year to year, what was the annual profit?

4. The values just determined are called *averages*, or *average numbers*. What is an average number? What processes of arithmetic are employed in averaging?

5. What does a physician mean when he says "My office calls average 12 per day"?

Averages offer a common and effective means of comparing values, and are used constantly in statistical work and in business. Thus the average output, expense and profit of a business, for past years, will guide a possible purchaser to a decision, or aid an owner in remedying defects. The average monthly rainfalls, in a certain farming section, will guide an intending purchaser in his decision as to possible crops. Other illustrations will be found throughout the book.

Ordinarily, the *average*, *mean*, or *normal*, is obtained by a process of addition, followed by division. At times, the addition may be mechanically accomplished, as in the case where a number of castings are placed on a foundry scale and weighed at one operation, the total weight being then divided by the number of castings to determine the average weight.

In determining averages care must be used to include all influencing factors.

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ILLUSTRATION. On April 1, Brown deposits, in a bank that pays interest on average monthly balances, the sum of $2000; and on April 21, reduces this sum to $1600. Determine his average balance for April.

Solution. The average is *not* $2000 + 1600 divided by 2, for the individual balances existed for different times. $2000 was the balance for 20 days, and $1600 of 10 days. There were 20 daily balances of $2000 and 10 of $1600. The average is, therefore,

$$\frac{20 \times \$2000 + 10 \times \$1600}{30} = \frac{\$56000}{30} = \$1866.67 -.$$

EXERCISE.

NOTE. These sentences are taken at random from news reports and general publications. Determine the missing values.

1. "The Lusitania made 598, 606, 612 and 603 knots in four successive days, averaging — knots per day."

2. "England records a West Coast train from London to Aberdeen, September, 1895, 540 miles in 512 minutes— — miles per hour."

3. "In 1895 the number of depositors in the savings banks of the country was 7,696,000, having total deposits of $3,093,236,119, or an average of —— each."

4. "The distance is 192 miles, but I can average 12 miles an hour. If I leave at 7 o'clock to-morrow, that would get me there by ——."

5. "Those cattle ought to average 940 lb. each, or —— lb. for the lot of 37."

6. "The output with the old machinery averaged 46,000 yd. per 10 hour day. With the new equipment, it is 48,000 yd. in 9 hours. The average hourly output has increased —— yd."

7. "Mr. Newbold testified that the improvement had increased their average daily output from 143.2 tons to 158.5 tons—an increase of —— tons per year." (1 year = 300 days.)

EXERCISE.

NOTE. Each example shows a different application of the *average*.

1. In a history test, the individual marks obtained were: 47, 90, 87, 82, 100, 100, 96, 90, 40, 35, 82, 87, or a class average of —. (2 dec. places.)

2. Twenty pupils in a class were present on the twenty-one school days in March, four were absent one-half day, three for one day, two for three days, and one for five and one-half days. Find the average daily attendance.

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3. In an Eastern city, having a normal maximum temperature for July of 76.5° a hot wave caused maxima, for successive days, of 90°, 93°, 95.5°, 89°, 96°, 98°, 101°, 92°, an average of —° above normal.

4. During November, 1907, individual rainfalls were .45 in., 2.15 in., .08 in., 1.37 in., and .78 in. Was the rainfall excessive as compared with (4.88 in.) the normal for past years?

5. The Rankin Mfg. Co. pays 60 men $2.15 per day; 80 men, $3 00; 12 men, $4.50; and 8 men, $5.75. How does its average daily wage compare with that of the Morgan Co., which has an average for the same class of work of $3.225?

6. Find the average daily and monthly circulation of the newspaper whose circulation statement is printed on page 12.

7. Find the average speed on these long-distance runs:

Course.	Distance.	Hours and Min.	Average.
Jersey City and Chicago.....	912 mi.	17:41	
Jersey City and Pittsburgh...	443	8:41	
New York and Buffalo.....	440	8:15	
New York and Chicago.....	979.52	18:00	
London and Edinburgh	393	7:45	
London and Glasgow.....	401.50	8:00	
Paris and Bayonne.....	486.25	8:59	

8. Compare the profits of the company mentioned below for the last year with the average for the past five years.

NET PROFITS OF THE CROPLEIGH WHEEL COMPANY.

1906	$17,246	1908	$18,076.50	1910	$20,132.61
1907	21,032	1909	16,548	1911	23,328.46
				1912	24,736.59

9. Explain the difference in meaning of these expressions:

(*a*) "126 pieces of castings, averaging 84 pounds each."

(*b*) "126 pieces of casting weighing 92 pounds each."

(*c*) "45 pieces of castings weighing 2746 pounds."

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CHAPTER VIII.

THE EQUATION.

INTRODUCTORY EXERCISE.

1. Use the word "equal" in comparing the cost of two articles.

2. Use the same word in expressing the cost of one article.

3. Use it in comparing the time necessary to do two pieces of work.

4. Use the word in expressing the time it takes to go from Philadelphia to New York, by rail.

5. Use it in comparing the size of two objects, one greater by one-half than the other.

An equation is formed by using the equality sign (=) between two equal numbers or expressions, called the members or sides of the equation.

ILLUSTRATIONS.

My business for May = that for April.
The cost of 5 bbl. at \$6 = the cost of 6 bbl. at \$5.
The quotient × the divisor = the dividend.

Certain self-evident truths (axioms) aid us in the practical use of the equation:

1. Things equal to the same thing are equal to each other.
2. The *equality* of two expressions is not destroyed:
   (*a*) By adding equals to each term.
   (*b*) By subtracting equals from each term.
   (*c*) By multiplying each term by the same factor.
   (*d*) By dividing each term by the same factor.

ILLUSTRATIONS.

1. If one barrel of apples cost \$3 and one barrel of sweet potatoes costs \$3, then—

The cost of 1 bbl. apples = the cost of 1 bbl. sweet potatoes.
If $a = 6$ and $b = 6$, then $a = b$.

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2. If the cost of 1 bbl. apples = cost of 1 bbl. sweet potatoes, then
   (*a*) Cost of 1 bbl. apples + \$4 = cost of 1 bbl. potatoes + \$4.
   (*b*) Cost of 1 bbl. apples − \$1 = cost of 1 bbl. potatoes − \$1.
   (*c*) Cost of 6 bbl. apples = cost of 6 bbl. potatoes.
   (*d*) $\frac{\text{Cost of 6 bbl. apples}}{2} = \frac{\text{cost of 6 bbl. potatoes}}{2}$.

3. If $a = b$, then—
   (*a*) $a + x = b + x$.
   (*b*) $a - y = b - y$.
   (*c*) $6a = 6b$.
   (*d*) $a/4 = b/4$.

Transposition. A term in an equation may be changed from one side to the other provided its algebraic sign is changed from plus to minus, or from minus to plus.

ILLUSTRATION.
If $x + 10 = y$
$x + 10 - 10 = y - 10$. (Subtract 10 from each side.)
$x = y - 10$. That is, the + 10 on the left has been changed to a − 10 on the right.
In a similar way, the equation "$a - b = c$" may be written "$a = c + b$."

If in an equation there occurs an expression whose value is not given, the value may be determined, in most cases, by transposition. This is termed *solving the equation.*

ILLUSTRATION.
Let $a + 6 = 9 + 4$, to find the value of a; then
$a = 9 + 4 - 6$ (transposing 6), and
$a = 7$.

In handling "signs," it is essential to have their order of rank, or precedence, understood. Of the four signs, +, −, × and ÷, the multiplication and division signs have precedence over those of addition and subtraction.

ILLUSTRATION.
Let $a = 52 - 6 \times 8$. The multiplication sign is considered first, and we have $a = 52 - 48 = 4$.

The signs × and ÷ have equal weight, and terms containing them are reduced to simpler form by performing each

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operation as its sign is met. The signs of addition and subtraction also have equal weight.

ILLUSTRATIONS.

$9 \times 8 \div 3 = 72 \div 3 = 24.$

$4 + 7 - 3 = 11 - 3 = 8.$

NOTE. The signs of addition and subtraction refer to all following values until the next plus or minus sign. Thus in "$3 + 8 \times 6 \div 2 + 8$," the "$+$" refers to the result of "$8 \times 6 \div 2$." The expression equals $3 + 48/2 + 8 = 35$.

EXERCISE.

1. Illustrate the five axioms, using the equation $a + 8 = b - 4$.
2. Find the "cost" from the equation "The cost of a house + \$275 = \$8450 − \$90."
3. Find the value of 1 A. from the equation: Cost of 21 A. = \$640.50.
4. Solve for x, if $x - 70 + 4 = 20 \times 6 \div 5$.
5. Dividend $\times 8 = 64$. What is the dividend?
6. Solve: $2 + 4 - 7 = a \times 6 \div 2 - 5$.
7. A number + 16 = 128. Find the number.

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CHAPTER IX.

UNITED STATES MONEY.

INTRODUCTORY EXERCISE.

1. What is your own definition of a coin?

2. What denominations of coin have you handled? What others can you name?

3. Read the wording on a one-dollar bill. What is the difference between it and a silver dollar? What makes the paper money of value?

4. What causes the difference in size between a silver dollar and a gold quarter eagle?

5. Why should not the Government coin iron, as money, in place of gold?

6. In what ways would a man be inconvenienced, who had much property but absolutely no money? Could he get along comfortably without money?

7. What is the disadvantage in trading one kind of property for another—instead of using actual money to pay for the desired article?

One function of a modern government is to provide a system of money or currency to serve as a "medium of exchange." A primitive people may use anything as a standard, as corn, wampum, horses. As commerce grows, and one who desires an article cannot meet directly a distant producer who has it, property must pass through several hands; the primitive property standards then become difficult of use, and also subject the users to loss and fraud. Then the government, as a representative of all the people, adopts as standard something easily transferable, and puts its mark upon it with such care and in such a way that to every holder a certain quantity may represent the same value. Thus gold is the standard of this country, and the coming standard of the world, because high values are imputed to small quantities of it, making it easy of transfer, and because, when coined, exact uniformity of

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weight and value may be given artificially to each piece. Small denominations are coined in metals of less value than gold, because otherwise the coin would be too small to handle and it would be easily lost. Such coin, in sufficient sums, may be exchanged for gold. Likewise, principally for convenience in transferring large sums, paper certificates are issued in exchange for gold and silver and are payable on demand in coin. The currency now in use in the United States consists of metal coins, and paper notes and certificates.

UNITED STATES CURRENCY.

METAL.

Gold.		*Silver.*		*Nickel.*	
Double eagle	$20.00	Dollar	$1.00	Nickel	$.05
Eagle	10.00	Half dollar	.50	*Bronze.*	
Half eagle	5.00	Quarter dollar	.25	Penny (one cent piece)	.01
Quarter eagle	2.50	Dime	.10		

PAPER.

1. Gold certificates. Denominations: $20, upward to $20,000. Issued for gold deposited in the United States Treasury, and payable in gold on demand.

2. Silver certificates. Denominations: $1.00, $2.00, $5.00, $10.00, $20.00, $50.00, $100.00. Issued for silver deposited in the Treasury, and payable, on demand, in the same metal.

3. United States notes (Greenbacks). Denominations: $10, $20, $50, $100, $500, $1000. They are a part of the debt of the United States and consist of promises to pay *in coin* the number of dollars specified on demand.

4. National bank notes. Denominations $5, and upward to $1000. They are issued by national banks, under government supervision, and are redeemable in *lawful money*.

NOTE. *Legal tender* is a term applied to money that may be legally offered in payment of debts. Gold and silver coin of standard weight, and gold and silver certificates, are legal tender for all debts. United States

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and national bank notes are legal tender, except for duties on imports and interest on the national debt.

The Government has five *mints*, or coin factories, where gold or silver bullion is received for coinage, and seven *assay offices*. The assay offices, as well as the mints, receive deposits of gold and silver, and also manufacture refined bars of standard purity. The bars, when returned to the depositors, are used in the arts, or are exported. The metal left in bars far exceeds in value the metal coined. The mints are situated at Philadelphia, San Francisco, New Orleans, Carson City and Denver. The assay offices are scattered throughout the country at the main points of metal production and importation. Some idea of the volume of business may be obtained from these figures for a recent year:

COINAGE OF THE UNITED STATES.

Denomination.	Pieces.	Value.
Gold		
Double eagles	1 495 035	$?
Eagles	1 494 795	
Half eagles	1 559 435	
Quarter eagles	142 509	
Total	?	?
Silver.		
Half dollars	1 621 700	?
Quarter dollars	4 422 700	
Dimes	20 998 431	
Total	?	?
Minor.		
Nickels	46 047 950	?
Pennies	89 588 480	
Total	?	?
Total Coinage	?	?

EXERCISE.

Find the missing values in this and the following tables.

Worn or abraded coins are collected through the banks and

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are recoined. The loss through abrasion is considerable, as the following table shows:

RECOINAGE OF UNCURRENT COIN.

(Six recent years.)

	Face Value.	New Coin.	Loss.
1	$3 832 280.69	$3 613 021.59	?
2	3 333 437.06	3 141 548.04	
3	3 008 747.98	2 829 890.71	
4	2 828 384.90	2 656 104.21	
5	1 964 476.11	1 839 219.24	
6	1 414 963.90	1 322 834.27	
Total, 6 yr.	?	?	?
Average	?	?	?

While the total amount of money in circulation seems very large, the actual amount of money in circulation per individual of our population seems remarkably small, and shows to what extent we carry on transactions by means of checks and other forms of credit, without actually handling currency.

MONEY IN CIRCULATION.

(Five recent years.)

	Population.	Money in Circulation.	Circulation per Capita.
1	84,662,000	$2,736,646,628	$32 32
2	86,074,000	2,772,956,455	?
3	87,496,000	3,038,015,488	?
4	88,926,000	3,106,240,657	?
5	90,363,000	3,102,355,605	?

As has already become evident, the "money value" is a common factor in a large part of the arithmetic calculations incident to our daily life, yet in the majority of cases the actual calculations are based on dollars and decimals thereof—and no knowledge of denominations is essential to solutions. The following classification of United States money is common,

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although the dime and eagle are usually omitted, being considered simply the names of coins.

TABLE OF UNITED STATES MONEY.

10 mills	= 1 cent.	Expressed c., ct., ¢, $.01.
10 cents	= 1 dime.	" d. or $.10.
10 dimes	= 1 dollar.	" $1.00.
10 dollars	= 1 eagle.	" E.

When, as a result of a calculation (as, for example, the determination of cost or wage) a payment of money must be made, knowledge of denominations of coins and certificates becomes of distinct value. A cashier, a bank teller, or a pay clerk, must have extended knowledge even of issues of coins and notes and of the minute details that distinguish the genuine from counterfeits. Moreover, such employees must be able to make change quickly and must be able to plan out and arrange for the quantities of coins of different denominations necessary during each day for the easy transaction of business.

In connection with the cash payments, many notes and checks are handled. These, and similar papers, usually have the money value stated twice, once in words, as briefly as possible, and once in figures. Portions of a dollar are usually expressed in figures, fractionally. See page 323.

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CHAPTER X.

MAKING CHANGE.

Speed and accuracy in change making and in making money payments are essential in an increasing number of positions.

Making change is a process of addition. The cashier adds mentally to the amount of the purchase each coin or note he takes from his tray, until he reaches the total of the sum given in payment. In most cases he does not know the exact amount of the change returned.

ILLUSTRATION. A $10.00 note is given in payment for a purchase of $2.37. The cashier takes from his tray three pennies, a dime, a half-dollar, a two-dollar note and a five-dollar note, saying aloud, if dealing with a customer, "Thirty-eight, thirty-nine, forty, fifty, three dollars, five, ten dollars."

QUESTION. Why is this better than a subtraction method?

The careful salesman, on receiving a note, states its value to the one who gave it to him, before sending it to the cashier, in order to avoid argument when the change is returned. In the above case, the salesman might remark, "You gave me $10.00."

QUESTION. State some simple misunderstanding that might arise from neglect of this precaution.

ORAL EXERCISE.

NOTE. In practising change-making, name the sub-totals until the amount is reached. When not reciting, watch the totals given to determine the currency denominations used. Suggest other combinations.

1. Name the common United States coins, certificates and notes.

2. Make change for the following:

Purchase.	Payment.	Purchase.	Payment.	Purchase.	Payment.
$5.47	$10.00	$15.43	$20.00	$.01	$.10
3 19	20.00	.08	.25	.18	.50
.67	2.00	1.32	5.00	3.21	3.50
.11	.50	.21	.50	.83	5.00
1.14	5.00	.43	2.00	2.17	4.00

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3. In each of the above cases, name the denomination of coin or notes given in payment.

4. Why should not a customer give a salesman $4.27 for a purchase of $3.09?

Make change for:

5. $10.00, given in payment for two chairs @ $1.17.

6. $5.00, in payment for May gas bill, $2.53.

7. $10.00, in payment for one pair of shoes, $3.75.

8. 50c, in payment for 17 1c stamps.

NOTE. Cashiers may have to break notes into smaller denominations, or to make specified payments.

9. Change a $1 00 bill into nickels and dimes.

10. "Break up" a $10.00 bill.

11. Pay a check for $124, in denominations of $5.00 and under.

12. Count out $15.42 for a pay envelope.

13. Pay your milk man, $3.41.

14. Pay John Brewer $700 in Treasury notes.

15. Pay John Smith $445 in gold coin.

FOR DISCUSSION.

1. Do all businesses require, for change-making, the same proportions of coins of each denomination?

2. Inquire of employees of ten distinctive businesses what coins are most needed for change-making. Report to class.

3. What businesses receive more change than they need?

4. Name businesses receiving large quantities of pennies, nickels and dimes.

5. Find out the proportions of coins required by a commercial bank in meeting the demands of its customers for change.

6. At what seasons of the year do banks make special preparations for supplying small change and bills?

7. Investigate and report on money machines, such as coin trays, cash registers, change-making machines, etc.

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CHAPTER XI.

POSTAGE.

INTRODUCTORY EXERCISE.

1. Is postage a tax?

2. Are stamps used on all mail matter?

3. Why do the stamp denominations differ in part from our coin denominations?

4. What government stamps are issued for other objects than the direct payment of postage?

5. Name some limitations as to acceptable mail matter?

6. How does the modern postal service help business?

7. What is "rural free delivery"?

8. Why is the regular postage rate in a country the same, irrespective of distance the matter is carried?

9. What is the "parcels post"?

The Post Office Department is one of the greatest of business organizations under government control. It is not run for profit, but its enormous expenses are met, largely, by postage and money order receipts. Some idea of its cost may be obtained from these figures for five recent years:

POST OFFICES AND POST ROUTES.

Year.	Post Offices Number.	Extent of Post Routes Miles.	Revenue of Department Dollars.	Expended for Transportation.		Total Expenditure of Department.	Deficit.
				Domestic Mail.	Foreign Mail.		
1	65,600	478,711	167,932,782	72,944,352	2,895,756	178,449,779	?
2	62,659	463,406	183,585,006	77,471,917	2,988,849	190,238,288	?
3	61,158	450,738	191,478,663	78,174,988	2,982,732	208,351,886	?
4	60,144	448,618	203,562,383	80,901,899	2,804,170	221,004,102	?
5	59,580	447,998	224,128,637	81,709,433	3,164,254	229,977,224	?
Total			?	?	?	?	?

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POSTAL INFORMATION.

1. Domestic Mail Matter.

Postage rates apply also to mail matter for Canada, Canal Zone, Cuba, Guam, Hawaii, Mexico, Philippines, Porto Rico, Republic of Panama, Tutuila, and the U. S. postal agency at Shanghai. The domestic rate, for letters only, applies to Germany, Great Britain and Ireland, and Newfoundland.

First Class. Letters and sealed matter: 2c for each ounce or fraction. Postal cards and post cards: 1c each. Limit of weight, 4 pounds.

Second Class. Newspapers and periodicals: 1c per 4 ounces or fraction. No weight limit. Publisher's rate, 1c per pound.

Third Class. Books and miscellaneous printed matter: 1c per 2 ounces or fraction. Limit of weight, 4 pounds, except for a single book.

Fourth Class. (Parcels-Post.) All matter not in other classes. Seeds and plants, 1c for 2 oz. For other matter the country is divided into unit areas and eight zones. The first zone includes territory within a radius of 50 miles of any unit area. The radius of the second zone is 150 miles, of the third zone, 300 miles, etc. Postage rates: Under 4 ounces, 1c per ounce; over 4 ounces, for delivery in first zone, 5c for first pound or fraction, and 3c for each additional pound.

For each additional mile of delivery there is an increase in the rate. Special stamps must be used. Limit of weight, 11 pounds. Consult your postmaster as to rates and size limitations.

Special Rates. For registering for safe delivery, 10c in addition to regular postage. Indemnity limit, $50. For "special delivery" or immediate delivery within the carrier-delivery limit of city free delivery and within one mile of any other United States post office, 10c in addition to postage.

Notes.

Matter that is harmful, libellous or threatening, which has to do with lotteries or fraudulent schemes, is unmailable.

At least 2c postage must be prepaid on first class matter. All other matter must be prepaid in full. Stamps are not affixed to publications mailed by the publishers, which are weighed in bulk. Stamps need not be affixed to third and fourth class matter consisting of at least 2000 identical packages. In these cases, a special "permit" is printed on the wrapper to show that postage has been paid.

Postage Stamps and Postal Cards.

The denominations of stamps are 1, 2, 3, 4, 5, 6, 8, 10, 15 and 50 cents; and 1 dollar. Stamped envelopes are issued of different sizes, for denominations of 1, 2, 4 and 5 cents. Postal cards are issued for 1c, single,

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and for 2c, reply. Private post cards must be within fixed limits as to size and must have a 1c stamp attached.

2. Foreign Mail Matter.

Rates and Regulations.

Letters and Sealed Matter. 5c for first ounce; 3c for each additional ounce or fraction. See Domestic Rates, for certain countries.

Postal Cards. 2c, single; 4c, double.

Commerical Paper. 1c for each 2 ounces or fraction; not less than 5c per package.

Printed Matter. 1c for each 2 ounces or fraction.

Samples of Merchandise. 1c for each 2 ounces or fraction, but at least 2c per packet.

Parcels-Post. 12c per pound. There are so many conditions as to such mail that it is advisable to consult your postmaster before sending package. Conditions are not uniform for all countries.

(For other information see United States Official Postal Guide, or a booklet on Postal Information, issued for public distribution.)

EXERCISE.

Classify this mail matter. Determine rate and cost of postage. Destination, the United States unless otherwise stated.

1. A 2 oz. letter to London.
2. 40½ oz. sealed circulars.
3. 18 oz. merchandise samples to Berlin.
4. Issue of 60,000 4-oz. magazines.
5. 8 souvenir postals to London.
6. 1½ lb. of gunpowder.
7. Mdse., 3 lb. 9 oz., to Zurich.
8. 8½ oz. commercial paper to Paris.
9. 9 oz. of seeds.
10. 12 oz. seeds, registered.
11. A 6 lb. package to Bremen.
12. 3 lb. 7 oz. of merchandise. (First zone.)
13. A Christmas card, 1¼ oz.
14. 4750 lb. of newspapers.
15. 9 oz. of proof sheets.
16. A book weighing 50 oz.

17. Estimate the total postage on: 21 letters; 150 circulars, single page; 4 pkg. merchandise, respectively 9 oz., 21 oz., 16 oz., 13 oz.; 1 pkg. seeds, 14 oz.; 2 registered letters; 5 special delivery letters; 25 postals. (Pkg.—first zone.)

18. Estimate the publisher's postage bill for his December issue of 147,456 copies, weighing 11 oz. each.

19. Compute postage on the following, and state the simplest stamp denominations to use: (*a*) On a 4-oz. letter, by special delivery. (*b*) On a 10-oz. registered package of merchandise. (Second zone.)

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(*c*) On a 5½ oz. sample of merchandise. (Fifth zone.)
(*d*) On a book weighing 47 oz.
(*e*) On a ½ oz. registered letter to London.

Most business organizations, sending out hundreds or thousands of pieces of mail matter per day, keep accurate postal records, such as the following:

POSTAGE REPORT.

For the month ending January 31, 190–.

Day of Mo.	Total Bought.	Balance.	On Hand.	Departments						Total for the Day.
				Purchase.	Advertising.	Sales.	Acct'ng.	Miscellaneous.		
1	145 20	25 —	?	9 16	7 25	111 40	6 15	3 10		?
2	115 90	?	?	4 90	5 40	92 85	3 27	1 09		?
3	90 —	?	?	8 52	12 82	47 32	4 52		96	?
4	200 —	?	?	6 10	9 75	105 86	8 21		72	?
5	120 —	?	?	16 82	13 40	116 45	11 06		52	?

NOTE. At the close of the day the mailing clerk enters under each department its cost for postage. The "Total Bought" column shows each day's purchases; "Balance" shows the quantity from the previous day. Together, they equal the amount on hand. The subtraction of the total for the day determines the new balance.

Two forms follow, which are designed to show even the denomination of stamps used, and a classification of the mail matter (page 60).

	DENOMINATIONS.											The Postage Account			
	1c	2c	3c	4c	5c	6c	8c	10c	15c	50c	1.00				
On hand	54	110	86	90	45	150	200	300	152	40	19	On hand			
Received	700	500	—	400	100			400		100		Rec'd			
Total	?	?	?	?	?	?	?	?	?	?	?	Total			
Used	528	602	44	128	46	51	40	161	8	45	7	Used			
Balance	?	?	?	?	?	?	?	?	?	?	?	Bal.			
Value of bal.	?	?	?	?	?	?	?	?	?	?	?				

EXERCISE.

Complete the extension of the above forms, and of the form on page 60.

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DAILY POSTAGE REPORT.

For November 20, 190— J. C. Parker, Mailing Clerk.

Number of Pieces Stamped—and Rates.

1c	2c	3c	4c	5c	6c	7c	8c	9c	10c	11c	12c	13c	14c	15c	16c	20c	25c	50c	Higher	Totals
454	596	32	124	46	31	8	40	12	21	16	96	8	4		45	10		5	2	

Cost of Postage.

4.54	11.92	.96	?	?	?	?	?	?	?	?	?	?	?	?	?	?	?	?	1.47	?

(See page 59.)

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CHAPTER XII.

PAYMENT FOR SERVICES.

INTRODUCTORY EXERCISE.

1. Name advantages and disadvantages of being a wage-earner.

2. What payment for services does a private householder make?

3. Name typical cases in which compensation is spoken of as a *wage* As a *salary*.

4. What name is given the payment of a lawyer? Of a physician? How do these payments differ from wages or salary?

The modern organization of business demands a careful accounting of all sources of expense. Of these, the cost of labor is a leading item and requires special attention. Some idea of the sums paid for services may be obtained from the following recent Census figures for a few selected industries:

Industry.	No. of Establishments.	Wage-Earners.		Average Wage.
		Av. Number.	Wages.	
Agricultural implements.....	648	47,394	$25,002,650	$?
Boots and shoes.....	1316	149,924	69,059,680	?
Bakery products.....	18227	81,284	43,179,822	?
Carriages, etc.....	4956	60,722	30,878,229	?
Clothing, men's.....	4504	137,190	57,225,506	?
Foundry products.....	9423	402,914	229,869,297	?
General iron and steel works.....	605	242,640	141,426,506	?
Silk, silk goods.....	624	79,601	26,767,943	?

Methods of payment vary, and the calculations involved are often so numerous and technical as to rank this class of computation as an independent subject. With millions of wage-earners, it is evident that the subject affects, directly or indirectly, our entire population.

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In business, the recompense paid, or stipulated to be paid, to persons, at regular intervals, for services rendered, is called *salary* or *wage*. In general, a stipend based on long intervals (i. e., year or month) is called a salary. Recompense reckoned by brief intervals (i. e., week, day, or hour) is usually called a wage. There are many exceptions to this classification. In some businesses, payment for distinctively mental labor is termed *salary* and manual labor *wage*. Thus a clerk in an office receives a salary, while a high-class mechanic, working at a machine, may receive a far higher wage.

In manufacturing establishments payment for services is frequently based on the quantity of work done. This work is called "piece work."

Salaries.

Salaries are usually paid monthly or semi-monthly. The rate is commonly expressed in dollars per year. A month is considered one-twelfth of a year, but in many cases, pay for a fraction of a month is based on the exact number of days in the month. (How would one day's earnings in February compare with one day's in January?) Payments are usually made by check or in cash. Sometimes employees are credited on the books and may draw funds as needed. Usually a pay roll is prepared and is signed by each person when he receives payment. A pay roll is shown on the opposite page.

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EXERCISE.

The Rankin Manufacturing Co.

We, the undersigned, hereby acknowledge that we have received from *John T. Warner*, Pay Clerk of The Rankin Manufacturing Co., the sum set opposite our respective names in full for services rendered during the month ending June 30, 19—.

No.	Name.	Position.	Time.	Annual Salary.	Salary Advanced.	Salary Due.	Ck No.	Signature.	Remarks
1	Robert Maynard	President	1 mo.	8000—		666 67	89	Robert Maynard	
2	John C. Carr	Vice-Pres.	1 mo.	6000—		500 —	90	John C. Carr	
3	Henry M. Downs	Manager	20 days	5000—		?	91	Henry M. Downs	Resigned 6/20
4	James Brown	Secretary	1 mo.	4000—	100 —	233 33	92	Jas. B. Brown	
5	C. T. Morgan	Treasurer	10 days	5000—				C. T. Morgan	
			20 days	4000—		361 11	93		Promoted 6/20
6	Henry A. Carll	Chief Clerk	1 mo.	3000—		250 —	94	Henry A. Carll	
					100 —	?			

Explain the use of each column. Which receipt for payment is incorrect? Give reasons for entries in the "salaries advanced" column. What effect do advanced payments have on the pay roll? How is the "salary due" of James Brown determined? Explain the computation for C. T. Morgan?

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EXERCISE.

Reckon salaries for June, in the following cases:

1. Annual salaries of $1200, $1600, $900, $2500, $450, $840.

2. Promoted from $1200 to $1300 on June 20.

3. Resigned from a $900 position to take effect on June 4.

4. Appointed at $960 to take effect June 4.

5. Rule up and extend the March pay roll for the general office force of a factory. Assume salaries and positions. Use names of pupils in the class. Have the pupils sign the pay roll. See that signatures are correct. Illustrate promotions, dismissals, appointments and "salaries advanced."

6. Prepare a similar pay roll for the teaching force of a school.

7. Special topic. (For individual pupils.) Secure blank forms of pay rolls that are in use in typical establishments in your neighborhood. Submit a report showing how the forms are adapted to special needs, and illustrating any unusual entries or calculations.

WAGES.

Wages are usually paid by the week and are quoted at a week, day or hour rate. Some workers are paid by the piece system, the amount of wage depending on the amount of work done. In other cases, a *bonus* of some form is added for extra efficiency. Many establishments pay one and one-half, or double rates, for overtime and necessary Sunday work.

In many small organizations, the pay rolls in use contain a receipt clause and a column for signature. In large organizations, the pay roll is simply a record sheet. Since relatively few workmen keep business bank accounts, employees are paid in cash, the pay envelopes being prepared beforehand; or in orders on the treasurer of the organization, payable on presentation. A more complicated pay roll is shown opposite.

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EXERCISE.

The Rankin Manufacturing Co.

We the undersigned hereby acknowledge that we have received from *James Brown*, Pay Clerk of the Rankin Manufacturing Co., the sum set opposite our respective names in full for services rendered during the week ending August 30, 19—.

No.	Name.	Ck.	Time in Hours. S. 24	M. 26	T. 27	W. 28	T. 29	F. 30	S 25	Total Hours.	Rate per Hour.	Wages Advanced.	Wages Due.	Signatures.	Remarks.
46	Gibson, W. C.	✓	8	8	8	8	8	8		48	60		28 80	W. C. Gibson	
47	Walker, I. M.	✓	8	8	6	8	8	8		46	50		?	I. M. Walker	
48	Parker, S. C.	✓						8		8	46		?	S. C. Parker	Engaged
49	Carter, James	✓	8	6	8	4				?	45		?	James Carter	Discharged
50	Wilson, Henry	✓	8	8	8	10	10	10	4	10	84			Henry Wilson	
										48	42		?	P. M. Denton	
51	Denton, P. M.	✓	8	8	8	8	8	8		48	38		?		
52	Walton, W. R.	✓	8	8	8	8	8	8		24	38			W. R. Walton	
										24	32	5 —	?	Chas. Douglas his X mark	Promoted 8/27
53	Douglas, Chas.	✓	8	8		8	8	8		?	30	2 50	?		
54	Sanford, Paul	✓	8	8	8	8				?	30		?	Paul Sanford	Resigned
56		✓										?	?		
57		✓													

Find the missing values in this pay roll.

How does this form differ from the salary pay roll?

The first sub-column under "time in hours" stands for Saturday. Why?

How many hours constitute an ordinary working day in your locality? How many in this pay roll? What number will appear most frequently in the "time in hours" column? Why is regularity of working hours insisted upon?

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From the "time in hours" and "remarks" columns explain how each person worked, and how his wage was determined? (*Note.* In the above case, Sunday time and overtime is paid at double rates.)

Do you notice anything unusual in the signatures?

ORAL EXERCISE.

Reckon wages for the week ending March 9, as follows:

1. 36 hr. at 15c; 42 hr. at 18c; 48 hr. at 20c; 47 hr. at 27c; 39 hr. at 19c; 42 hr. at 30c; 54 hr. at 33⅓c; 46 hr. at 25c.

2. Reckon wages in the following cases, basing calculation on a 54 hour week, with double pay for overtime: 42 hr. at 15c; 54 hr. at 30c; 54 hr. at 32c; 48 hr. at 17c; 60 hr. at 25c; 56 hr. at 30c; 45 hr. at 35c.

EXERCISE.

Rule up and extend a pay roll for the current week for some shop, factory or contracting firm. Add to the pay roll a column to show workman's position or department. Follow directions in example 5, page 64.

SCHEDULE OF CHANGE DESIRED.

By .....

Address .....

	$	
In 1-cent pieces	....	14
" 5-cent "	1	40
" 10-cent "	1	60
" 25-cent "	2	75
" 50-cent "	14	50
" Silver Dollars	....	—
In Notes or Certificates—		
$1	41	—
2	36	....
5	20	....
10	100	....
20	....	....
50	....	....
100	....	....
Total	217	39

When the pay envelope method is used the pay clerk computes, or if he is an expert he approximates, the quantity of each coin denomination required to enable him to "make-

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up" each employee's exact "amount due." Sometimes this coin classification is incorporated in the pay roll. Often a currency memorandum similar to that shown opposite is prepared, to be presented at the bank when the check for the total of the pay roll is cashed.

EXERCISE.

1. Prepare a denomination slip for this series of payments: $4.78, $9.62, $11.21, $11.45, $12.00, $12.65, $13.11, $14 29, $14.43, $15.80.

2. Prepare a currency memorandum for the pay roll shown on page 65.

3. Prepare a wage pay roll and currency slip for a contractor's force for the current week. The force consists of 1 foreman, 4 drivers, and 17 loaders. One man is discharged and two are hired during the week; one is out sick for one day.

The pay clerk who is obliged to handle a large pay roll based on fixed rates, often buys or computes wage tables as aids to rapid work. These give the wages due for any number of hours or half hours, at given rates per week. The original computations are made with absolute accuracy, and are then written in the table to the nearest cent.

EXERCISE.

THE RANKIN MANUFACTURING COMPANY.

$12.00 — 48 hours.

Hour.	Wage.	Hour.	Wage.	Hour.	Wage.	Hour.	Wage.	Hour.	Wage.	Hour.	Wage.
½	13	8½	2 13	16½	4 13	24½	6 13	32½	8 13	40½	10 13
1	25	9	2 25	17	4 25	25	6 25	33	8 25	41	10 25
1½	38	9½	2 38	17½	4 38	25½	6 38	33½	8 38	41½	10 38
2	50	10	2 50	18	4 50	26	6 50	34	8 50	42	10 50
2½	63	10½	2 63	18½	4 63	26½	6 63	34½	8 63	42½	10 63
3	75	11	2 75	19	4 75	27	6 75	35	8 75	43	10 75
3½	88	11½	2 88	19½	4 88	27½	6 88	35½	8 88	43½	10 88
4	1 00	12	3 00	20	5 00	28	7 00	36	9 00	44	11 00
4½				20½							

1. Complete the above table. (The upper half only is given.)

2. What error would result from constructing the table by computations based on the approximate value for the first half hour?

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3. Read off a man's wage, at the given rate, for 37 hours, 21 hr., 13 hr., 12½ hr., 19 hr., 36½ hr., 50 hr. (double), 52 hr.

4. How might the table be used for a wage rate of $6.00 per week? Of $18.00?

5. Construct a table for a wage of $15.00 and a week of 54 hr.

6. Construct a 48 hour wage table showing overtime at double rate up to 60 hours.

TEACHER'S NOTE. Assign each pupil a different rate. Have tables prepared and exchanged for testing.

In some factories and large stores time clocks are used to record the arrival and departure of employees. In some systems, the employee inserts his *key* in the clock, thus automatically recording his number and the time. In other systems, the employee inserts a card in the clock, the time being stamped automatically. Generally, the form is used to show whether or not the employee is on time—small fractions of hours being ignored. In other organizations, a few moments' tardiness may cause the docking of the employee to the extent of one-half, or one hour.

FACTORY EXPENSE.

EMPLOYEE NO. 276

WORK R 626-P

DATE 5/27

DAY		IN	OUT	IN	OUT	TOTAL
S	A.M.					
	P.M.					
S	A.M.					
	P.M.					
M	A.M.	6 58	11 59			
	P.M.	12 58	5 02			
T	A.M.	6 59	8 40	11 05	12 10	
	P.M.					
W	A.M.	Finished				
	P.M.					
T	A.M.					
	P.M.					
F	A.M.					
	P.M.					

TIME ?

RATE $18.40

COST ?

TICKET A.

EXERCISE.

1. Compute the cost of labor on the piece of work shown by ticket A. Reckon to the nearest one-fourth hour. The rate given is the rate per week.

2. Compute the time of eight men for Monday, and determine the wages due at 31½c per hour:

No.	In.	Out.	In.	Out.
1	6:59	12:01	1:00	4:59
2	7:01	12:02	1:01	5:03
3	6:58	11:59	12:58	5.01
4	6:59	10:01	2:00	5:02
5	——	——	12:59	5:03
6	6:58	12:00	12:58	3:02
7	6:56	12:01	12:57	4:59
8	6:50	12:02	12:55	5:04

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Premiums or Bonus Work. The premium plan of labor payment is intended to reduce the time of production in factory work, by the payment of a *bonus* for extra efficiency. Usually, as a result of hundreds of experiments, a fixed average time for a certain operation is determined. The employee is then told that his employer will share with him any saving in working time as compared with this average. For example, the offer might be "one-half of the wage saved." If the average time to make an article is 10 hours, and the wage $4.00, the employee earns 20 cents extra for every hour saved. The table below shows the effect of the plan.

Time Consumed in Hours.	Wage per Piece.	Premium.	Total Cost of Work.	Workman's Hr. Rate.	Workman's Day Rate.
10	$4.00	—	$4.00	$.40	$4.00
9	3.60	.20	3.80	.422	4.22
8	3.20	.40	3.60	.45	4.50
7	2 80	?	?	?	?
6	?	?	?	?	?

NOTE. Often a set number of pieces or operations is required per hour, and any increase in quantity is paid for at a special rate. Often, if the wage is stated as a price per piece, the wage rate is increased for a quantity over a certain fixed number.

EXERCISE.

1. Complete the above form.

2. Prepare a similar table showing one-half hour time reductions from 6 hours to 4 hours, the 6-hour standard rate being $3.60.

3. A workman, whose wage is 38c per hour, reduces his working time on a stove from 4½ to 3 hours, earning a premium of 16c per hour. He raises his wage for a 9-hour day to $—.

4. The standard number of pieces required of a polisher per hour is 12. The rate is 5c each, or 6c for extra efficiency. He averages 13½ per hour, for an 8-hour day, earning $—.

BILLS FOR PROFESSIONAL SERVICE.

Professional men, as doctors, and lawyers, and many independent artisans, charge *fees* for their work, and render *bills for services.* As a rule the computations are very simple.

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LAWYER'S BILL.

Cleveland, Ohio, Dec. 21, 19—.

B. C. Student & Co.,
No. 848 Payne Ave.,
To Charles Rawlingson, Dr.,
Attorney at Law.

For professional services rendered in suit against Salton & Bro.	185	00
Court Costs,	25	00
Stenographer, 26 hr. at 30c	?	

NOTE. Bills for the service of an agent are considered in the chapter on Agency.

EXERCISE.

1. Draw up a lawyer's bill for services. Fee, $275; court costs, $30; stenographer, 17 hr. at 32c; registration fee, $3.50.

2. Draw up a cabinet maker's bill for remodelling an antique sideboard. Material, $1.75; delivery, 50c; labor $—.

3. Draw up an original bill for services.

4. From the doctor's memorandum card illustrated on the following page, showing house treatments (numerals), and office treatments (O), draw up a bill. The rates are $2 and $1 respectively, for house treatments and office visits. Supply names.

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RECORD OF VISITS.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	TOTAL
JAN.																																
FEB.																																
MAR.																																
APR.																																
MAY						O				O										O											
JUNE		1				O								O																	
JULY																						O									
AUG.			1		1	1	2	2	1	2	1	1		1				1					O					O			
SEPT.										O											O										
OCT.				O									1						1	2	1				1				1		
NOV.										1													O								
DEC.																			O										O		

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CHAPTER XIII.

BUSINESS TERMS AND ACCOUNTS.

All business is an exchange of one kind of value for another kind. The values exchanged may be general property, funds, or services. In this connection, the business man employs two terms: (1) *Debit*, as an expression of value received or due; and (2) *credit*, an expression of value delivered or owed.

ILLUSTRATIONS. (1) A dealer gives $200 (a credit) for a horse (debit). (2) The dealer pays his clerk $20 salary. The debit, or value received, is *services;* the credit is $20 cash. (3) He buys 40 bbl. of flour from Norton, giving his note for 30 days. The debit is *merchandise;* the credit a *note payable.* Notice that in every transaction the debits equal the credits.

Accounts are collections of debits or credits relating to a particular person or thing. Thus the *Cash Account* is a record of all cash received or paid out. Other accounts with which the average person meets are: Merchandise, Expense and Personal.

If, in the course of business, a transaction, or a series of transactions, produce more than they cost, a *gain* results; if the sum spent exceeds the returns, a *loss* results. The excess of gains over losses, for an entire business, or for a certain series of transactions, is termed the *net gain;* the excess of losses over gains is a *net loss.* Any property owned by, or owed to, a person is termed his *resource;* any amount he owes is termed his *liability.* If resources exceed liabilities, a person has a *net capital,* or *present worth,* equal to the difference. If liabilities are greater, the excess represents his *net insolvency.* Properly kept accounts show the resources, liabilities, losses and gains of a business.

The *Cash Account* shows the debits and credits of cash, the

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difference, or *balance*, representing the amount of cash on hand, as follows:

Cash.

19—		RECEIVED			19—		PAID		
Oct.	17	Invested	2000	—	Oct.	18	Office furniture	182	60
	21	J. Smithson	140	—		20	Merchandise	520	15
	22	Merchandise	485	60		24	Wages	30	—
						31	Balance	???	
			2625	60				2625	60

NOTE. The *balance* is entered on the smaller side, so that the sides "total" the same. The source of receipt or payment is sometimes entered in the explanatory space.

The *Merchandise Account* shows the cost of merchandise bought (debits) and the receipts from sales (credits). In general, explanations are not written. Here, however, they show, among other things, the people who owe, or are owed, for merchandise. The balance shows a loss, if the debit side exceeds the credit, or a gain, if the credit side is in excess. See below.

Merchandise.

19—		MDSE. REC'D.			19—		MDSE. DELIVERED.		
Jan.	2	Goods on hand	845	20	Jan.	5	Cash	362	50
	5	C. B. Kane	278	40		8	Smith's Note	40	—
	16	W. B. Powers	840	60		15	A. B. Brown	120	—
	18	Cash	500	—		29	Cash	14	60
	31	Gain	???	—		31	Inventory	2146	20
			2683	30				2683	30

NOTE. If unsold goods are on hand their cost value (inventory) is entered on the credit side. This is equivalent to subtracting the inventory from the debit side, in order to find the cost of the goods sold, and thus to determine the gain.

The *Expense Account*, as illustrated on the following page, shows the cost of running a business:

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Expense.

19—					19—				
Feb.	2	Postage	15	20	Feb.	28	Inventory	2	40
	6	Coal	30	—			(postage)		
	15	Rent	60	—		28	Inventory	8	50
	21	Salaries	38	50			(coal)		
	26	Freight	12	47		28	Loss	???	
			156	17				156	17

A *Personal* account shows the amounts owed to, or by, the person whose name appears in the title.

Henry Brown, 1456 K St., N. W.

19—					19—				
Apr.	5	Mdse.	240	—	Apr.	10	Discount	5	—
	8	"	165	—		10	Cash	400	—
			405	—				405	—
Apr.	20	Mdse.	176	50	Apr.	25	Cash	48	—
							Balance	???	
			176	50				??	

NOTE. Whenever the two sides of a personal account are equal, it is ruled up to show no balance or debt. If debits exceed credits the person named owes the firm the balance; if the credits are in excess the firm owes the person named.

The *Proprietor's or Partner's Account* shows his relation to the business. The credits usually represent investments and gains; the debits show the losses and withdrawals. A credit excess represents the present worth, or net capital; a debit excess, the net insolvency. A proprietor's account is shown on the next page.

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S. P. Mason, Proprietor.

19—					19—				
Jan.	11	Cash withdrawn	85	60	Jan.	1	Cash Invest.	4000	—
						1	Real Estate	8000	—
		Present Worth	?				Invest.		
						31	Gains	865	31
			12865	31				12865	31

ORAL EXERCISE.

1. What are debits? Credits? What is an account?

2. When is an account debited? When credited?

3. What sum of cash was received, according to the cash account? What sum paid out? What sum was on hand on October 21? For what was cash paid out during October?

4. From the Merchandise account it is evident that, $? worth of merchandise was bought during January, and that, $? was sold. $? was sold for cash; $? for a note; and $? to be paid for later by ?.

5. Henry Brown's account shows that he bought merchandise on what dates? For which purchases has he paid in full? For which has he evidently paid in part?

6. Name the debit and the credit in each of the following transactions:

(*a*) The firm buys merchandise for cash, $200.

(*b*) The firm sells A. B. Bolton, merchandise, $400, for which he agrees to pay in ten days.

(*c*) The firm pays cash for office furniture, $120.

(*d*) The firm receives cash for some real estate. (Real Est. Ac.)

(*e*) The firm pays cash in settlement of a gas bill.

7. Classify the following as resources, liabilities, losses and gains:

(*a*) The balance of the Cash Account.

(*b*) A personal account with debits of $500 and credits of $400.

(*c*) Merchandise account with debits, $600, credits, $700. No goods on hand.

(*d*) Expense account with debits of $46.50 and credits of $13.00.

(*e*) Merchandise on hand, $125.

8. What is the present worth, if resources are $12,800 and liabilities $2500?

9. What is the net loss if gains are $560 and losses $764?

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EXERCISE.

1. Copy the Cash Account on page 73 through Oct. 24. Add these entries: Oct. 25, received for merchandise, $250; Oct. 26, paid for horse and wagon, $185; Oct. 27, lost $2; Oct. 29, received from C. B. Drew, $89.60. Balance the account.

2. Alter the Merchandise Account as follows: Goods unsold on Jan. 31, $2003.15; amount paid by A. B. Brown, Jan. 15, $36.40. Balance the account.

3. Find the balance again, assuming that all goods had been sold. What does the account show?

4. Write an account for D. Poynter, Proprietor, showing investments on Jan. 1, of cash, $2500; merchandise, $3867; and real estate, $4500. Withdrawals, Jan. 10, $265 cash; Jan. 18, $1645 in merchandise. Net gain, Jan. 31, $211.46.

5.

NAME	ADDRESS	FILE NO.
John C. Brewer,	*Memphis Tenn.*	*161*

19—					91—				
Jan.	16	Order No. 3620	360	—	Jan.	17	30 d. note	360	—
	21	4125	214	85		31	By cash	214	85
	30	4356	561	40	Feb.	9	" "	245	—
Feb.	3	4581	311	27		20	60 d. note	900	—
	19	4936	1216	03		20	By cash	316	03
	21	5011	1008	60		25	Overcharge	17	25
	26	5203	65	40		28	Balance due	?	
			?					?	

(*a*) Brewer still owes $?

(*b*) Between what dates did Brewer owe nothing?

(*c*) Between what dates did he owe the least amount?

(*d*) How do his cash payments compare with his payments by note?

(*e*) Explain the credit item for Feb. 25.

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CHAPTER XIV.

ADVERTISING.

Modern advertising, or solicitation through public notice, really dates from the insertion of such notices into the magazines in 1864. In that year, the most prominent monthly contained, in its October number, three and one quarter pages. During this period advertising rates rose enormously, as did the number and circulation of periodicals. Advertising extended in heavy volume to newspapers, and then to bill boards, complicated catalogues, street car signs, show window displays and demonstrations. In a recent year it was estimated that $600,000,000 was spent in these main sources of advertising.

Practically everyone reads advertisements; and many people, regularly or on some special occasions, advertise. The cost of advertising is one of the heaviest expenses of many a modern business. The methods of computing costs and returns, however, are based on the simplest arithmetical processes.

INTRODUCTORY EXERCISE.

1. What is advertising?
2. Name at least five ways of advertising merchandise in print.
3. Name other ways of advertising.
4. What is a trade mark? Is it a form of advertising?
5. What effects have climate, season and prosperity on advertising matter?
6. Is the paper of largest circulation necessarily the best advertising medium?
7. What is the value of a "catch phrase" or title?
8. Why do not local dealers advertise in magazines?

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9. Does the character of newspaper advertising differ from that of magazines?

10. Can you name any article brought into general use by advertising?

11. Some advertisements give no dealer's name. Why?

12. What need has the salaried man or private householder to advertise?

13. *Special.* Bring to class specimen advertisements, securing cost of insertion, where possible.

Wall Signs, for advertising purposes, are painted on blank walls of buildings, fences, bulletins, etc. Contracts are usually for the year, including painting, and exclusive of the varying commissions or fees for rental. For bulletins, the price is quoted per running foot.

EXERCISE.

1. A manufacturer of breakfast foods rents for $200, a wall space of 40 ft. by 24 ft., and pays 6c. per sq. ft. for the sign. Find the entire cost.

2. Lawton & Co. rent spaces as follows: 26′ by 20′ for $250; 20′ by 18′ for $4 per sq. ft.; 32′ by 21′ $2.50 per sq. ft. The charge for placing signs is $4.25 per sq. ft. What is the entire cost?

3. I contract for six months, for a 20′ sign on a standard bulletin board (10′ high) at 45 c per running foot per month. What is the cost of advertising?

4. What is the cost of a one-year contract for 60 feet of bulletin space at 55 c per foot per month?

Posters for advertising purposes are shown on bill boards. These boards are designed to accommodate posters four sheets high and one to six sheets wide. Complete posters are spoken of as 4s, 8s, 16s, etc., according to the number of sheets. The sheets measure 28 in. high by 42 in. wide. A bill-poster's space is rented by the month of four weeks, or the year of forty-eight weeks. Prices are usually quoted for four weeks' display.

EXERCISE.

1. Find the dimensions in feet and inches of 4, 8, 12 and 16 sheet posters.

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2. Allowing for 1/10 renewals of damaged sheets, how many sheets are required for 450 24-sheet posters?

3. To post 100 16-sheet posters for ten weeks cost a cereal company 16 c per sheet per month, or $——?

4. Extend the following:

Estimate of the Cost of Two Thousand 16-sheet Posters.

Number required	2000	
For renewal, 1/5	?	
For 16 s. at $269 per M	——	$——
Freight and drayage		26.45
To be posted 12 weeks		
560 in small towns at 14 c	$——	
1200 in specified cities at 16 c	——	
240 in San Francisco at 12 c	——	——
		$——

Street Car Posters are printed on cards, usually 11 in. by 21 in., though some of other size are used. The rates average about 40 c per card per month, on yearly contracts, 45 c on contracts for six months, and 50 c on three months contracts.

EXERCISE.

1. The manufacturer of a toilet soap contracts for 3500 cards at $47.50 per M., paying $18.50 for the design, and $16.00 for the necessary plates, a gross cost of $——.

2. He distributes 3/5 of his supply (on six months contracts) throughout Middle West cities at prices given above. This cost him $——?

3. The balance he distributes in Pennsylvania and New York on a one year contract, at the above rate, except for 80 for which the price is 35 c. Find the cost of the year contract?

4. Prepare a statement showing the entire cost to the manufacturer.

Newspaper and periodical advertising absorbs the larger portion of the money spent on publicity. Periodical advertising is usually the more costly. The common measure of space is the agate line, of which fourteen equal one inch of column space. One periodical charges $6.00 per insertion per agate line, or $4000 for a full page of $9\frac{3}{8}''$ by $14\frac{1}{4}''$.

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For a standard magazine page, $5\frac{1}{2}''$ by $8''$, rates may run up to $500 per insertion.

Extensive general advertisers usually engage agencies to prepare copy, select publications, and arrange for insertion. The agency is paid either a lump sum or a commission. In some cases the commission is paid by the publisher.

EXERCISE.

1. From the following table find to two decimal places the average number of copies issued for each member of our population.

Newspaper and Periodical Circulation.
(Round numbers).

Year	Population	Gross Circulation	Copies per capita.
1850	23,000,000	426,000,000	?
1860	31,000,000	928,000,000	?
1870	39,000,000	1,509,000,000	?
1880	50,000,000	2,068,000,000	?
1890	63,000,000	4,681,000,000	?
1900	76,000,000	8,168,000,000	?
1910	92,000,000	13,105,000,000	?

Find necessary values for the following:

2. The recent holiday numbers of ten standard magazines contained respectively, 192, 181, 175, 171, 150, 150, 148, 193, 138, and 127 pages of advertising, a total of —— pages, and an average of —— pages.

3. In the same issue, the agate lines of advertising were 43,776, 41,496, 39,900, 39,047, 34,980, 34,392, 34,088, 33,306, 32,172 and 27,132 lines. Total lines ——. Average lines ——.

4. Five great railway systems recently spent for advertising in one year, respectively; $348,457; $150,647; $84,335; $147,564; $251,532—an average of $——.

5. A half-page advertisement in a monthly, $448 per page less one-fifth for time, costs a dealer $—— for twelve insertions.

6. The magazine referred to above contained 168 pages of advertising in a given number. Estimate the minimum income. Why might the income be larger?

7. A novelty desk pad is advertised by its maker in a 20-line space for 3 months, in five monthlies, having line rates as follows: $3.50, $2.75, $2.50, $2.00 and $1.20. Compute the cost.

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NOTE. Agencies receive net or gross rates from publishers. They charge advertisers the net rate, plus a commission; or, if under the gross rate agreement, charge the advertiser this rate, receiving a commission or rebate from the publisher.

8. Complete this statement.

NEW YORK, N. Y. Nov. 30, 19 .

Mr. BROWN C. NORTON

To J. F. Jamison, Dr.

Advertising Agent.

			Net	Gross
The Johnstone Weekly	¼ p.	12 times		5400
The World Magazine	1 col.	12 times		2700
The Hour	¼ p.	12 times	3880	
Every Moment	1½ col.	12 times	7650	
The Monthly	1 p.	12 times	3648	
Literary Magazine	1 p.	12 times		960
Livelihood	1 col.	12 times	1260	
			?	?
Commission 1/10			?	?
				?

In newspaper advertising, which is largely local, many dealers make contracts for a fixed amount of space per annum, to be used as required. This space is usually expressed in *line measure*. For "want" advertising and minor notices, rates vary with the advertising matter. The rate cards of newspapers are often very complicated. Opposite is a small section of one.

Extract from a Newspaper Rate Card.

General Display — Agate Line.

Run of paper 50c.

Special portion, last page, opp. editorials, etc. 55c.

SPACE DISCOUNTS. ONE YEAR LIMIT.

5000 lines	1/20 off.	20000 lines	3/20 off.
10000 "	1/10 "	40000 "	1/5 "

CLASSIFICATION RATES.

Agate Line.

Amusements	50c line.	Public notices	20c line.
Art sales, etc	20c "	Railway notices	15c "
Auctions	20c "	Railway time-tables, daily by the month	20c line.
Birth, marriage, death notices	$1.00 ea.	Railway time-tables, daily by the year	15c line.
Financial notices	45c line.		

Reading notices, financial page, $2.00.

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WANT ADVERTISEMENTS.

Per Line.....1 time	3	7		Per line.....1 time	3	7	
For sale.....18c	25c	40c		Lost and Found...8c	20c	35c	
Help wanted..... 8	20	35		Positions wanted..5	10	15	

NOTE. Eight words to the line: five words if set in caps.

ORAL EXERCISE.

Use above rates where none are stated in problem. Find the cost of:

1. 60-line advertisement, run of paper.
2. 28-line advertisement, general display.
3. 5-inch advertisement, special position on last page.
4. 100-line advertisement, special position surrounded by reading matter—the charge being 5c. extra per line.
5. 5000 lines, contracted for 55c.
6. 20,000 lines, contracted for 35c.
7. 10,000 lines, quoted rates.
8. 4-inch public notice.
9. Three insertions of a four-line Lost and Found notice.
10. One month's insertion of a 56-line railway time table.
11. A six-inch reading notice on the financial page.
12. Seven insertions of a six-line "position wanted" advertisement.
13. A ten-inch advertisement of a circus.

EXERCISE.

1. A 12-in. advertisement in a Chicago newspaper @ $4.92 per in. costs $? per insertion, or for 26 insertions @ $3.92 per inch, per insertion, $?

2. A Seattle dealer contracted for 12,000 lines @ 49c. During January he inserted advertisements as follows: 120 lines, 65 lines, 140 lines, 210 lines, 560 lines (twice running), 28 lines. What is his bill for the month?

3. A 70-line double column advertisement in a Kansas City paper, @ $1.40 per inch, costs how much for two insertions?

4. Compute the cost of this advertisement at quoted rates.

WANTED—EXPERIENCED SALESWOMEN IN
books, stationery, handkerchiefs, notions, neckwear, toys, dolls, umbrellas, &c. Also bright young women from 8th grade and high school.
Box 462, Boston, Mass.

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5. An agency agrees to insert the above "want ad" in 103 city papers for $35.00 for 21 words, and $11.65 for each additional 7 words. This will cost $?

Advertisers watch the returns from advertising and discard publications that fail to bring customers. Returns are traced in many ways. For example, an advertiser may alter his street number, writing it "234 State Street" in one periodical, and "236 State Street" in another; or he may offer a different bargain in each magazine; or he may insert in the advertisement a reply-slip, properly keyed with a number or letter for identification. The replies are then classified, and the papers yielding the *lowest cost per inquiry*, the greatest number of orders, or amount of sales, are rated as the best advertising media.

EXERCISE.

1. The form on p. 84 illustrates a cost per inquiry card. It is arranged as a calendar showing the number of inquiries received on each date. Why are no replies received on Jan. 8, 15, etc.? Each horizontal line total represents what? The vertical total? Extend the form and find all values.

2. To trace advertisements month by month, and to see which of a series of advertisements *draws* best, the key in a magazine may be changed in each issue. For example, the magazines may be numbered from 10 to 99. To them is affixed a number from 1 to 12 to show the issue. Magazine No. 10, for a February issue would be represented by 102, because February is the second month. The form on p. 85 is made out for June, and shows on the left the classification of calendar replies. The blocks on the right (1 to 12) are a classification of replies by the advertisements that called them forth. Find the totals and explain the form in full.

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Paper	Advertisement	Key No.
THE MONTHLY	No. 65 Library Fittings	M. C.-2
Price Per Issue	**Price Per Month**	**Lines** **Inches**
$168.—	$168.—	56 = ?

Display Cost per Inquiry (Divide cost by No. replies) per mo.	JAN.	FEB.	MAR.	APR.	MAY	JUNE	JULY	AUG.	SEPT.	OCT.	NOV.	DEC.
	?	?	?	?	?	?	For 6 mo. = $.					

No of Inquiries

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	TOTAL
JAN.	—	42	61	73	98	112	101	—	96	73	54	40	30	26	—	27	23	16	14	10	—	—	8	3	2	—	—	3	—	—	1	?
FEB.	13	56	93	99	—	123	111	92	90	86	73	—	92	84	72	75	63	50	—	61	43	—	35	21	16	—	5	0	—	—	—	?
MAR.	21	40	62	73	—	98	105	116	80	72	73	—	79	72	65	60	63	41	—	52	39	31	28	20	11	—	12	6	3	8	2	?
APR.	14	—	55	82	116	120	136	91	—	112	88	93	90	67	53	—	70	62	61	40	38	12	—	32	16	5	10	—	3	—		?
MAY	78	59	71	91	98	97	—	110	47	70	38	86	54	—	43	60	81	28	29	21	—	30	22	17	13	12	8	—	3	—	6	?
JUNE	51	63	72	—	114	108	111	93	87	98	—	90	63	92	47	80	39	—	63	45	32	41	26	14	—	12	1	8	—	5		?
TOTAL																																?

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LAWRENCE AUTOMOBILE HORN MANUFACTURING CO.

ADVERTISING RECORDS.

Advertisement A23. Inquiries for June, 19—.

Key	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
566	12	10	9		8	9	6	5	8	4		13	6	7	8	2	9		5	8	4	10	6	7		8	9	0	2	1	
506	5	8	9		2	9	7	8	4	3		8	2	9	7	9	8		4	5	4	6	3	4		8	2	9	4	5	
426	4	7	8		2	9	6	5	4	3		8	2	4	7	3	6		7	4	13	9	8	5		4	3	8	2	4	
596	5	14	7		9	9	10	9	8	10		12	9	10	8	9	10		12	13	10	9	8	7		8	5	8	9	8	
416	14	13	12		10	12	10	9	8	7		6	5	8	9	6	5		8	4	8	3	9	8		7	6	5	4	0	

CLASSIFICATION BY ISSUES OF YEAR.

	1	2	3	4	5	6	7	8	9	10	11	12	TOTA
	3	16	24	40	10	83							?
	0	1	0	16	58	77							
	0	0	0	5	?	89							
	0	0	5	8	24	?							
	0	1	0	4	5	186							
												Total	?

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CHAPTER XV.

FACTORS AND MULTIPLES.

Thus far, attention has been given to numbers, their expression, the units of which they are composed and the fundamental operations involving them. Certain properties common to numbers should now be noted.

1. *Any number may be considered as the product of two or more factors.* The process of determining these factors is termed factoring.

ORAL EXERCISE.

1. Define a factor.
2. Name two factors of 35, 64, 120, 42, 18, 168.
3. $2 \times 3 \times ? = 48$.
4. Name three factors of 36, 160, 54, 846.
5. 2 is a factor of which of these numbers: 68, 45, 32, 19, 127, 388?
6. Name some factor that is *common* to 48 and 36; 42 and 280.
7. Name two factors of 17, 29, 41, 53, 13. What is common about the factors of these numbers?

Classified according to factors, integral numbers are *even* if two is a factor, and *odd* if two is not a factor.

Integral numbers are *prime numbers* if the number itself and *one* are the only factors. A series of numbers are *mutually prime* if one is the only common factor. All numbers not prime are *composite* and contain two or more factors.

ORAL EXERCISE.

1. Name and classify the numbers from 1 to 150, as even, odd, prime or composite.
2. Name four groups of numbers that are mutually prime.
3. May the factors of a number be composite?

NOTE. There are certain tests of divisibility that are useful in factoring and general computation. Thus a number is divisible by:

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2, if it is even.
3, if the sum of its digits is divisible by 3.
4, if the number represented by its two right hand figures is divisible by 4.
5, if the right hand figure is 5 or 0.
6, if even, and the sum of its digits is divisible by 3.
7, (No simple test).
8, if the number expressed by the three figures to the right is divisible by 8.
9, if the sum of the digits is divisible by 9.
10, if the last figure is 0.

EXERCISE.

1. Suggest tests of divisibility by 100, 1000, 10000, 20, 90, 18, 12.
2. If a number is divisible by 8, it is necessarily divisible by ?, ?, or ?
3. If a number is divisible by 12, it is necessarily divisible by ?, ?, ?, or ?
4. 9,684,324 is divisible by what factors under 13?

Using divisibility tests, determine at least two factors of:

5. 144. 7. 1728. 9. 2240. 11. 5280. 13. 4589.
6. 320. 8. 19,200. 10. 63,360. 12. 12,345. 14. 12,726.

EXERCISE.

Reduce to prime factors:

1. 7668.

Being even, 2 is a factor. The resulting factor, 3834, is even, and is therefore again divisible by 2; the sum of the digits of 1917 (18) is divisible by 3, a prime factor. The remaining factors are found by inspection.

Check $2 \times 2 \times 3 \times 3 \times 3 \times 71 = 7668$.

Solution.

2	7668
2	3834
3	1917
3	639
3	213
	71

2. 246. 4. 4225. 6. 1728. 8. 2496.
3. 389. 5. 4128. 7. 63,360. 9. 7884.

Find the prime factors common to:

10. 378 and 864.

Since 7 and 16 have no common factor, division need not be carried further. It is evident that the factors common to 378 and 864, are 2, 3, 3 and 3.

Solution.

2	378	864
3	189	432
3	63	144
3	21	48
	7	16

11. 984 and 676. 12. 1425 and 5000. 13. 672 and 396.

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The product of the common factors (in Ex. 10, $2 \times 3 \times 3 \times 3$) of two or more numbers is termed their *highest common factor* (h.c.f.). The highest common factor was once of extreme importance in complicated fractional work, but it is now little used, owing to the substitution of decimals for all but simple fractions.

EXERCISE.

Find the highest common factor of:

1. 678 and 420. **2.** 5280 and 320. **3.** 546 and 384.

EXERCISE.

Factoring is often employed to simplify and shorten the processes of multiplication and division. This is done by rejecting common factors from numbers that stand in the relation of dividend and divisor.

1. Why is the quotient of 5×8 divided by 5×4 the same as $8 \div 4$?

2. Divide $4 \times 7 \times 8$ by $3 \times 7 \times 9$. Reject the common factor and divide.

What is the effect of dropping the common factor?

3. Divide $12 \times 42 \times 18 \times 6$ by $6 \times 8 \times 9 \times 11$.

Solution.

$$\frac{\overset{3}{\cancel{12}} \times 42 \times \overset{2}{\cancel{18}} \times \cancel{6}}{\cancel{6} \times \underset{2}{\cancel{8}} \times \cancel{9} \times 11} = \frac{3 \times 42}{11} = \frac{126}{11} = 11\tfrac{5}{11}$$

Indicate the products and division as shown. Cancel the common factor, 6. Cancel the factor 9 of the divisor with the 9 in 18 of the dividend, leaving 2. Cancel the 2 from dividend and from 8 in the divisor, leaving 4. Cancel 4 from the divisor and from 12 in the dividend, leaving 3. Multiply the remaining factors.

4. $14 \times 8 \times 64$ divided by $11 \times 16 \times 4 \times 7 = ?$

5. $3 \times 21 \times 84 \times 9 \div 69 \times 8 \times 6 \times 14 = ?$

6. $\dfrac{52 \times 16 \times 5 \times 11}{13 \times 28 \times 12 \times 15} = ?$

7. The output of a cracker factory is 840 doz. packages per hour, for a nine-hour day. This is equivalent to how many cases of 8 cartons, containing six packages each?

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8. 420 cases of 4 doz. cans each, are equivalent to how many packages of each containing 6 cans?

9. At the rate of $9.60 per doz., what is the cost of five units?

10. In a given year the coal consumption for heating an office building was 1825 tons. The price paid was $3.90 per ton. What was the average daily cost for fuel?

The *multiple* of a number is any integral number of times the number. A *common multiple* of two or more numbers is one that is an integral number of times each of the numbers. The least number that is such a multiple, is termed the least common multiple (l.c.m.).

EXERCISE.

1. Name multiples of 8, 5, 21, 4.

2. 12 is a common multiple of what smaller numbers?

3. Name a common multiple of 15 and 25.

4. Name two common multiples of 4 and 6. What is the least common multiple?

5. Is the product of two numbers a multiple of them?

6. Is it ever the least common multiple?

7. Find the l c. m. of 36, 84 and 96.

To be a common mutiple the number must contain the prime-factors of each of the given numbers.

ILLUSTRATION.

Resolve 36, 84 and 96 into their prime factors. The multiple must have the factors "2" five times because 96 contains that number; the factor "7" once, because 84 contains it; and two "3's" because both 36 and 96 have them.

The least common multiple is therefore $2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 7$, or 2016.

Solution.

$36 = 2 \times 2 \times 3 \times 3$
$84 = 2 \times 2 \times 3 \times 7$
$96 = 2 \times 2 \times 2 \times 2 \times 2 \times 3$

If the factors are not determined by inspection, arrange the numbers in a horizontal line and divide by factors common to any two numbers.

Bring down undivided quotients that are not exactly divisible. Continue until

Solution.

2	36	84	96
2	18	42	48
3	9	21	24
	3	7	8

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all quotients are mutually prime. The l. c. m. is equal to the product of the divisors and the final quotients.

$$\text{l. c. m.} = 2 \times 2 \times 8\ (2 \times 2 \times 2) \times 3 \times 7 = 2016.$$

Find the l. c. m. of:

8. 4, 9, 6, 2. **10.** 4, 20, 30, 18. **12.** 5, 35, 60, 75.

9. 9, 12, 15. **11.** 4, 2, 8, 32. **13.** 3, 6, 5, 92, 18.

14. A steamship company has two steamers leaving New York, the first making a round trip in 10 days and the second in 16 days. How often do they reach New York at the same time?

15. A manufacturing company has on the road, four drummers, each following circuits that take respectively 7, 6, 4 and 12 days. How often can the manager confer with the four men together?

16. Two wheels of 20′ and 16 ft. circumference are geared together. How often do the wheels complete their revolutions at the same instant?

17. A novelty manufacturer uses cloth in strips of 3″, 4″, 9″, 6″ and 12″ in width. What least width of cloth should he buy in order that he may cut it into any of the given widths without waste?

FOR DISCUSSION.

1. Find the h. c. f. and l. c. m. of any two numbers. Compare the product of these values with the product of the numbers. Explain.

2. The dollar of our currency is a multiple of what other denomination of coin?

3. Name some of our systems of measures that are based on a multiple system.

4. What are the advantages of multiple over mutually prime measures?

5. Show that in determining the l. c. m., all numbers that are factors of other given numbers may be disregarded.

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CHAPTER XVI.

REDUCTION OF FRACTIONS.

A *fraction* is one or more of the equal parts into which a whole may be divided. One of the equal parts is a *fractional unit.*

All fractions may be expressed as *common* fractions by two numbers or terms separated by a horizontal line. The *denominator* (lower term) names the parts or size of the fractional unit; the *numerator* (upper term) shows the number of fractional units in the given fraction. In reading, the numerator is invariably read first.

ILLUSTRATION. $\frac{\text{five}}{\text{eighths}}$ $\frac{\text{(numerator)}}{\text{(denominator)}} = \frac{5}{8}$ = five-eighths.

INTRODUCTORY EXERCISE.

1. Name the *unit* in the expressions: 15 lb., 7 da., 3 wk., 1/2, 5/6, four-ninths, 7/8 yd.

2. Which term of the common fraction is missing in the decimal fraction? How is it expressed?

3. Which is greater, 1/4 or 1/8? How is the value of a fraction affected by increasing its denominator?

4. Arrange in order of value: 1/5, 1/98, 1/72, 1/5000.

5. Can 1/3 lb. and 1/4 A. be compared as to value?

6. How many thirds is 1? Which is greater, 1/3 or 2/3? How is the value of a fraction affected by increasing the numerator?

7. Arrange in order of value: 3/9, 7/9, 11/9, 5/9.

It is evident that any increase in the numerator or decrease in the denominator increases the value of the fraction, and that the value is decreased by decreasing the numerator or increasing the denominator. Moreover, if two fractions have like numerators the larger fraction has the smaller denomina-

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tor; if they have like denominators, the larger fraction has the larger numerator.

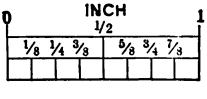
Common fractions are *proper* fractions if the numerator is less than the denominator; otherwise they are improper fractions.

Fractions are *compound,* if each term is fractional (e. g., $\frac{\frac{1}{4}}{\frac{3}{8}}$). A *mixed* number is an expression containing an integer and a fraction (e. g., $6\frac{5}{8}$).

The tendency in all business and general computation is toward the use of decimals in place of complicated or irregular fractions, so that only the simple fractions are in common use. The business series covers halves, thirds, fourths, fifths, sixths, eighths, twelfths, sixteenths and twentieths. To these our main attention will be given.

EXERCISE.

1. This is one inch of a ruler divided into eight parts. Name the unit and the smallest fractional unit.



2. In the whole there are how many halves, fourths, eighths?

3. Multiply numerator and denominator of 1/2 by 4. How is the value affected?

4. Divide numerator and denominator of 6/8 by 2. How is the value affected?

5. Why is value unaffected? (*Suggestion.* Note independent effect of first dividing numerator and then denominator.)

It is evident that a change of expression by multiplying or dividing both terms of a fraction by the same number, causes no change in value. A fraction is *reduced to lower terms* when smaller numbers are found to express the same value; it is *reduced to higher terms* when larger numbers are used.

ORAL EXERCISE.

1. Reduce to thirds: 8, 17, 21, 14, 8, 64.

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2. Reduce to 16ths: 3, 4, 12, 9, 21, 5.

3. Reduce to 12ths: 7, 13, 5, 3, 12, 15.

4. Reduce to 20ths: 3/4, 5/4, 7/5, 3/10, 3/2.

ILLUSTRATION. 3/4 = ?/20. $20 \div 4 = 5$; $1/4 = 5/20$;

$$3/4 = \frac{3 \times 5}{20} = 15/20.$$

5. Reduce to 16ths: 3/4, 5/8, 5/2, 7/8, 9/4.

6. $7/6 = ?/36$; $5/8 = ?/40$; $7/3 = ?/18$; $3/7 = ?/49$.

7. Is there any limit to the increase of size of denominator in reduction?

8. Reduce to improper fractions: $3\frac{1}{4}$, $5\frac{1}{3}$, $16\frac{3}{4}$, $27\frac{1}{8}$.

ILLUSTRATION. In 3 there are 3×4 fourths. In $3\frac{1}{4}$ there are 12/4 + 1/4 or 13/4.

9. Reduce to 16ths: $1\frac{3}{4}$, $2\frac{5}{8}$, $12\frac{1}{2}$.
(Suggestion. Reduce first to improper fractions.)

10. $3\frac{1}{2} = ?/12$; $1\frac{5}{8} = ?/24$; $7\frac{1}{4} = ?/20$; $9\frac{1}{2} = ?/6$.

EXERCISE.

1. Reduce 76 to a fraction with a denominator of 842.

2. Reduce 117 to 43ds.

3. $3/7 = ?/427$; $118/142 = ?/710$.

4. Reduce: $16\frac{8}{24}$ to 24ths; $192\frac{7}{15}$ to 45ths.

It is often advisable, in computations, to *simplify* a fraction by reducing it to lower terms. In final results, proper fractions are usually expressed in lowest terms, and improper fractions are expressed as whole or mixed numbers. It has been noticed that the value of a fraction is not affected by dividing numerator and denominator by the same number. Hence reduction is possible by continued reduction by common factors. When the reduced terms are mutually prime, the fraction is reduced to its lowest terms.

ILLUSTRATION. Reduce 585/675 to its lowest terms.

Analysis and Solution. By inspection, 5 is a common factor of both terms. $\frac{585 \div 5}{675 \div 5} = \frac{117}{135}$

By inspection, 9 is seen to be a second common factor.

This really amounts to dividing the terms by their greatest common divisor. $\frac{117 \div 9}{135 \div 9} = \frac{13}{15}$

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ORAL EXERCISE.

1. Reduce to 8ths: 3/4, 2, 10/16, 15/24.
2. Reduce to 12ths: 3/4, 5/6, 18/24, 12/36, 40/60.
3. Reduce to lowest terms: 15/18 cu. yd., 45/69, 720/2400 T.
4. Reduce to lowest terms: 160/4, 325/5, 56/8, 120/3.
5. Reduce to lowest terms, or to mixed numbers: 9/2, 15/4, 73/3 45/12 lb., 52/8 bu., 162/16.

EXERCISE.

1. Reduce to lowest terms: 56/124, 128/264, 985/1200, 172/465.
2. Reduce to whole or mixed numbers: 725/80, 392/56, 1982/1250.
3. Simplify: 285/16 T., 3924/5280 mi., 453/144 sq. ft.

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CHAPTER XVII.

FRACTIONS—FUNDAMENTAL PROCESSES.

Addition of Fractions.

Since only like numbers can be added, it follows that only *fractions* or *parts* of like units can be added. Moreover, since the denominator of a fraction is but a name showing the relative size of the fractional unit, only *similar fractions*, those having the same denominator, can be added. If fractions differ only in denominator, addition is possible after reduction of the fractions to a common denominator. In the cases commonly arising in everyday usage, the least common denominator is usually determined by inspection.

Illustration. Add 3/5 and 5/6.

Solution. By inspection, the common denominator is 30.

3/5 = 18/30.

5/6 = 25/30.

3/5 + 5/6 = 18/30 + 25/30 = 43/30, or $1\frac{13}{30}$.

ORAL EXERCISE.

Which of these groups can be added without intermediate steps? Which cannot be added at all?

1. 4/5 + 8/5 + 5/5.
2. 1/4 + 1/2 bu.
3. 5/4 ft. + 2 lb.
4. 1/4 + 1/5 + 1/6.
5. 1/4 yd. + $3\frac{1}{3}$ yd.

Find the sum of:

6. 1/4, 3/4, 2/4, 9 fourths.
7. 1/3 yd., 2/3 yd., 5/3 yd., 3/3 yd.
8. 3/16, 4/16, 7/16, 11 sixteenths.
9. 1/5, 3/5, 7/5, 4/5, 11/5, 9/5.
10. 2/3 + 4/5 + 9/15 (= ?/45).
11. 1/3, 1/4.
12. 3/4 ft., 5/8 ft
13. 1/2, 3/4, 5/8, 1/16.
14. 3/8, 4/5, 7/10.
15. 7/12, 2/3, 1/4.
16. 1/2, 2/5, 7/10.
17. 3/20, 4/5, 1/2.
18. 1/3, 3/2, 5/6, 1/12.
19. 1/3, 2/9, 5/6.
20. 3/4 ft., 2/3 ft., 5/2 ft.

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EXERCISE.

NOTE. If the numbers to be added are mixed numbers, the integers and fractions may be added separately and the two results combined. This, however, is not necessary with very simple numbers.

1. $1\frac{1}{2}$, $1\frac{3}{4}$, $5\frac{3}{4}$.
2. $3\frac{1}{2}$, $5\frac{3}{4}$, $6\frac{5}{8}$.
3. $32\frac{1}{5}$, $7\frac{3}{5}$, $8\frac{3}{5}$.
4. 5/12, $1\frac{7}{24}$, 1.5.
5. $3\frac{1}{4}$, $4\frac{1}{8}$, $12\frac{7}{12}$.
6. $4\frac{11}{17}$, $85\frac{6}{51}$.

7.
$214\frac{5}{8}$
$319\frac{1}{16}$
$429\frac{3}{4}$
$296\frac{1}{2}$

8.
$3296\frac{3}{4}$
$562\frac{1}{8}$
$14729\frac{5}{12}$
$862\frac{1}{2}$
$928\frac{7}{12}$
$662\frac{3}{4}$

9.
$12921\frac{1}{3}$
$1828\frac{5}{6}$
$1346\frac{3}{4}$
$462\frac{3}{8}$
$18\frac{3}{5}$

NOTE. In totalling lengths of cloth, the exponents, 1, 2, 3, are used to represent one-fourth, two-fourths and three-fourths respectively. Thus $76^{2}\frac{1}{4}$ yd. are $762\frac{1}{4}$ yd.

10. 27 pieces of percale are sold a customer on an order. Find the total quantity if the respective piece lengths are: $44, 40, 43, 42^{1}, 41^{2}, 40^{3}, 42^{1}, 42^{3}, 43^{3}, 43^{1}, 43^{3}, 44^{3}, 42, 43^{1}, 42^{3}, 43^{1}, 42^{3}, 42^{1}, 41^{2}, 40^{3}, 44^{1}, 43^{1}, 42^{3}, 43^{2}, 42^{1}, 43^{1}, 40^{3}$.

11. Find the cost of thirty-one pieces of Scotch cheviot, @ 37c per yd., the pieces measuring respectively: $38^{1}, 37^{1}, 37^{3}, 39, 39^{2}, 40^{1}, 42^{3}, 40^{1}, 38^{3}, 39^{1}, 40, 38^{2}, 39^{1}, 40, 41, 39^{2}, 38^{3}, 39^{3}, 40, 40^{1}, 40^{2}, 39^{3}, 38^{3}, 39^{1}, 39^{2}, 40^{1}, 39^{3}, 39^{2}, 39^{1}, 40^{1}, 41^{2}$.

EXERCISE.

1. Find the sum of any two fractions that have the same numerator. Study the solution and suggest a short method for adding any two such fractions.

Add:

2. 1/8, 1/3.
3. 1/9, 1/4.
4. 1/11, 1/16.
5. 4/9, 4/21.
6. 5/8, 5/11.
7. 2/3, 2/9.

SUBTRACTION OF FRACTIONS.

Since subtraction and addition are reverse processes, the principles of units and common denominators apply as in addition.

ILLUSTRATION. Find the difference between 3/7 and 1/6.

Solution. $3/7 = 18/42$. $1/6 = 7/42$.

$3/7 - 1/6 = 18/42 - 7/42 = 11/42$.

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EXERCISE.

(Solve mentally if possible.)

Find the difference between:

1. 1/4 and 1/5.	**5.** 7/3 and 4/5.	**9.** 7/8 yd. and 2/3 yd.
2. 6/7 and 1/4.	**6.** 4/3 ft. and 3/4 ft.	**10.** 7/40 and 3/20.
3. 3/8 and 2/9.	**7.** 7/4 and 4/5.	**11.** 1/3 and 1/4.
4. (1/4+1/8) and 3/4.	**8.** 7/8 and 7/20.	**12.** 6/9 and 6/11.

NOTE. If mixed numbers are given, whole numbers and fractions may be subtracted separately, except that in case the fraction of the subtrahend is greater than the fraction of the minuend, it may be necessary to reduce an integral unit of the subtrahend to fractional units.

Find the value of:

13. $5\frac{1}{4} - 3\frac{3}{4}$.	**18.** $5\frac{4}{5} - 2.1$.
14. $6\frac{1}{8} - 1\frac{5}{16}$.	**19.** $388\frac{3}{4} - 172\frac{3}{16}$.
15. $2\frac{3}{4}$ yd. $- 1\frac{1}{2}$ yd.	**20.** $32\frac{7}{20} + 87\frac{1}{4} - 23\frac{3}{16}$.
16. $1\frac{3}{4} - 5/6$	**21.** $582\frac{1}{2} - 96\frac{1}{2}$.
17. $17\frac{7}{10} - .8$.	

MULTIPLICATION OF FRACTIONS.

INTRODUCTORY.

'5/8' may be read in the form '5 eighths.'
16 times 5 eighths are ? eighths.
One-eighth of 16 is ? ; five-eighths of 16 are ?.

It is evident that the product of a fraction by a whole number is equal to the product of the numerator and the whole number, divided by the denominator. Thus, either fraction or whole number may be taken as multiplier.

ILLUSTRATION. Multiply 5/6 by 124.

Solution. $124 \times 5/6 = \frac{124 \times 5}{6} = \frac{620}{6} = 103\frac{1}{3}$.

EXERCISE.

(Solve mentally if possible.)

1. Multiply 320 by each of these fractions: 1/4, 1/2, 3/4, 5/8, 4/5, 3/2, 3/16.

2. Find the following fractional parts of 48: 1/3, 3/4, 5/6, 7/8, 2/3, 7/6.

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3. Find 2/3 of 7, 3, 9, 18, 42, 6, 27, 49, 56, 129.

Multiply:

4. 425 by 2/5.	**8.** 2/7 by 147.	**12.** 320 rd. by 9/16.
5. 25 by 3/4.	**9.** 2/9 by 117.	**13.** 142 by 2/3.
6. 37 by 5/8.	**10.** 5/8 bu. by 726.	**14.** 7/12 by 30 × 9.
7. 3/8 by 5280.	**11.** 5/6 by 8 × 42.	**15.** 364 T. by 4/3.

If one factor is a mixed number, it may be reduced to an improper fraction, although direct multiplication is common.

ILLUSTRATION. Multiply 36 by $2\frac{1}{5}$.

Solution (1). $36 \times 2\frac{1}{5} = 36 \times 11/5 = \frac{36 \times 11}{5} = 396/5 = 79\frac{1}{5}$.

Solution (2). 36

$2\frac{1}{5}$

$7\frac{1}{5}$ (1/5 of 36)

72

$79\frac{1}{5}$

EXERCISE.

(Solve mentally if possible.)

Find the product of:

1. $16 \times 1\frac{1}{4}$.	**5.** $165 \times 1\frac{1}{5}$.
2. $424 \times 7\frac{1}{8}$.	**6.** $3\frac{1}{2} \times 16$ T.
3. $21 \times 4\frac{1}{7}$.	**7.** $2\frac{3}{4} \times \$120$.
4. $4\frac{1}{2} \times 160$ ft.	**8.** $168 \times 68\frac{9}{16}$.

Find the cost of:

9. $7\frac{1}{2}$ yd. dress goods @ \$3; $8\frac{1}{2}$ yd. lining @ 17c; 3/4 yd. velvet @ \$3.50.

10. 420 bu. corn @ $42\frac{1}{2}$c; 652 bu. wheat @ $81\frac{1}{2}$c; 1900 bu. oats @ $32\frac{1}{2}$c.

FRACTIONS MULTIPLIED BY FRACTIONS.

INTRODUCTORY.

1/2 in. = ? fourths of an inch = ? eighths.

1/4 of 1/2 in. = 1/4 of ? eighths, or ? eighth.

The product of two fractions is evidently a fraction whose numerator is the product of the given numerators and whose denominator is the product of the given denominators. The computation may be simplified, in some cases, by cancellation.

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ILLUSTRATIONS. Multiply 3/85 by 5/6.

Solution 1. $3/85 \times 5/6 = \frac{3 \times 5}{85 \times 6} = \frac{15}{510} = 1/34.$

Solution 2. By cancelling common factors.

$$3/85 \times 5/6 = \frac{\overset{1}{\cancel{3}} \times \overset{1}{\cancel{5}}}{\underset{17}{\cancel{85}} \times \underset{2}{\cancel{6}}} = \frac{1}{17 \times 2} = \frac{1}{34}.$$

ORAL FACILITY EXERCISE.

Find the product of.

1. $1/4 \times 1/3$.
2. $1/2 \times 3/4$.
3. $3/8 \times 5/2$.
4. $4/5 \times 7/12$.
5. $5/6 \times 6/7$.
6. $3/2 \times 8/9$.
7. $3/8 \times 8/3$.
8. $3/8 \times 5/6 \times 4/5$.

EXERCISE.

Simplify

1. 3/4 of 9/24 of 7/15 of 852 bu.
2. $3/84 \times 5/72 \times \frac{124}{15} \times \$8290.$
3. $\frac{129}{4} \times 84/36 \times 21/22.$

PRODUCTS OF MIXED NUMBERS.

With large integral numbers and small fractions, multiplication by parts is common. In simple cases, the mixed numbers may be reduced to improper fractions and the products obtained as in the previous case.

ILLUSTRATIONS. (1) Multiply $429\frac{4}{5}$ by $127\frac{1}{3}$.

Solution.

$$\begin{array}{rl} 429\frac{4}{5} & \\ 127\frac{1}{3} & \\ \hline \frac{4}{5} & (1/3 \times 3/5) \\ 143 & (1/3 \times 429) \\ 76\frac{4}{5} & (127 \times 3/5) \\ \left.\begin{array}{r} 3003 \\ 858 \\ 429 \end{array}\right\} & (127 \times 429) \\ \hline 54702\frac{4}{5} & \text{Ans.} \end{array}$$

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(2) Multiply $4\frac{1}{3}$ by $6\frac{3}{8}$.

Solution. $4\frac{1}{3} = 13/3$. $6\frac{3}{8} = 51/8$.

$$13/3 \times 51/8 = \frac{13 \times \overset{17}{\cancel{51}}}{\underset{1}{\cancel{3}} \times 8} = \frac{221}{8} = 27\frac{5}{8}.$$

EXERCISE.

(Solve mentally if possible.)

Multiply:

1. $1\frac{1}{2}$ by $2\frac{3}{4}$.
2. $2\frac{3}{4}$ by 3/8.
3. $1\frac{2}{3}$ by $3\frac{1}{7}$.
4. $1\frac{1}{3}$ by $1\frac{3}{4}$.
5. $4\frac{1}{2}$ by $1\frac{5}{8}$.
6. $1\frac{1}{3}$ by $2\frac{3}{4}$.
7. $4\frac{3}{7}$ by $15\frac{1}{4}$.
8. $7\frac{3}{5}$ by $14\frac{2}{16}$.
9. $234\frac{1}{2}$ by $65\frac{1}{4}$.
10. $345\frac{3}{4}$ by $168\frac{1}{8}$.
11. $1420\frac{5}{6}$ by $1728\frac{3}{4}$.
12. $182\frac{1}{7}$ by $14\frac{8}{11}$.

Short Methods.

Few fractional short methods, aside from cancellation, have a broad use. The most valuable method is that applying to mixed number factors having the fraction "$\frac{1}{2}$."

Case I. Integers alike. Example. Multiply $4\frac{1}{2}$ by $4\frac{1}{2}$.

Analysis and Solution.

By parts, $4\frac{1}{2} \times 4\frac{1}{2} = 4 \times 4 + 4 \times 1/2 + 1/2 \times 4 + 1/2 \times 1/2.$

$= 4$ fours $+ 2$ halves of $4 + 1/4.$

$= 5$ fours $+ 1/4$

Condensed solution.

$$4\frac{1}{2} \times 4\frac{1}{2} = 5 \times 4 + 1/4 = 20\frac{1}{4}.$$

In general, *multiply the integer by the next higher integer and add 1/4.*

Case II. Integers unlike. Example. Multiply $8\frac{1}{2}$ by $11\frac{1}{2}$.

Analysis and Solution.

By parts, $8\frac{1}{2}$ by $11\frac{1}{2} = 11 \times 8 + 11 \times 1/2 + 1/2 \times 8 + 1/2 \times 1/2.$

$= 88 + 1/2\,(11 + 8) + 1/4.$

$= 88 + 9\frac{1}{2} + 1/4 = 97\frac{3}{4}.$

In general, *add to the product of the integers one-half their sum, plus 1/4.*

EXERCISE.

Multiply:

1. $12\frac{1}{2}$ by $12\frac{1}{2}$.
2. $8\frac{1}{2}$ by $8\frac{1}{2}$.
3. $15\frac{1}{2}$ by $15\frac{1}{2}$.
4. 9.5 by 9.5.
5. $6\frac{1}{2}$ by $6\frac{1}{2}$.
6. $6\frac{1}{2}$ by $8\frac{1}{2}$.
7. 10.5 by $12\frac{1}{2}$.
8. $14\frac{1}{2}$ by $16\frac{1}{2}$.
9. $30\frac{1}{2}$ by $30\frac{1}{2}$.
10. $15\frac{1}{2}$ by 5.5.
11. $46\frac{1}{2}$ by $46\frac{1}{2}$.
12. $124\frac{1}{2}$ by $61\frac{1}{2}$.

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DIVISION OF FRACTIONS.

1. Fractions by Integers.

INTRODUCTORY.

12 ft. ÷ 3 = how many feet?

12 thirteenths ÷ 3 = how many thirteenths? = ?/13.

15/16 ÷ 5 = 1/5 of 15/16 = 1/5 × 15/16 = ?

It is possible, evidently, to divide the fraction by dividing the numerator, or multiplying the denominator by the integral divisor.

The *reciprocal* of a number is 1 divided by that number. Multiplying the denominator by the integral divisor is equivalent to multiplying the fraction by the reciprocal of the divisor.

Illustration. Divide 18/23 by 6.

Solutions.

(1) $\frac{18}{23} \div 6 = \frac{18 \div 6}{23} = \frac{3}{23}$.

(2) $\frac{18}{23} \div 6 = \frac{\overset{3}{\cancel{18}}}{23 \times \underset{1}{\cancel{6}}} = \frac{3}{23}$.

(3) $\frac{18}{23} \div 6 = \frac{\overset{3}{\cancel{18}}}{23} \times \frac{1}{\underset{1}{\cancel{6}}} = \frac{3}{23}$.

EXERCISE.

(Solve mentally if possible.)

Divide:

1. 28/37 by 4.	5. 3/8 by 5.	9. 5/8 by 15.
2. 5/8 by 6.	6. 21/25 by 7.	10. 142/56 by 14.
3. 2/3 by 12.	7. 2/3 by 7.	11. 36/89 by 18.
4. 24/15 by 6.	8. 5/9 by 8.	12. 22/45 by 33.

2. Division of Mixed Numbers by Integers.

Since the mixed number may be reduced to an improper fraction, the method does not differ from that just shown. It is common, however, to divide directly.

Illustration. Divide $54\frac{2}{3}$ by 6.

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Solutions. (1). $54\frac{5}{9} = \frac{491}{9}$. $\frac{491}{9} \div 6 = \frac{491}{9} \times \frac{1}{6} = \frac{491}{54} = 9\frac{5}{54}$.

(2) $6\overline{)54\frac{5}{9}}\ 9\frac{5}{54}$

54

$\frac{5}{9}$ (5/9 ÷ 6 = 5/9 × 1/6 = 5/54)

$\frac{5}{9}$

EXERCISE.

(Solve mentally if possible.)

Divide:

1. $1\frac{2}{3}$ by 3.	**5.** $16\frac{1}{2}$ by 5.	**9.** $382\frac{4}{5}$ by 26.
2. $12\frac{3}{4}$ by 4.	**6.** $4\frac{2}{3}$ by 3.	**10.** $193\frac{7}{10}$ by 15.
3. $27\frac{2}{3}$ by 9.	**7.** $12\frac{5}{16}$ by 4.	**11.** $672\frac{2}{3}$ by 12.
4. $8\frac{2}{5}$ by 3.	**8.** $2321\frac{1}{4}$ by 8.	**12.** $458\frac{1}{16}$ by 15.

Fractional Divisors.

INTRODUCTORY.

In 4 there are how many thirds? How many groups of 2 thirds?

$$4 \div \frac{2}{3} = \frac{4 \times ?}{2} = ?$$

Evidently the quotient of an integer by a unit fraction is the product of integer and denominator. If the fractional divisor is not a unit fraction, its terms may be inverted (the reciprocal taken) and the quotient is then the product of the dividend (fractional or integral) and the reciprocal fraction.

Illustrations. (1) Divide 425 by 5/6.

Solution. 425 ÷ 5/6 is equivalent to 425 × 6/5.

$$\overset{85}{\cancel{425}} \times \frac{6}{\cancel{5}} = 510.$$
$$1$$

(2) Divide $\frac{9}{16}$ by $\frac{3}{4}$.

Solution. $\frac{9}{16} \div \frac{3}{4}$ is equivalent to $\frac{9}{16} \times \frac{4}{3}$.

$$\frac{\overset{3}{\cancel{9}}}{\underset{4}{\cancel{16}}} \times \frac{\cancel{4}}{\cancel{3}} = \frac{3}{4}.$$

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EXERCISE.

(Solve mentally if possible.)

Divide:

1. 1/2 by 2/3.
2. 2/3 by 1/5.
3. 3/4 by 4/5.
4. 1/6 by 1/7.
5. 2/5 by 5/6.
6. 3/7 by 4/11.
7. 5/8 by 7/12.
8. 3/8 by 9/16.

NOTE. Simplify the process in the following by reducing to a common denominator.

9. 3/4 by 7/8.
10. 2/3 by 5/6.
11. 5/12 by 1/3.
12. 426 by 3/4.
13. 129 by 2/3.
14. 3545 by 5/18.
15. 3/15 by 5/36.
16. 5/8 by 17/140.
17. 256 by 19/24.

MIXED NUMBER DIVISORS.

Since mixed numbers may be reduced to improper fractions, this case reduces to that last given. If the fractions are simple it is often advisable to reduce to a common divisor.

EXERCISE.

(Solve mentally if possible.)

Find the quotient of:

1. $1\frac{1}{2} \div 1\frac{1}{3}$.
2. $2 \div 1\frac{1}{2}$
3. $1\frac{1}{4} \div 1\frac{1}{3}$.
4. $2\frac{1}{4} \div 3\frac{1}{4}$.
5. $5/2 \div 1\frac{1}{3}$.
6. $5\frac{1}{3} \div 4\frac{1}{3}$.

Divide:

7. $546\frac{2}{3}$ by $1\frac{1}{3}$.
8. $672\frac{11}{16}$ by $14\frac{1}{2}$.
9. $129\frac{1}{3}$ by $66\frac{2}{3}$.
10. $582\frac{1}{3}$ by $13\frac{1}{3}$.
11. $416\frac{2}{3}$ by $12\frac{1}{2}$.
12. $6546\frac{1}{4}$ by $2\frac{1}{4}$.
13. $599\frac{1}{4}$ by $7\frac{1}{4}$.
14. $59\frac{5}{21}$ by $1\frac{1}{3}$.

CONVERSION OF FRACTIONS.

Owing to the growing use of the decimal fraction, it is necessary to have facility in expressing fractions in either common or decimal form. It is evident that the denominator of a decimal fraction, though not expressed, is always determined by the decimal point.

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ILLUSTRATION. Express .0825 as a common fraction.

Solution. .0825 = 825 ten-thousandths = $\frac{825}{10000} = \frac{33}{400}$.

EXERCISE.

Express as common fractions reduced to lowest terms:

1. .125.	**5.** 18.	**9.** .096.	**13.** .19275.
2. .25.	**6.** 1.54.	**10.** .392.	**14.** .088.
3. .33⅓.	**7.** 7.45.	**11.** .875.	**15.** 62.045.
4. .0084.	**8.** 375.	**12.** .0625.	**16.** 1.0458.

The reverse process of converting common fractions to decimal fractions is far more common than that just given. It is performed by fractional reduction, or, more commonly, by executing the represented division.

ILLUSTRATION. Reduce 28/85 to the nearest decimal of three places.

Solution. Carry out the represented division.

$$\begin{array}{r} .329 \quad \text{Ans.} \\ 85\overline{)28.00000} \\ \underline{25\,5} \\ 2\,50 \\ \underline{1\,70} \\ 800 \\ \underline{765} \\ 35 \end{array}$$

EXERCISE.

(Solve mentally if possible.)

1. Convert into decimal form: 1/2, 1/3, 1/4, 1/5, 1/6, 1/7, (.14$\frac{2}{7}$), 1/8, 1/9, 1/10, 1/11, 1/12, 1/15, 1/16.

2. Reduce to decimals: 3/4, 2/3, 5/8, 3/8, 1/30, 5/6, 2/9, 5/16.

NOTE. Common measures are frequently expressed as decimals of other measures.

3. Express 1 ft. as a decimal of 1 yd.; 1 qt. as a decimal of 1 gal.

EXERCISE.

Reduce to equivalent decimals:

1. To two decimal places with fractional remainder: 3/12, 5/11, 5/24, 11/25, 13, 16, $1\frac{1}{16}$, 17/48, $5\frac{3}{64}$, $8\frac{1}{15}$.

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2. To the nearest four-place decimal: 47/52, 5/14, 6/28, 9/32, 21/56, 9/125, 12/250, 13/80, 11/60.

3. Express 100′ as a decimal of a mile (5280′).

4. Mechanics often use conversion tables, giving fractions of inches as decimals of a foot. Complete the following table for each 1/8″ up to one foot, computing to four decimal places.

Inch.	Fraction of Foot.	Decimal.
1/8	1/96	.0104
1/4	1/48	.0208

QUESTIONS FOR DISCUSSION.

1. Why are not 7ths and 9ths common business fractions?

2. What has led to the general use of the common business fractions?

3. Are our common measures adapted to the business fractions?

4. What fractions are used in measuring cloth? Give similar illustrations of the use of particular fractions in specific trades or professions.

5. Why are decimals used in place of fractions, in statistical tables?

6. Name some fractions that are simpler, for business use, than their equivalent decimals?

7. What is the shortest method of comparing the relative value of several irregular fractions?

8. Why is the value of a fraction altered by *adding* the same amount to both numerator and denominator?

9. What is true of the product of two reciprocal fractions? Why?

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CHAPTER XXII.

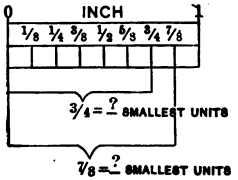
FRACTIONAL RELATIONS OF NUMBERS.

It is often necessary to determine the *fractional relations* of numbers. There are two common cases:

I. To Find What Part One Number Is of Another.

INTRODUCTORY.

8 is what part of 12? Reduce to lowest terms. Which number represents the part? Which number represents the whole to which the part is referred?



2 sevenths is what part of 4-sevenths?

3/4 in. = ?/8 in. 3/4 in. = what part of 7/8 in.?

It is evident that the part that one number is of another is expressed as a fraction having for a numerator the number expressing the part, and for the denominator that *whole* to which it is referred. This is equally true of integral, fractional, decimal, abstract or concrete numbers.

Illustration. Example. I buy 3/8 interest in a business from one who owns 4/5 interest in it. What part of his holdings do I buy?

Solution. $\frac{\text{(Part) } 3/8}{\text{(Whole) } 4/5}$ = relation.

$$\frac{3/8}{4/5} = 3/8 \div 4/5 = 3/8 \times 5/4 = 15/32 \text{ the part bought.}$$

EXERCISE.

(Solve mentally if possible.)

What part of:

1. 14 is 22?
2. 11 is 15?
3. 18 yd. is 7½ yd.?
4. 1/2 is 1/3?
5. 14 lb. is 6½ lb.?
6. 7 bu. is 3½ bu.?

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7. $2.50 is 70 c? 9. 4/11 is 13/16? 11. 5.01 is 1.29?

8. 3/4 is 5/8? 10. $326.50 is $420? 12. 1728 is 1264?

13. The stands at a ball park have a seating capacity of 13,500, of which 3200, or ?/?, are grand stand seats; 1/4 are bleachers and the balance, or ? seats, are in the pavilion.

14. In a certain coal mine an investigating commission discovered that for every ton of coal secured for market, 6.45 tons are wasted. What part is wasted?

15. Compare the speed of two trains, one of which covers four miles in 3½ minutes, and the other 7 miles in 6 minutes.

16. 726 tickets were taken up at the entrance to a hall seating 1200. The hall was approximately what part filled?

Note. When an approximate relation is called for, find the nearest simple fractional relation.

II. To Find the Whole When a Part Is Known

INTRODUCTORY.

If 16 ft. equal 1/5 of a quantity, find 5/5 of it.

If 16 ft. equals 4/5 of a distance, find 1/5 of the distance. Find the entire distance.

Thus the *whole* may be determined by finding the value of a fractional unit of the given part, and multiplying this value by the number of fractional units in the whole.

Illustration. 3/8 is 2/3 of what number?

Analysis and Solution.

3/8 = 2 thirds of a number.

1/2 of 3/8, or 3/16 = 1 third of the number.

3 × 3/16, or 9/16 = 3 thirds of the number, or the number itself.

In brief: If 3/8 = 2/3 of a number, the number = 3/2 of 3/8, or 9/16.

EXERCISE.

Find missing values:

1. 16 = 1/2 of —.
2. 35 = 5/6 of —.
3. 42 bu. = 3/7 of —.
4. $5.60 = 4/5 of —.
5. 2/3 = 1/2 of —.
6. 1/2 = 2/3 of —.
7. 750 lb. = 2/3 of —.
8. 4280 = 1/4 of —.
9. 192 lb. = 4/3 of —.
10. 127 = 3/4 of —.
11. 2420 lb. = 3/15 of —.
12. $752 = 3/8 of —.
13. 592 = 4/13 of —.

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14. A railway speed of 14 miles in 18 minutes is equivalent to a speed of how many miles per hour?

15. A steam shovel excavated 47½ cu. yd. of earth in 4½ minutes. This was at the rate of how many cubic yards per hour?

16. A yield of 26 bushels on a trial plot of 2/5 of an acre is equivalent to a yield of how many bushels per acre?

17. A certain cloth shrinks 1/32 in dyeing. To have 930 yd. of finished product, one must dye how many yards?

18. A man who offered me $750 for a 3/8 interest in a certain patent, evidently valued the patent at $—.

19. A contractor has completed 2/5 of a grading contract in 48 working days. At the same rate, how many days work remain?

20. A manufacturer replaces his old machines with new ones of one-fifth greater productive power. He runs on a ten hour day basis. His new machines should give him his former weekly output with a saving of how many hours.

ORAL FACILITY EXERCISE.

Find the single term equivalent to each group of two or more terms. Distinguish amounts less than zero by the term minus, thus $3 - 4 = -1$.

1. *Halves and thirds.* Two is what part of three? Three of two? Find 1/2 + 1/3, 1/3 + 1/2, 1/3 of 1/2, 1/3 − 1/2, 1/2 − 1/3, 1/2 − 2/3, 1/3 ÷ 1/2, 1/2 ÷ 1/3, 1/3 ÷ 2/3, 1/2 ÷ 2/3, 1/3 × 1/2, 1/2 × 2/3, 2/3 of 1/2, 2/3 divided by 1/3, 1/2 of .4, 2/3 of 15, 6 = 1/2 of ?, 12 = 1/3 of ?, 1/2 = 1/3 of ?, 1/3 = 1/2 of ?, 1/3 is what part of 1/2, 1/2 is what part of 2/3?

2. *Thirds, sixths and twelfths.* Find 1/3 + 1/6, 2/3 − 5/6, 1/6 − 2/3, 1/6 ÷ 1/3, 5/6 ÷ 5/3, 2/3 ÷ 1/6, 2/3 ÷ 4/6, 1/3 of 1/6, 2/3 × 5/6, 1/3 + 1/16 + 1/12, 1/6 ÷ 1/12, 1/3 ÷ 1/12, 7/12 ÷ 1/3, 5/12 ÷ 5/6, 5/6 − 5/12, 1/3 of 2.4, 5/6 of 72, 7/12 of 18, 4 = 1/6 of ?, 1.5 = 5/12 of ?, 6 = 1/12 of 1/2 of ?, .1/6 is what part of 1/12? Of 1/3? Of 5/6? 5/6 is what part of 2/3? Of 7/12? 1/6 is 1/12 of ? 5/12 is 1/3 of ?

3. *Halves, fourths, eighths and sixteenths.* Find 1/2 + 3/4, 3/4 + 5/8, 7/8 − 1¼, 9/16 − 5/8, 1/4 ÷ 3/16; 3/4 of 3/16, 5/8 × 1/2, 5/8 of 3/4, 1/16 − 3/8, 5/16 ÷ 3/8, 3/8 of 5/16, 1/4 + 1/8 ÷ 1/16, 1/2 times 7/16, 1/2 ÷ 9/16, 1/2 × 15/16. 1/2 is what part of 7/8? Of 15/16? Of 5/4? 3/4 is what part of 3/8? Of 15/16? Of 1⅜? What is 1/2 of 3.6? Find 3/4 of 64. What is 5/8 of 12? Find 3/16 of 80. 11/16 of 3.2 = ? 14 = 1/2 of ? 10 = 5/16 of ? 12 = 3/8 of ? 12 = 9/16 of ? 1/2 = 3/4 of ? 1/8 = 1/2 of ? 5/8 = 3/4 of ? 9/16 = 1/8 of ? 9/16 = 3/4 of ?

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4. Use the answer, at each step, as the basic value for the next computation.

$4 + 12 = ?$ Add 8. Divide by 2. Multiply by 1/2. Divide by 1/3. Multiply by 2/3. Divide by 4. Increase by 2/3 of itself. Multiply by 4/5. Divide by 2/3. Multiply by 3/4. Change to eighths. What is the final result?

TEACHER'S NOTE. Similar original exercises should be prepared by the teacher and given to the class, in order to cultivate a high degree of accuracy in handling the simple business fractions.

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CHAPTER XXIII.

ALIQUOT PARTS.

Numbers that exactly divide a given number are termed its aliquot parts. Thus 1, $1\frac{1}{2}$, 1/5, 2, 3, etc., are aliquot parts of 6.

ORAL EXERCISE.

1. Name 6 aliquot parts of 12. What fractional part is each aliquot part?

2. Find 1/4 of 200; of 10. Find 1/8 of 10, of 100, of 1000, etc.

3. What parts of \$1.00 are 50c, 10c, $33\frac{1}{3}$c, 20c, 2c?

4. Which of these numbers are aliquot parts of 10: 1, $1\frac{1}{2}$, 2, $2\frac{1}{2}$, 3, $3\frac{1}{3}$, $3\frac{1}{2}$, 4?

5. Complete the following table:

ALIQUOT PART.

Number.	1/2	1/3	1/4	1/5	1/6	1/8	1/9	1/10	1/12	1/15	1/16	1/20
1.												
10.												
100.	50	$33\frac{1}{3}$	25	20	$16\frac{2}{3}$	$12\frac{1}{2}$	$11\frac{1}{9}$	10	$8\frac{1}{3}$	$6\frac{2}{3}$	$6\frac{1}{4}$	5
1000.												

1. MULTIPLICATION.

Aliquot parts are frequently used to simplify multiplication and division. The aliquot parts of 10c and \$1.00 are especially useful in business.

INTRODUCTORY.

1. Express as aliquot parts of \$1.00, 25c, $12\frac{1}{2}$c, $2\frac{1}{2}$c, 4c.

2. How does the cost of several articles @ 25c compare with their cost @ \$1.00?

3. The cost of 48 yd. @ 25c = the cost of 48 yd. at what price?

4. 522 × $33\frac{1}{3}$c = 522 × \$1/3 = 1/3 of \$? = \$?

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5. The cost of 522 articles @ $1.00 is $? The cost at 33⅓c is what part of the cost at $1.00?

It is evident that the cost of an article at any aliquot price of one dollar is its simple fractional part of the cost at one dollar.

ILLUSTRATION. Find the cost of 568 yd. cloth @ 12½c per yd.

Analysis and Solution. 12½c = $1/8.

Cost of 568 yd. @ 12½c = Cost of 568 yd. @ $1/8
= 1/8 of $568 = $71.

NOTE. Notice that in some of the following examples the aliquot factor is the figure representing the quantity rather than the price.

EXERCISE.

Find cost of:

1. 840 yds. wash silk @ 25c; at 33⅓c.
2. 1620 yds. plaids @ $1.25; at 33⅓; at 50c.
3. 580 yds. cotton at 6⅔c, at 6¼c.
4. 864 yds. prints at 12½c; 16⅔c, 20c.
5. 164 yds. lining at 12½c, 10c, 8⅓c
6. 422 yds percale at 25c, 20c, 16⅔c, 12½c.
7. 360 doz eggs at 12½c, at 25c.
8. 560 lb. lard at 10c; 8⅓c, 12½c.
9. 48 lb. chocolate @ 25c, 16⅔c, 33⅓c.
10. 450 lb. tapioca @ 6¼c, 8⅓c.
11. 62 hats @ $2.50 (aliquot part of $10).
12. 156 T. old iron @ $16⅔ ton.
13. 1520 A. @ $12.50 per A.
14. 1500 blank books @ 25c.
15. 196 doz. buttons @ 16⅔c.
16. 124 pr. shoes @ $1.25 ($1+25c).
17. 2500 pineapples @ 12½c.
18. 1425 posts @ 33⅓c.
19. 625 yds. wire netting @ 25c yd.
20. 364 lbs. coffee @ 33⅓c.
21. 48 lbs. tea @ 50c; @ 33⅓c.

NOTE. The principle of interchanging multiplier and multiplicand is frequently employed. Thus the cost of 25 yd. @ 88c is equivalent to the cost of 88 yd. @ 25c.

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Compute the cost of:

22. $33\frac{1}{3}$ yd. @ 78c.
23. $16\frac{2}{3}$ yd. lace @ 96c.
24. 25 bu. beans @ \$2.60 (260c)
25. 50 bu. clover seed @ \$4.20.

ILLUSTRATIVE EXERCISE.

$100 \times 720 = ?$

25×720 = what part of 100×720? $25 \times 720 = \frac{100}{?} \times 28 = ?$

Suggest the short method.

It is evident that the aliquot method greatly simplifies computation, often substituting oral for written work.

ILLUSTRATION. Multiply $672 \times 33\frac{1}{3}$.

Solution-aliquot.	Solution-regular.
$672 \times 33\frac{1}{3}$ = 1/3 of 672×100..	672
$100 \times 672 = 67200$.	$33\frac{1}{3}$
$67200 \div 3 = 22400$	224
	2016
	2016
	22400

ORAL EXERCISE.

1. Suggest short methods for multiplication by $12\frac{1}{2}$, $6\frac{1}{4}$, $6\frac{2}{3}$, 25, 50, $16\frac{2}{3}$.

ILLUSTRATION. To multiply by $12\frac{1}{2}$, multiply by 100 and divide by 8. Why?

Find the products of:

2. 25×36.
3. $33\frac{1}{3} \times 42$.
4. $12\frac{1}{2} \times 184$.
5. $6\frac{2}{3} \times 960$.
6. $16\frac{2}{3} \times 640$.
7. 20×85.
8. $33\frac{1}{3} \times 456$.
9. $6\frac{1}{4} \times 64$.
10. $6\frac{2}{3} \times 90$.
11. 50×588.
12. 25×692.
13. 25×12.4.
14. $8\frac{1}{3} \times 72$.
15. $3\frac{1}{3} \times 420$.
16. $12\frac{1}{2} \times 8.48$.
17. $2\frac{1}{2} \times 880$.

WRITTEN EXERCISE.

Find the products of:

1. 426×25
2. $33\frac{1}{3} \times 2672$.
3. $4892 \times 12\frac{1}{2}$.
4. $6\frac{2}{3} \times 968$.
5. $6\frac{1}{4} \times 884$.
6. 25×672.

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7. 2.5×876.
8. 250×968.
9. $25 \times 48/49$.
10. $33\frac{1}{3} \times 66\frac{2}{3}$.
11. $6\frac{2}{3} \times \$1.50$.
12. $6\frac{1}{4} \times \$6.40$.
13. 25×8.004.
14. $2\frac{1}{2} \times 8.4$.
15. $3\frac{1}{3} \times .0696$.
16. $333\frac{1}{3} \times 6.0096$.

MULTIPLES OF ALIQUOT PART.

The fraction representing an aliquot part has a numerator of ——?

Find 1/2 of 100; 1/8 of 100; 3/8 of 100; 1/3 of 100; 2/3 of 100; 1/4 of 100; 3/4 of 100.

$12\frac{1}{2} \times 248 =$ ——. $37\frac{1}{2} =$ —— $\times 12\frac{1}{2}$.

$37\frac{1}{2}$ = what part of 100. $37\frac{1}{2} \times 248$ = what part of 100×248?

Suggest short methods for multiplying by multiples of aliquot parts.

ORAL EXERCISE.

1. Multiply 963 by $66\frac{2}{3}$.

Solution. $66\frac{2}{3}$ = 2/3 of 100. $66\frac{2}{3} \times 963$ = 2/3 of $963 \times 100 = 32100$. Note that the fractional part of 2/3 may be taken before or after multiplication by 100. Also one may multiply by 100 and subtract 1/3 of product.

Multiply:

2. 124×125.
3. 68×75.
4. $42 \times 66\frac{2}{3}$.
5. $16 \times 37\frac{1}{2}$.
6. $24 \times 66\frac{2}{3}$.
7. $24 \times 62\frac{1}{2}$.
8. $48 \times 87\frac{1}{2}$.
9. $54 \times 83\frac{1}{3}$.
10. $7\frac{1}{2} \times 640$.
11. $66\frac{2}{3} \times 12$.
12. 125×86.
13. $168 \times 37\frac{1}{2}$.
14. $96 \times 112\frac{1}{2}$.
15. 1220×150.
16. 84×75.
17. $328 \times 37\frac{1}{2}$.

WRITTEN EXERCISE.

1. 75×874.
2. $66\frac{2}{3} \times 960$.
3. $37\frac{1}{2} \times 8480$.
4. $62\frac{1}{2} \times 7244$.
5. $87\frac{1}{2} \times 920$.
6. $83\frac{1}{3} \times 97.3$.
7. $7\frac{1}{2} \times 4520$.
8. $3\frac{1}{3} \times 872$.

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9. 125×1846.
10. $133\frac{1}{3} \times 276$.
11. $112\frac{1}{2} \times 488$.
12. $233\frac{1}{3} \times 606$.
13. $333\frac{1}{3} \times 942$ qt.
14. $137\frac{1}{2} \times 840$ bu.
15. $183\frac{1}{3} \times 6960$.
16. $166\frac{2}{3} \times 825$.

ILLUSTRATIVE EXERCISE.

1. Find 1/4 of a dollar, 3/4 of it, 2/3, 3/8, 5/8, 4/3, 7/8, 3/40.

2. What part of a dollar are: 25c, 75c, $66\frac{2}{3}$c, $7\frac{1}{2}$c, 15c, 70c, \$1.25, \$2.50?

3. The cost of 240 yd. @ 75c is what part *less* than the cost at \$1.00? How many times the cost at 25c?

4. Suggest two short methods for finding the cost at 75c.

It is evident that the cost at some multiple of an aliquot price is obtained by taking the aliquot fractional part of the total cost at \$1.00.

ILLUSTRATION. Find the cost of 726 yd. @ $66\frac{2}{3}$c.
Solution. 726 yd. @ \$1.00 cost \$726. $66\frac{2}{3}$c = 2/3 of \$1.00.
2/3 of \$726 = \$484, cost.

ORAL EXERCISE.

1. Suggest short methods for finding the cost @ 80c, $37\frac{1}{2}$c, $\$1.16\frac{2}{3}$, $83\frac{1}{3}$c, \$1.25.

Find the cost of:

2. 290 lbs. @ 40c, 10c, \$1.50.
3. 42 yds. @ $66\frac{2}{3}$c, 50c, 25c.
4. 85 yds. @ 80c, 10c, 50c.
5. 96 lb. @ $62\frac{1}{2}$c, 75c, $83\frac{1}{3}$c.
6. 24 T. @ \$7.50, \$2.50, \$8.00.
7. 136 qt. @ 25c, $37\frac{1}{2}$c, 75c.
8. 48 oz. @ $3\frac{1}{3}$c, $6\frac{2}{3}$c, $66\frac{2}{3}$c, $\$1.33\frac{1}{3}$.
9. 120 sq. yds. @ $7\frac{1}{2}$c, 40c, 75c, $37\frac{1}{2}$c, $66\frac{2}{3}$c.
10. 90 cu. yds. @ $\$1.33\frac{1}{3}$, $\$2.66\frac{2}{3}$, $\$15.66\frac{2}{3}$.
11. 16 T. @ \$2.50, \$25, \$3.75, \$6.25.
12. 45 ft. @ 20c, 40c, \$1.20, \$1.60.
13. 75 yd. @ \$1.24, \$3.36, \$2.40, \$.88.
14. $12\frac{1}{2}$ T. @ \$4.80, \$5.40, \$6.00, \$12.00.
15. $37\frac{1}{2}$ yds. @ 64c, 72c, 90c, 24c.

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DIVISION.

INTRODUCTORY EXERCISE.

1. $1.00 will buy how many pounds @ 25c; @ 33⅓c; @ 50c?

2. $1.00 will buy how many yd. of prints @ 25c? $240 will buy 240 times —— yd. or —— yd.

3. Suggest a method of finding quantity when cost and aliquot price are given.

4. 37½c = what part of a dollar? $840 ÷ 37½c = $840 ÷ $? = 840 × ? = ——

5. Suggest a method of division when the price is a multiple aliquot price.

It is evident that by substituting the aliquot fraction, the operation is reduced to that of division by fractions, and thus to multiplication.

ILLUSTRATION. $360 will buy —— articles @ 33⅓c; or —— articles @ 37½c.

Solution. (1) 33⅓c = $1/3. $360 ÷ $1/3 = 360 × 3/1 = 1080, the no. of articles that can be purchased.

(2) 37½ = $3/8. 360 ÷ 3/8 = 360 × 8/3 = 960, the number of articles that can be purchased.

ORAL EXERCISE.

1. State rules for division by 37½c, 45c, 75c, 66⅔c, $1.16⅔.

2. Suggest methods of checking such divisions.

	Cost.	Price.	Quantity.		Cost.	Price.	Quantity.
3.	$72	25c	?	9.	$5.20	$1.25	?
4.	40	12½c	?	10.	48	.37½	?
5.	46	16⅔c	?	11.	60	1.33⅓	?
6.	45	6⅔c	?	12.	54	1.50	?
7.	21	8⅓c	?	13.	918	2.25	?
8.	126	50c	?	14.	468	.75	?

WRITTEN EXERCISE.

1. $758 will buy — lb. @ 80c.
2. $296 will buy — yd. @ 37½c.
3. $540 will buy — bu. @ 62½c.
4. $925 will buy — A @ $12.50.

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5. $5296 will buy — bu. @ 75c.
6. — cu. yd. @ 62½c cost $1600.
7. — oz. @ 7½c cost $52.
8. — lb. @ 16⅔c cost $615.50.
9. — lb. @ 12½c cost $6040.
10. — lb. @ $2.75 cost $810.

ILLUSTRATIVE EXERCISE.

1. 1.25 is what part of 100. The quotient obtained by dividing by 25 will be —— times the quotient by dividing by 100.

2. 7620 ÷ 100 = ——. 12½ = —— of 100. 7620 ÷ 12½ = —— times 7620 ÷ 100.

3. Suggest short methods for division by aliquot parts of 100, of 1000.

4. 37½ is contained —— times in 100.

5. Explain: 9640 ÷ 37½ = 8/3 of 1/100 of 9640 = ——.

6. Suggest short methods for division by 62½, 66⅔, 7½, 3½, 166⅔, 125, 133⅓.

ORAL EXERCISE.

Find the quotient of:

1. 2500 ÷ 20.
2. 4200 ÷ 25.
3. 3850 ÷ 50.
4. 7200 ÷ 33⅓.
5. 240 ÷ 12½, by 16⅔.
6. 1200 ÷ 75.
7. 1600 ÷ 80.
8. 920 ÷ 33⅓.
9. 600 ÷ 66⅔.
10. 1200 ÷ 133⅓.
11. 800 ÷ 125.
12. 2760 ÷ 150.

WRITTEN EXERCISE.

1. 17800 ÷ 37½.
2. 52900 ÷ 62½.
3. 4890 ÷ 66⅔.
4. 82984 ÷ 1.33⅓.
5. 67500 ÷ 125.
6. 1284 ÷ 87½.
7. 84020 ÷ 6¼.
8. 94368 ÷ 116⅔.
9. 9218 ÷ 16⅔.

PRICE BY THE HUNDRED OR THOUSAND.

In the many cases where articles are sold by the hundred or thousand, computations are simplified by "pointing off."

ILLUSTRATION. (1) Compute the cost of 920 lb. lead @ $4.00 per c. 920 lb. = 9.2 cwt. $4.00 × 9.2 = $36.80 cost.

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EXERCISE.

(Solve mentally if possible.)

Compute the cost of:

1. 425 lb. @ $5.00 per c.
2. 675 lb. @ $12.50 per c.
3. 1250 yd. @ $45.00 per c.
4. 786 ft. @ $1.25 per c.
5. 1900 ft. @ $1.33⅓ per c.
6. 4580 lb. @ $5.50 per c.
7. 546 lb. fertilizer @ 85c per c.
8. 2450 ft. lumber @ $33.33⅓ per M.
9. 5260 ft. pine @ $40 per M.
10. 4690 ft. oak @ $50 per M.

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CHAPTER XX.

PROBLEM ANALYSIS AND SOLUTION.

In general, five steps should mark the analysis and solution of a problem.

1. A careful reading of the problem to determine what is given and what required.
2. Analysis, to determine the relations between what is given and what required.
3. Selection of process and of factors to be used.
4. The computation.
5. Checking—of reasoning and of computation.

A problem should be thoroughly understood before its solution is attempted. The process selected should be the simplest and most direct. The computation should be performed and set down in the briefest and most direct manner — factoring, cancellation and short methods being used to save time and energy. In the written statement, care should be taken to use symbols accurately, and to exercise judgment in the naming of important values.

There is no uniformity in the method of solution of problems. It is essential, however, that the selected solution should involve no unnecessary labor, and that the results obtained by it should be easily checked. All solutions of applied arithmetic problems require the application of common sense analysis, but this analysis is a simple matter, as the following illustrations will show.

ILLUSTRATIONS. (1) Andrews offers a 3/8 acre plot of land for $72. This price is equivalent to a price of how many dollars per acre?

Analysis and Solution.

1. Given the price of a part, to determine the price of the whole.

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2. If 3/8 of an acre cost \$72, 8/8 acres will cost 8 times one-third of \$72.

3. All given factors required for solution.

4. $8 \times \frac{24}{\cancel{72}} = \$192$. The value of one acre is \$192.

5. Check. 3/8 of \$192 = \$72.

(2) Working at the rate of 100 strokes per minute, a power pump delivers 400 gal. of water per minute. How much must the speed be increased to pump 30,000 gal. per hour?

1. Given present speed and output per minute, and required output per hour, to find speed increase.

2. If the output is 400 gal. per minute, it is 60 times 400 gal. per hour. The speed must be increased by the same fractional part that the required output is greater than the present output.

3. By multiplication, find the hourly output. Compare the two outputs to find increase. By fractional multiplication determine increase in speed.

4. *Solution.*

60×400 gal. = 24,000 gal., capacity per hour.

$\frac{30,000}{24,000} = \frac{5}{4}$. Required capacity is 5/4 of present capacity, or 1/4 greater.

1/4 of 100 strokes = 25 strokes. Therefore, the speed must be increased 25 strokes per minute.

5. Check. An increase of 1/4 in speed will increase the delivery per minute 1/4, or from 400 to 500 gal. A delivery of 500 gal. per minute = a delivery of 60×500 gal. per hour, or 30,000 gal.

NOTE. Numbers 1, 2 and 3 are usually distinct mental processes. They are not represented in paper solutions.

ORAL EXERCISE.

1. 32 is 4/7 of what number?

2. A pump that raises 6400 gal. per day of 8 hours will raise —— gal. if run 10 hours.

3. At 84c per lb., what is the cost of 3/4 lb. of pepper?

4. A factory is insured in one company for \$6000, and for \$4000 in a second. If damaged by fire to the extent of \$3600, what loss should each company pay?

5. An improvement to a certain steam engine is warranted to reduce the coal consumption 2/5. The present consumption being 15 T. per day,

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this means a saving of —— T., or a saving in money, at $3.40 per T., of $——.

6. A train running 1 mile in 4/5 of a minute is running at the rate of —— miles per hour?

7. The report of a cannon is heard 20 sec. after firing. How far distant is the cannon if sound carries 65,000 feet per minute?

8. Divide 10 into two parts, one being three times the other.

EXERCISE.

1. A 5 inch line is used on paper to represent a distance of 60 feet. The paper length is what part of the true length?

2. By improvements in machinery. the output of a factory is increased 3/4. If, at the same time, the working time is reduced from 10 to 9 hours, what is the net increase in output?

3. A blend of tea consists of two grades, the first forming 3/8 of the mixture. By increasing the proportion of second grade 1/4, what part remains first grade?

4. A pipe discharges 32½ gal. per sec. How many minutes will be necessary to discharge 10,000 gal.? What is its discharge rate per hour?

5. A dredge averages 1¼ cu. yard of material per minute. At that rate, it will remove how many cu. yd. in a working day of 8 hours.

6. How much water must be added to a solution containing 1/10 ammonia, to make the solution 1/30?

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CHAPTER XXI.

INVOLUTION AND EVOLUTION.

I. INVOLUTION.

The products obtained by taking any number two or more times as a factor are termed *powers* of that number. The process of finding the powers of numbers is termed *involution.*

If the number is taken twice, the product is termed the *square* of the number; if taken three times, the *cube* of the number; if taken more times, the corresponding power of the number, as the *fourth power, fifth power,* etc.

Powers are expressed by writing the number with a small raised figure or *exponent,* showing the times it is taken as a factor.

ILLUSTRATIONS.

2×2 is written 2^2.

$2 \times 2 \times 2 \times 2$ is written 2^4.

Since the exponents represent the number of times the given number is taken as a factor, the product of two powers of the same number may be *represented* by writing the number with the sum of the given exponents as the exponent of the product. This principle is of value, also, in reckoning high powers of numbers.

ILLUSTRATIONS.

(1) Represent the product of 2^3 and 2^4.

Solution. $2^3 = 2 \times 2 \times 2.$

$2^4 = 2 \times 2 \times 2 \times 2.$

$2^3 \times 2^4 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^7$

Short method. $2^3 \times 2^4 = 2^{3+4} = 2^7.$

(2) Find the value of 3^8.

Solution. $3^8 = 3^{4+4} = 3^4 \times 3^4.$

$3^4 = 3 \times 3 \times 3 \times 3 = 81.$

$3^8 = 3^4 \times 3^4 = 81 \times 81 = 6561.$

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ORAL EXERCISE.

Determine the value of:

1. 3^3.	**3.** 5^3.	**5.** 8^3.	**7.** 12^3.
2. 2^6.	**4.** 4^3.	**6.** 4^4.	**8.** 10^5.

9. Find the squares of 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 20, 25, 100, 120, 400, .2, .5, .12, .006, 1/4, 3/8, 2/3.

10. Show that a square must end in 0, 1, 4, 5, 6, or 9.

11. Show that a cube may end in any of the digits.

EXERCISE.

Compute, by the simplified method, the value of:

1. 16^3.	**3.** 6.05^4.	**5.** $(\frac{5}{6})^6$.
2. 121^4.	**4.** $.0036^6$.	**6.** $(\frac{7}{4})^3$.

II. EVOLUTION.

The process of determining a number from one of its powers is termed *evolution.* If the number is obtained from its square it is termed the *square root;* if obtained from the cube it is termed the *cube root.* Roots of higher powers are named from the powers, as *fourth root, fifth root,* etc.

ILLUSTRATIONS. The square of 6 is 6×6, or 36. 6 is the square root of 36.

The cube of 4 is $4 \times 4 \times 4$, or 64. 4 is the cube root of 64.

Roots are expressed with the symbol $\sqrt{\ }$ written over the number of which the root is to be found. The power to be taken is shown by a numeral, except in the case of the square root.

ILLUSTRATIONS. The square root of 225 is expressed $\sqrt{225}$, but the square root equals 15.

The cube root of 729 is expressed $\sqrt[3]{729}$, but it equals 9.

INTRODUCTORY EXERCISE.

1. Name at sight the equal factors or square roots of 4 ($4 = 2 \times 2$; square root is 2), 9, 16, 25, 36, 64, 81, 100, 10,000, 121, 144.

2. Find the squares of each digit. How many places in each case?

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3. Find the squares of 1, 10, 100, 10,000, .1, .01, .001. How do the numbers of places in the squares compare with those in the roots?

It is evident, from these examples, that the number of places in any square is twice, or one less than twice, the number of places in the root. This fact is important, as it is possible, by pointing off the square from the decimal point into periods of two places, to determine the number of places in the root.

The method of determining the square root may be understood, now, by studying the reverse process, using the method of multiplication by parts.

EXAMPLE. Find the square of 86.

Solution.

$$86 = 80 + 6.$$

$$\begin{array}{rr} 80 + 6 & 86 \\ \underline{80 + 6} & \underline{86} \\ (80 \times 6) + 6^2 & 516 \\ \underline{80^2 + \ (80 \times 6)} & \underline{6880} \\ 80^2 + 2(80 \times 6) + 6^2 = & 7396 \end{array}$$

It is evident that the square of a number, composed of tens and units, is equal to the square of the tens plus twice the product of the tens multiplied by the units plus the square of the units. It is evident, also, that any number may be divided into two parts and its square determined in exactly the same way. This composition of the square is made use of in determining the root.

ILLUSTRATIONS. (1) Compute the square root of 1296.

Solution.

$$\begin{array}{rrl} & & 12'96(30 + 6 = 36, \text{ root.} \\ & 30^2 \ = & \ 9\ 00 \\ & & \overline{3\ 96(6} \\ \text{Trial div.} & 2 \times 30 = & 60 \\ & 2 \times 30 \times 6 = & 360 \\ & 6^2 = & 36\ \ 3\ 96 \\ & & \ \ \ \ 0 \end{array}$$

Analysis.

Since there are 4 places in the square there are 1/2 of 4, or 2 places in the root. Point off by 2's. 12 represents the ten's place in the root, and

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96 the unit's place. By trial, the largest square in 12 is the square of 3. ∴ the largest square of the tens is 30^2, or 900. Subtracting the square of the tens, there are left 396 units. Since the square of the tens has been subtracted, there is left in the perfect square twice the product of tens and units plus the square of the units. As a trial divisor, use twice the tens, or 60. 60 is contained in 396 six times. Try 6 as the required unit. $2(30\times6)=360$; $6\times6=36$. The sum is 396. Evidently 6 is the required unit, and 36 the required root.

Check. $36\times36=1296$.

(2) Find the square root of 1.6384.

Solution.

1.63'84(1.28 root.
1^2 1
$2\times1.0=2.0$, trial, 63(0.3
$2\times1.0\times.2=40$
$.2^2=.04$ 44
$2\times1.20=2\,40$, trial, .19 84(0.8
$2\times1.20\times.08=.1920$
$.08^2=.0064$ 19 84
Check. $1.28\times1.28=1.6384$

or

Treat as whole number.

1.63'84(1.28 root.
$1^2=1$ 1
Trial, 2×10 63
$2\times10\times2=40$
$2^2=4$ 44
Trial 2×120 19 84
$2\times120\times8=1920$
$8^2=64$ 19 84
0

Analysis.

Point off by 2's, each way from the decimal point. If a perfect square, the root will contain one whole number and two decimals. 1 is the nearest square of the unit period and the first figure of the root. Subtract the unit and bring down the first decimal group. Twice 10 tenths, or 20 tenths is the trial divisor. The resulting quotient, 3, is found to be too large, so the next lower number, 2, is taken as the next figure of the root. Complete the square, considering 1 as the ten, and .2 as the units. Subtract the value of the completed square from .63 leaving .19 and bring down final group. As a trial divisor, use twice the portion of the root already found, or 2.40, obtaining the final figure, .08. Complete the square.

Thus, to determine the square root:

1.—Beginning at the decimal point separate the number, each way, into groups of two figures. (The outermost period on the left may have only one place.)

2.—Find the greatest square of the left hand period. Its root is the first figure in the required root.

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3.—Subtract the square from its period and bring down the next period. As a trial divisor use twice the root value already found, annexing a cypher. Considering the quotient as the new unit, complete the square by taking twice the product of the tens multiplied by the units plus the square of the units.

4.—Subtract this value from the last remainder; bring down the next group, and proceed as before.

NOTE. (1) In the case of fractions, either take the square root of numerator and denominator separately, or reduce the fraction to the decimal form.

(2) When the remainder, at any stage, will not contain the trial divisor, write a cypher in the root and bring down the next group. If there are no further periods, periods of two cyphers may be added to the original number and the root determined to any desired number of decimal places.

Since the product of a second power by itself is a fourth power, it is evident that the square root of a fourth power is the square of the number. If the square root of this square is then taken, the result will be the fourth root of the number.

EXERCISE.

Find the square root of:

1. 256
2. 7225.
3. 7744.
4. 16,641.
5. 1025.9209
6. 52.9984.
7. 134,944.81.
8. .000,000,494,209.
9. 29,506,624.
10. 329,968,821 (2 dec. pl.).
11. .008,907,386,72 (5 dec. pl.).
12. 56,847.02938176 (4 dec. pl.).
13. $\frac{196}{7569}$
14. $\frac{2209}{3969}$
15. $\frac{17,161}{23,104}$
16. $\frac{85,264}{1,580,049}$

17. Reduce the following to decimals and take the approximate square root to the third decimal place: (*a*) 5/14; (*b*) 7/88; (*c*) 1/21.

By continued square root, find:

18. The fourth root of 10,098, 039, 121.
19. The fourth root of 3.8416.

CUBE ROOT.

INTRODUCTORY EXERCISE.

1. What is the cube of any number? What is the cube root of the cube?
2. Find the cubes of the digits. What is the largest number of places in the results?

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3. Find the cubes of 1, 10, 100, 1000, .1, .01. How many places in each cube?

It is evident that the cube root of a number is one of its three equal factors. It is evident, also, that each period of three places in the cube stands for one place in the root.

As in the case of square root, the method of determining the cube root may be understood from a study of the reverse process.

ILLUSTRATION. Find the cube of 46.

Solution. Let 46 be written $40' + 6$.

$$\begin{array}{rr}
 & 40 + 6 \quad 46 \\
 & 40 + 6 \quad 46 \\
\hline
 & (40 \times 6) + 6^2 \quad 276 \\
40^2 + & (40 \times 6) \quad 1840 \\
\hline
40^2 + 2(40 \times 6) + 6^2 & \quad 2116 \\
 & 40 + 6 \quad 46 \\
\hline
(40^2 \times 6) + 2(40 \times 6^2) + 6^2 & \quad 12696 \\
40^2 + 2(40^2 \times 6) + & (40 \times 6^2) \quad 84640 \\
\hline
40^3 + 3(40^2 \times 6) + 3(40 \times 6^2) + 6^3 = & \quad 97336
\end{array}$$

From a study of the product it is evident that the cube of a two-place number equals the cube of the tens plus three times the product of the square of the tens multiplied by the units, plus three times the product of the tens multiplied by the square of the units, plus the cube of the units. It is now a simple matter to reverse the process and determine the cube root.

ILLUSTRATION. Find the cube root of 97,336.

Solution.

$$\begin{array}{rrl}
 & & 97'336(46 \\
 & 4^3 & 64 \\
\text{Trial} & 4800 & \overline{33}\,336 \\
3 \times 40^2 \times 6 = & 28800 & \\
3 \times 40 \times 6^2 = & 4320 & \\
6^3 = & 216 & \overline{33}\,336 \\
\text{Check.} \quad 46 \times 46 \times 46 = & 97{,}336. &
\end{array}$$

Separate into periods of three places. The nearest cube root of the tens group is 4. Subtract the cube of 4 from 97 and bring down the next group. As a trial divisor use three times the square of the tens, obtaining 6 as a trial unit. The 33,336 units must be made up of three times the product of the square of the tens multiplied by the units, plus three times the tens multiplied by the square of the units, plus the cube of the units. These values are

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determined separately and totalled as shown. Since they total 33,336, the 6 is the correct unit value of the root, and 46 is the cube root of the number.

As in the case of square root, by always considering the figures of the root, which have been determined at any point in the solution, as tens, it is possible to determine the next figure of the root.

ILLUSTRATION. Find to three decimal places, the cube root of 32.890033664.

Solution.

	3^3	
Trial,	3×30^2	
	$3\times30^2\times2$ =	5400
	$3\times30\times2^2$ =	360
	2^3 =	8
Trial,	3×3200^2	
	$3\times3200^2\times3$ =	92160000
	$3\times3200\times3^2$ =	86400
	3^3 =	27

32.890'033'664(3.203 + root.
27
5 890(2
5 768
122 033 664(3
92 246 427
29 787 237 remainder.

NOTE. In trying for the third figure of the root, the trial divisor proves larger than the new dividend. Therefore a cypher is written in the root, another period brought down, and a new trial divisor used.

It is evident that the steps to be followed in obtaining a cube root are:

1.—Beginning at the decimal point, separate the number, each way, into periods of three places each.

2.—For the first root figure, take the cube root of the greatest cube contained in the left hand period. Subtract the cube, bringing down the remainder and the next period of three places.

3.—For succeeding root figures, use as a trial divisor, three times the square of the root values already obtained, considered as *tens*. Use the quotient so obtained as a trial root value and complete the cube based upon it. If it proves too large reduce the unit value by 1. Complete the cube, subtract from the last dividend and bring down the next

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period. Continue this process until the periods are exhausted, or the desired number of decimal places in the root have been obtained.

EXERCISE.

Find the cube root of:

1. 941,192.
2. 9.300746727.
3. 147,197,152.
4. 25 (4 dec. pl.).
5. 1,083,201,548.
6. 4/5, by reduction to decimal.
7. 8.0378843 (4 dec. pl.).
8. 1007.65 (5 dec. pl.).
9. .001205 (to 3 dec. pl.).
10. $\frac{729}{9261}$
11. $\frac{12,167}{10,095,813}$
12. .0000000678 (3 dec. pl.).

GRAPHIC ILLUSTRATIONS OF SQUARES AND CUBES.

NOTE. If deemed advisable, this section may be taken after the chapter on practical measurements.

1. SQUARES AND SQUARE ROOT.

It has been shown numerically (page 123) that the square of any number composed of tens and units is equal to the square of the tens plus twice the product of the tens multiplied by the units plus the square of the units. This may be shown graphically, as follows:

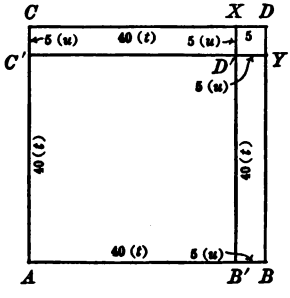

ILLUSTRATION. Compute the square of 45.

Solutions.

(1)	(2)
40+5	45
40+5	45
$5\times40+5^2$	225
$40^2+\quad 5\times40+$	180
$40^2+2\times5\times40+5^2=2025$	2025

(3) To any scale, lay off 40, and then 5, in one straight line. The square on this line, $ABCD$, is evidently the square of 45. Construct the square of 40, $AB'C'D'$, extending the sides meeting at D', to their intersection with the square of 45. This divides the square, as shown, into a square of 40, two rectangles measuring 40×5, and a small square of 5. The square of 45 plainly contains the square of the tens (40), the square of the units

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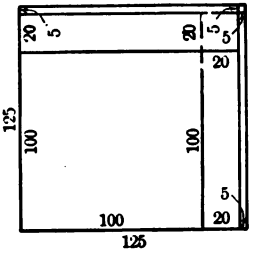
(5) and twice the product of units and tens ($2\times40\times5$). But 45 might have been any number of t tens and u units. Substituting t and u for 40 and 5 respectively, it is evident, from the figure, that the square of $t+u = t^2+2tu+u^2$.

In a number containing more than two places, as 125, the principle still holds. $125=100+20+5$. As shown by the diagram, the square of $100+20 = 100^2+2\times100\times20+20^2$. Now assuming 120 as twelve tens, the square of 125 is evidently, $120^2+2\times5\times120+5^2$. It is evident that each square, from the inmost, becomes in turn the square of the "tens" for the determining of the square of the "tens plus units."



In the reverse process of obtaining the square root of a number (see page 124) it should be evident, now, that in taking as the highest figure of the root the square root of the greatest square contained in the left hand group of two figures, one obtains the edge of the innermost square. Considering this as the "tens" value, the next place is computed as the "unit," thus obtaining the next larger square. Then this square is taken in turn as the square of the "tens" and a new "tens plus units" square is obtained, and so on.

In the process of computing the square root, at each step, after the square of the known "tens" is subtracted from the total square, the remainder equals $2\times$tens$\times$units plus the square of the units. The new unit is unknown, but, by dividing the remaining area by the known factor of the large term, "2 tens," as a trial divisor, there is obtained an approach to the exact unit. This unit is then tested to see if it is exact.

2. Cubes and Cube Roots.

It has been shown numerically (page 126) that the cube of any two-place number equals the cube of the tens plus three times the product of the square of the tens multiplied by the units, plus three times the product of the tens multiplied by the square of the units, plus the cube of the units. As in the case of squares, this may be shown graphically as well as numerically.

Illustration. Compute the cube of 15.

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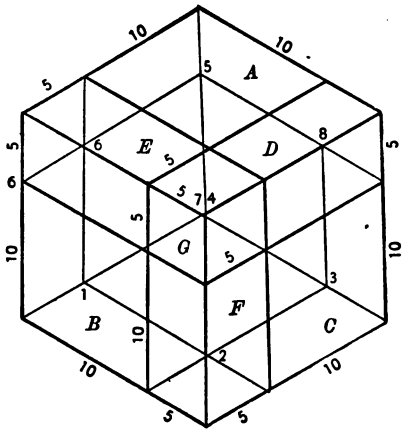
Solutions.

(1)	(2)
15	$10+5$
15	$10+5$
75	$5\times10+5^2$
15	$10^2+5\times10$
225	$10^2+2\times5\times10+5^2$
15	$10+5$
1125	$5\times10^2+2\times5^2\times10+5^3$
225	$10^3+2\times5\times10^2+5^2\times10$
$3375=10^3+3\times5\times10^2+3\times5^2\times10+5^3$	

(3) To any scale, construct a cube of one "ten" edge. Extending the edges meeting at one point, construct over the cube, a second cube of an edge equal to one "ten" and five "units." Assume the faces of the smaller cube extended to cut the faces of the larger. A study of the subdivided figure will show that the entire cube is made up of a cube of 10 edge; three rectangular solids, *A*, *B* and *C*, each measuring $10\times10\times5$; three rectangular solids, *D*, *E* and *F*, measuring $10\times5\times5$; and a cube, *G*, of 5 edge. Thus, as in the numerical statement, the cube contains the cube of the tens, plus three solids whose contents equal the square of the tens multiplied by the units ($10\times10\times5$) plus three solids whose contents equal the product of the tens multiplied by the square of the units ($10\times5\times5$) plus the cube of the units (5^3). Since this is general for two place numbers, or for numbers treated as tens and units, the cube, in general, will equal $t^3+3t^2u+3tu^2+u^3$.



Original cube 1, 2, 3, 4, 5, 6, 7, 8 shown by the *light* lines.

In reconstructing this cube from the known cube of the tens, it is evident that there must be added to the cube of 10, 3375 − 1000, or 2375, cubic units. These additions must be in the form of three solids having an outer surface area of 10×10, or 100 square units; three solids having an outer surface length of 10 and for width the unit value still unknown; and a cube having the unknown unit for an edge. All these additions have a uniform thick-

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ness equal to the desired unit of the root. Since the numerical value of the cubic contents, when divided by the numerical value of surface area, gives a linear measure as a quotient, the trial process in finding the second and succeeding figures in a cube root consists in dividing the cubic contents, remaining after the cube of the known root is subtracted by the surface area of the new additions, so far as known. The known part is $3t^2+3t$. Since this does not represent the complete outer surface area of the addition, it is used as a trial divisor, and the resulting quotient is tested to see if it is exact, or too large or small. It is tested of course in the formula $t^3+3t^2u+3tu^2+u^3$.

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CHAPTER XXII.

DENOMINATE NUMBERS.

Standard Measures.

The modern systematizing of business, and the simplifying of methods of handling merchandise, have had a marked effect on our systems of measures. It is rarely, now, that commercial quantities are expressed in several denominations. Instead, some measure of a table is selected as a unit or standard and quantities are expressed as multiples, decimals, or simple fractions of that unit. The unit varies for different trades, professions, or different classes of merchandise. Thus the civil engineer measures by units of 1000 ft., 100 ft., 10 ft., tenths of a foot, etc.; the mechanic, in inches and decimals. The contractor often estimates in cubic yards and decimals. Scales are now made to weigh in tons and decimals, and clocks to mark time in hours and hundredths. The grocer uses fractional pounds rather than ounces, as the dry goods dealer uses the yard and its fractions. Many milk dealers use the pint as a unit. Grain, and some vegetables, may be measured by the pound unit instead of by dry measure. Practically all forms of statistics use a decimal system based on a single unit.

The tendency toward unit measures is aided and reflected by the increasing sale of merchandise in unit packages, and by the increasing use of automatic weighing and computing machines, practically all of which work on a decimal basis.

The simplicity in actual use is reflected, naturally, in computations. Except in computations of time and English money, it is seldom necessary to handle three denominations of any table.

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Tables of Measures.

This section includes the standard tables in general use, and a few units and special values common to certain type businesses.

MEASURES OF CAPACITY.

Liquid Measure.

4 gills (gi.) = 1 pint (pt.).
2 pints = 1 quart (qt.).
4 quarts = 1 gallon (gal.).
. . .
1 gallon = 231 cu. in.

Barrels and hogsheads vary in size. For general estimates.
31½ gal. = 1 barrel (bbl.).
63 gal. = 1 hogshead.

Dry Measure.

2 pints = 1 quart.
8 quarts = 1 peck (pk.).
4 pecks = 1 bushel (bu.).
1 dry qt. = 67.2 cu. in.
1 bushel = 2150.42 cu. in.

A heaped bushel, used for corn in the ear, apples, etc., contains 2747.71 cu. in.

MEASURES OF WEIGHT.

Avoirdupois Weight.

16 ounces (oz.) = 1 pound (lb).
100 pounds = 1 hundred-weight (cwt.).
2000 pounds or 20 hundred weight = 1 ton (T.).

In measuring mining products and in custom house business, the *long ton* of 2240 lb. is used.

Troy Weight.

24 grains (gr.) = 1 penny-weight (pwt.).
20 pennyweights = 1 ounce.
12 ounces = 1 pound.

Troy weight is used in the measurement of the precious metals. Jewels, such as diamonds and pearls, are weighed by the carat which = 3.2 Troy grains. The term "carat" is also used to express the proportion of pure gold in composition metals. If pure gold, the metal is "24 carats fine." If one-half alloy, it is "12 carats fine," etc.

Comparative Table.

Troy and Avoirdupois Weight.
1 oz. Avoir. = 437½ gr.
1 lb. " = 7000 gr.
1 oz. Troy = 480 gr.
1 lb. " = 5760 gr.

Apothecaries' Weight.

20 grains = 1 scruple (sc. or ℈).
3 scruples = 1 dram (dr. or ʒ).
8 drams = 1 ounce (oz. or ʒ).
12 ounces or 5760 gr. = 1 pound (lb.).

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Special Unit Weights—Avoirdupois.

1 bbl. flour	= 196 lb.	In most states:	
1 bbl. salt	= 280 lb.	1 bu. potatoes	= 60 lb.
1 bbl. pork	= 200 lb.	1 bu. wheat	= 60 lb.
1 cu. ft. fresh water	= 62½ lb.	1 bu. corn	= 56 lb.
		1 bu. oats	= 32 lb.

COUNTING.

12 things = 1 dozen (doz.).
12 dozen = 1 gross (gro.).
12 gross = 1 great gross (g. gr.).
20 things = 1 score.

24 sheets (paper) = 1 quire (qr.).
20 quires, or
480 sheets = 1 ream (rm.).
(Commercially, 500 sheets are often used for a ream.)

MEASURES OF EXTENSION.

Linear or Long Measure.

12 inches (in.) = 1 foot (ft.).
3 feet = 1 yard (yd.).
5½ yards, or
16½ feet = 1 rod (rd.).
320 rods, or
5280 feet = 1 mile (mi.).

Sea Measure.

6 feet = 1 fathom.
120 fathoms = 1 cable length.
6086.7 feet, or about
1.15 miles = 1 knot, nautical or geographical mile.
3 knots = 1 league.

Surveyor's Long Measure.

7.92 inches = 1 link (lk.).
25 links = 1 rod.
4 rods, or
100 links = 1 chain (ch.).
80 chains = 1 mile.

City property is measured in feet and decimals thereof; large tracts of farm, or unimproved land by surveyor's measure.

Circular or Angular Measure.

60 seconds (″) = 1 minute (′).
60 minutes = 1 degree (°).
1 minute = 1 geographic mile.
90 degrees = 1 right angle.
360 degrees = 1 circumference.

This measure is used in geography, navigation and higher surveying, for the computation of differences of time, locations on the earth's surface, longitude and latitude; and for the measurement of angles.

Square Measure.

144 square inches (sq. in.) = 1 square foot (sq. ft.)

Surveyor's Square Measure.

625 square links = 1 square rod.
10 square rods = 1 square chain.

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Square Measure.

9 square feet = 1 square yard.
30¼ square yards = 1 square rod.
160 square rods = 1 acre (A.).
640 acres = 1 square mile.
100 square feet = 1 square.

Surveyor's Square Measure.

10 square chains = 1 acre.
640 acres = 1 square mile.
36 square miles = 1 township.

Cubic Measure.

1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.)
27 cubic feet = 1 cubic yard.
16 cubic feet = 1 cord foot (cd. ft.).
128 cubic feet or 8 cord feet = 1 cord (wood).
1 cubic yard = 1 load (earth).

MEASURE OF TIME.

60 seconds (sec.) = 1 minute (min.).
60 minutes = 1 hour (hr.).
24 hours = 1 day.
7 days = 1 week (wk.).
30 days = 1 commercial month.
12 months (mo.) = 1 year (yr.).
360 days = 1 commercial year.
365 days = 1 common year.
366 days = 1 leap year.
10 years = 1 decade.
100 years = 1 century.

365 days, 5 hours, 48 minutes, 49.7 seconds = 1 solar year.

Centennial years divisible by 400 and other years divisible by 4 are leap years.

MEASURES OF VALUE.

U. S. Money.

See pages 50 and 53.

Canadian Money.

The same as for the United States.
100 cents = 1 dollar = $1.

English Money.

4 farthings (far.) = 1 penny (d.).
12 pence = 1 shilling (s.) = $0.243 +.
20 shillings = 1 pound sterling (£) = $4.8665.

French Money.

100 centimes = 1 franc = $0.193.

German Money.

100 Pfennigs = 1 Mark = $0.238.

Mexican Money.

100 centavos = 1 peso = $0.498.

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DENOMINATE NUMBERS.

INTRODUCTORY EXERCISE.

1. Give illustration of abstract numbers, concrete numbers, like and unlike numbers, denominate numbers, simple and compound numbers. (See pp. 1 and 2.)

2. Name simple denominate numbers expressing: time, distance, area, value, weight.

3. Name compound denominate numbers expressing: cubic contents, liquid capacity, area.

4. Name a very common denominate measure used in each of these trades, businesses or professions: grocery, provision, oil, produce, dry goods, plumbing, mechanical engineering, contracting, banking, jewelry, coal and wood.

5. Name common denominate measures used in speaking of such things as—

(1) The distance from New York to Chicago.
(2) The distance across the street.
(3) The quantity of a meat order.
(4) The time required to do an errand.
(5) The amount of imports of the United States for last year.

6. Name the common measures used in your household and its trading.

7. What is the advantage of buying by some measure or fraction of it, over buying by two or more denominations of a thing?

8. What are the advantages to the retailer in selling goods in denominate packages, rather than in bulk?

It is evident from the tables that a number expressed in one denomination may be expressed in another denomination of the same table, and sometimes of other tables, without altering its value. The process of changing the form of expression of a quantity is termed *reduction ascending*, if the denomination chosen is larger than the original, and *reduction descending*, if it is smaller.

REDUCTION DESCENDING.

ILLUSTRATIONS. (1) Reduce 10 rds. 3 yd. to feet.

Analysis and Solution. (*a*) 1 rd. = 5.5 yd. 10 rd. = 55 yd. 55 yd. + 3 yd. = 58 yd. 1 yd. = 3 ft. 58 yd. = 3 × 58 ft. or 174 ft.

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(*b*) 1 rd. = 16.5 ft. 10 rd. = 10 × 16.5 ft. or 165 ft. 1 yd = 3 ft. 3 yd. = 3 × 3 or 9 ft. 165 ft. + 9 ft. = 174 ft.

(2) Reduce .45 gal. to lower denominations.

Analysis and Solution.

1 gal. = 4 qt. .45 of 4 qt. = 1.8 qt.
1 qt. = 2 pt. .8 of 2 pt. = 1.6 pt.
1 pt. = 4 gi. .6 of 4 gi. = 2.4 gi.
∴ .45 gal. = 1 qt. 1 pt. 2.4 gi.

ORAL EXERCISE.

Find missing values:

1. 3 gal. = —— qt.
2. 2 cd. = —— cu. ft.
3. 2 sh. 6 d. = —— d.
4. 2½ gal. = —— pt.
5. 3/4 bu. = —— pt.
6. 2 m. = —— rd.
7. 2 bu. = —— pt.
8. 1 gr. 3 doz. = —— units.
9. .4 pk. = —— qt.
10. 3.5 sq. yd. = —— sq. ft.
11. 14′ 6″ = —— ″.
12. 6½ score = —— units.
13. 1 mi. 40 rd. = —— rd.
14. 1.3 T. = —— lb.
15. £2.25 = —— sh.

EXERCISE.

Reduce:

1. 7.3 cu. yd. to cu. in.
2. 3⅝ T. to oz.
3. 5.6 gal. to gills.
4. 1.375 sq. mi. to A.
5. 7.475 mi. to ft.
6. Find the cost of 5¾ A. @ 5c per sq. ft.
7. What will it cost to excavate 942.5 cu. yd. @ 15c per cu. ft.?
8. Express 2 mi. in equivalent engineer's unit of 1000 ft.
9. Reduce 11 gt. gross to a fraction of 1000 units.
10. What part of 1 rd. are 6 in.?
11. What part of 1 mi. are 2 yd.?

REDUCTION ASCENDING.

ILLUSTRATIONS. (1) Express 729 ft. in higher denominations.

Solution. Since 3′ = 1 yd.
1 rd. = 5½ ft.
(mult. by .2) or
729 ft. = 44 rd. 2 yd.

3)729
5.5)243 = no. yd.
11)486
44 = no. rd.
2 yd. rem.

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(2) Express 2 ft. 9 in. as a decimal of a yard.

Solution. ∵ 2 ft. 9 in. = 33 in. and 1 yd. = 36 in.
∴ 2 ft. 9 in. = 33/36 of a yd. 33 ÷ 36 = .916 +.
∴ 2 ft. 9 in. = .916+ yd.

ORAL EXERCISE.

Reduce to the next higher denomination:

1.	52 ft.	**3.**	12 yd.	**5.**	65 in.	**7.**	8½ sh.
2.	27½ pt. (dry).	**4.**	27,000 lb.	**6.**	148 oz.	**8.**	50 cu. ft.

Find the cost of:

9. 2 lb. @ 6c per oz.
10. 5 qt. @ 8c per pt.
11. 4 yd. @ 5c per in.
12. 2 bu. @ 30c per pk.
13. 3½ gal. @ 5c per pt.
14. 3 pk. @ 60c per bu.
15. 40 ft. @ 30c per yd.
16. 900 lb. @ $36 per T.

EXERCISE.

(Decimals to 4 places.)

1. Express as decimals of a mile: 1 rd.; 1 ft.; 1 yd.; 7.35 in.; 8.4 yd.

2. Express as decimals of a bu.: 1 pk.; 3 qts.; 1 pt.; 3½ pt.

3. Express as decimals of a cu. yd.: 1 cu. ft.; 20 cu. in.

4. Express as decimals of a £: 1 sh.; 1 d.; 7 sh. 6 d.; £ 3 8 sh. 5 d.

5. Express as decimals of a T.: 11 lb.; 1 oz.; 745 lb.; 212 lb. 8 oz.

6. Express as decimals of an acre: 1 sq. rd.; sq. yd.; 96 sq. ft.; 45 sq. yd.

7. Reduce to higher denominations: 326,425 in.; 8429 ft. (dry); 11,927 cu. in.

8. What quantity (gallons) of olive oil must be bought to fill 90 gross of pint bottles?

THE FUNDAMENTAL PROCESSES. The fundamental processes, as applied to denominate or compound numbers, differ relatively little from the same processes in simple numbers. The main point of difference is in the substitution of an irregular reduction scale for the decimal scale.

ADDITION.

ILLUSTRATION. (1) Add 567 and 329. (2) Add 5 bu. 3 pk. 5 qt. and 3 bu. 2 pk 4 qt.

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Solutions.

(1)	(2)	(2) Condensed.
567	5 bu. 3 pk. 5 qt.	5 bu. 3 pk. 5 qt.
329	3 2 4	3 2 4
896	8 bu. 5 pk. 9 qt.	9 bu. 2 pk. 1 qt.
	or 9 bu. 2 pk. 1 qt.	(Reduction ascending.)

NOTE. (2) In the condensed form the reduction is performed mentally, as the total of each measure is obtained. Thus the total of the quart column is 9 qt. Since 9 qt. = 1 pk. 1 qt., the 1 quart is written in the sum, and the 1 peck is carried forward to the next column.

ORAL EXERCISE.

Find the sum of:

1.	2.	3.
5 gal. 2 qt. 1 pt.	6 bu. 1 pk.	1 T. 12 cwt.
2 1 1 1 g.	3 5 qt.	3 9

4.	5.	6.
15 cwt. 84 lb.	7 mi. 46 rd.	6 sq. yd. 5 sq. ft.
7 19	2 300	1 1
		2 8

7. To 3 gal. add consecutive pints.

EXERCISE.

1. Find the total if the items of a purchase from a London merchant are: £5 3 s. 6 d.; £ 18 11 s. 4 d.; £12 5 s. 8 d.

2. In St. Clair County the following stretches of country road have been macadamized in the past year. 1 mi. 160 rd., 2 mi. 145 rd., 8 mi. 176 rd., 1 mi. 46½ rd., a total of ?.

3. A lumberman, who already owns stumpage rights on 3 sq. mi. 260 A., buys rights to neighboring tracts of 84 A., 1 sq. mi. 52 A., 526 A. 40 sq. rd., and 64 A. 120 sq. rd. He then has rights to ? sq. mi. ? A. or to ? A. For the rights to lumber these new tracts, he paid $85 per A. or $? in all.

SUBTRACTION.

Subtraction involves reduction descending.

ILLUSTRATIONS.	(1)	(2)
	5 bu. 3 pk. 6 qt.	5 T. 7 cwt. 16 lb.
	2 1 3	2 5 46
Differences	3 bu. 2 pk. 3 qt.	3 T. 1 cwt. 70 lb.

Reduce 1 cwt. of the minuend to pounds. 116 lb. − 46 lb. = 70 lb.; 6 cwt. − 5 cwt. = 1 cwt., etc.

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ORAL EXERCISE.

Find the difference between:

1.	2.	3.	4.
5 bu. 3 pk.	5 gal. 2 qt.	16 cwt. 18 lb.	5 lb. 11 oz.
1 2	2 3	4 20	3 2

Find the difference between the larger and smaller quantity in ex. 1–5 of the last oral exercise.

EXERCISE.

1. What *check* can you suggest for compound addition and subtraction? Illustrate with examples.

2. On a contract for the laying of 752 cu. yd. of masonry, 15 cu. yd. 7 cu. ft. are laid one day, 31 cu. yd. 14 cu. ft. the second day, 68 cu. yd. 8 cu. ft. the third and 88 cu. yd. 11 cu. ft. the fourth. How much work remains to be done?

3. From 15.0739 mi. subtract 2 mi. 126 rd. 2 yd.

4. Make the extensions in this form:

	Gross Cost.	Discount.	Net cost.
	£ 16—5— 8	£ 2— 6—4	£ ———
	4—9— 3	8—7	———
	84—7—11	3—15—8	———
Totals	?	?	?

MULTIPLICATION.

That no new principle is involved is seen from this

ILLUSTRATION. Find the product of 3 bu. 2 pk. 1 qt. by 8.

Solution.	*Analysis.*
3 bu. 2 pk. 1 qt.	8×1 qt. $= 8$ qt. $= 1$ pk.
8	8×2 pk. $+ 1$ pk. $= 17$ pk. $= 4$ bu. 1 pk.
28 bu. 1 pk. 0 qt.	8×3 bu. $+ 4$ bu. $= 28$ bu.

EXERCISE.

(Solve mentally if possible.)

Multiply:

1. 3 ft. 2 in. by 6.

2. 1 yd. 2 ft. by 8.

3. 3 lb. 2 oz. by 30.

4. 2 pk. 3 qt. by 12.

5. 2 qt. 1 pt. by 8.

6. 3 rd. 27 links by 9.

7. 3 cwt. 60 lb. by 10.

8. 1 T. 4 cwt. by 20.

9. How much silver is required for five ornaments each containing 1 oz. 7 pwt.?

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10. A factory manager reduces the length of a metal bar in a machine he manufactures $3\frac{1}{2}$ in. This means a saving of how much metal in 12,000 bars?

11. Determine the cost of 75 yd. extra broadcloth, purchased at £1 2 s. 3 d. per yd.

12. A factory expert discovers that the length of a bolt used in the company's product may be reduced 1/4 in. This means a saving of how many feet if 250,000 bolts are used per year?

13. Compute the cost of a car load containing 60 bbl. of apples, each containing 2 bu. 2 pk., at 75c per pk.

DIVISION.

In division of denominate numbers the divisor may be either abstract or denominate.

ILLUSTRATIONS. (1) Compute the cost, per yd., of 80 yd. cloth invoiced at £180.

Solution (a).

80)£180
£ 2.25 = £2 5 s. (cost)

Solution and Analysis (b).

The cost per yd. is evidently 1/80 of £180. 1/80 of £180 = £2 and a remainder of £20. £20 = 400 s. 1/80 of 400 s. = 5 s. ∴ 1/80 of £180 = £2 5 s.

(2) At a price of £1 2 s. per yd., how many yards may be purchased for £89 2 s. ?

Solution.

£ 1 2 s. = 22 s.
£89 2 s. = 1782 s.
81 = no. of yd.
22)1782

The numbers being concrete, reduction to a common denominator is necessary.

ORAL EXERCISE.

Divide:

1. 6 bu. 3 pk. by 3.
2. 5 gal. 2 qt. by 2.
3. 3 T. 6 cwt. by 200.
4. 4 lb. by 8.
5. 3 pk. 6 qt. by 10.
6. 5 yd. 1 ft. by 4.
7. 3 sq. yd. 3 sq. ft. by 6.
8. 3 gal. by 2 qt.
9. 4 lb. by 8 oz.
10. 2 ft. 6 in. by 3 in.
11. 5 bu. 1 pk. by 1 bu. 3 pk.

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EXERCISE.

1. Divide 960 cu. yd. 5.4 cu. ft. by 11.5, checking by multiplication.

2. A carriage wheel revolves two times in 7 yards, or —— times in one mile.

3. Divide 5 bu. 7 pk. 3 qt. by 1 pk. 2 qt. 1 pt. Check the result by multiplication.

4. A dealer has 2 bbl., 31½ gal. each, of olive oil, to be bottled in half-pint bottles. He requires — bottles.

5. A newspaper press printed 57,384 copies in 2 hr. 5 min., at the average rate of — copies per minute.

Individual Original Work.

Report on "Common Measures Used in Business." Disregard text-book classifications, and find out what parts of tables are in actual common use. Classify under: (1) Common to business in general; (2) limited to particular trades; (3) non-text-book measures. Give illustrative examples.

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CHAPTER XXIII.

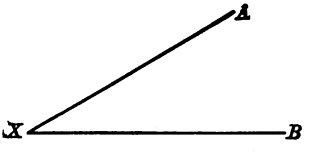
PRACTICAL MEASUREMENTS.

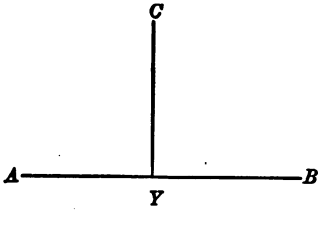
Many simple measurements in business and science are based on elementary geometric principles. A slight knowledge of some of these principles, and of the geometric figures to which they apply, is of value to everyone.

GEOMETRIC CONCEPTIONS—PLANE FIGURES.

A *geometric line* is considered to have extension but neither length nor breadth. A straight line is the shortest distance between two points. Lines are *parallel* if they are the same distance apart throughout their entire length.

An *angle* is the divergence of two lines having a common point. AXB is the angle of the lines AX and XB. If the





two lines meet so that the two angles on the same side of one line are equal, the angles are *right angles*, and the lines are *perpendicular*. Thus AYC and BYC are right angles and CY is perpendicular to AB. An *acute angle* is less than a right angle. An *obtuse angle* is greater than a right angle.

A *geometrical surface* has length and breadth but no thickness. A *plane*, or *plane surface*, is a level surface, such as that of still water. A figure in a plane is a *plane figure*.

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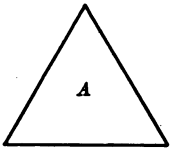
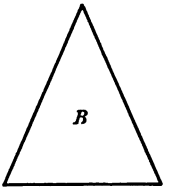
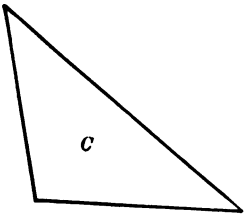
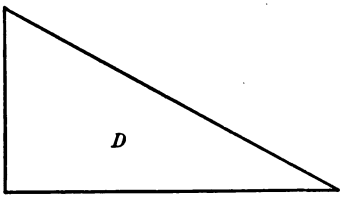
A *triangle* is a plane figure bounded by three straight lines. It is called *equilateral, isosceles,* or *scalene,* according as it has



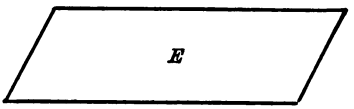


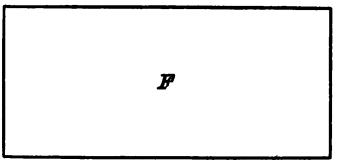
three sides equal (*A*), two sides equal (*B*), or no sides equal (*C*). A right angled triangle has one right angle (*D*).





A *quadrilateral* is a plane figure bounded by four straight lines. If the opposite sides of a quadrilateral are parallel, the figure is called a *parallelogram* (*E–H*). The parallelogram is a *rectangle* if it has four right angles (*F*). It is a *square* if the





sides are equal and the angles right angles (*G*). It is a rhombus, if the four sides are equal but the angles are not right angles (*H*).

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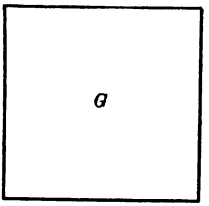
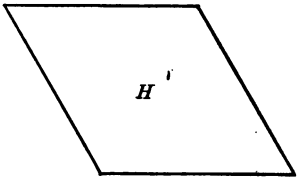
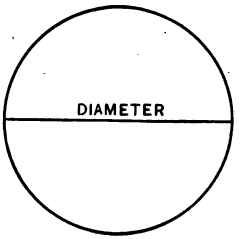




A *circle* is a plane figure bounded by a curved line, called the *circumference*, every point of which is equidistant from a point within called the *center*. The *diameter* is any straight line passing through the center and terminating in the circumference. The *radius* is one-half of the diameter. An *arc* is any part of the circumference, and is measured by degrees of angular measure.



The circumference is approximately 3.1416 (approximately $3\frac{1}{7}$) times the diameter. This factor is called by the Greek letter "π," (pronounced, "pi.") If the diameter is known, the circumference may be determined by multiplying it by 3.1416. If the circumference is known, the diameter is determined by finding the quotient of the circumference divided by 3.1416.

The *perimeter* of a plane figure is the distance around it. The *base* is the side on which it is assumed to rest. The *altitude* is the perpendicular distance from the base to the most distant point of the figure.

EXERCISE.

1. Draw four circles of 4″, 5″, 6″, and 8″ radius, respectively. In each case, measure carefully the circumference and divide by the respective diameters. Take the average of the resulting quotients. How close to the value of "π" do you approach? By computation, find your error in measurement of each circumference. To what is this error due?

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2. Compare the diameters and circumferences of two circles, one of which has a diameter of 10′ and the other a circumference of 42′.

3. What is the diameter of a circular path a mile in circumference?

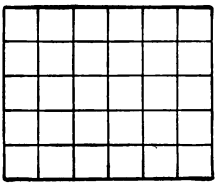
4. Compute the cost of the fencing around a circular park of 90 ft. radius, at $1.50 per foot.

5. In racing on a circular track, what gain may result from securing an *inside* position? Illustrate.

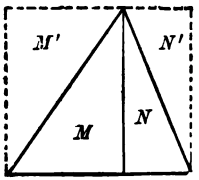
6. A man has 1000 ft. of movable fencing for a chicken run. Give dimensions of a square field, a rectangular field and a circular field that he may enclose.

Areas of Plane Figures.

The accompanying rectangle is divided by parallel lines into unit squares. How many unit squares in each row? How many rows? Which dimension shows the number of rows? Which the units per row? What is the total number of units? How obtained? Show that the number of square units would be the same if "5" were considered the base and "6" the altitude. It is evident that the *area* of the rectangle is equal to the product of its length and width—its two dimensions.



In the case of the parallelogram, by cutting off the section X and placing it in the position X', it is evident that the figure is equivalent in area to a rectangle having the same base and an equal altitude. Try this experimentally with a piece of paper.



In the case of the triangle, as shown by the figure, the area is equal to one-half the area of the rectangle erected on the same base, and with equal altitude. The area is equal, therefore, to one-half the product of its base and altitude.

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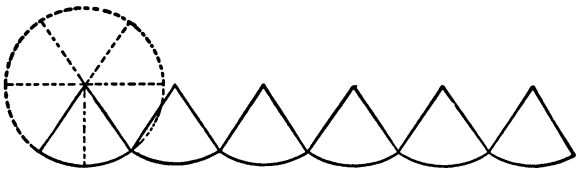
If a circle is divided as shown, and the parts rearranged as a series of approximate triangles, it is evident that the area will equal the sum of the areas of the triangles. If the triangles are made small enough they have altitudes equal to the radius of the circle. The sum of their bases equals the circumference



of the circle. The area of the circle, therefore, equals one-half the product of the radius by the circumference. Since the circumference equals $\pi \times$ the diameter ($2 \times$ radius) the area equals $\frac{1}{2}R \times 2R \times 3.1416$, or *3.1416* $\times$ *the square of the radius.*

The area of irregular right line figures may be obtained by dividing them into triangles and parallelograms and determine the areas of these parts.

ORAL EXERCISE.

Compute the areas of the following:

	Figure.	Altitude.	Base.
1.	Triangle	14 ft.	16 ft.
2.	Rectangle	24 ft.	2½ ft.
3.	Square	15 ft.	——
4.	Parallelogram	16 in.	2 ft.
5.	Rectangle	2½ ft.	4½ ft.
6.	Triangle	20 in.	1 ft. 6 in.
7.	Parallelogram	6½ ft.	2 ft. 3 in.

8. Find the difference between 3 sq. yd. and an area 3 yd. square.

9. How does a 5 in. square compare with an area of 5 sq. in.?

10. Name two factors of 360. Find two dimensions for a rectangle of 360 sq. ft. area. Find other dimensions for a second rectangle of the same

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area. How many rectangles have the same area? If one dimension is 4′ what is the other? If one dimension is 12 ft. what is the other?

11. A series of rectangles have a common area of 480 sq. ft. Find the bases if the altitudes are respectively 4, 16, 12, 6, 8, 15, 20, 40, 1/2 feet. Compare the perimeters. Do rectangles of the same area have the same perimeter?

12. Give the dimensions of several rectangles that have a common area of 1200 sq. in.

13. To what is the area of a triangle equal? If the area and one dimension are known, how is the other determined?

14. Find the altitudes of a series of triangles having a common area of 180 sq. ft. and bases of 4, 3, 20, 9, 18 and 30 ft., respectively.

15. Name the bases and altitudes of ten triangles that have a common area of 720 sq. in.

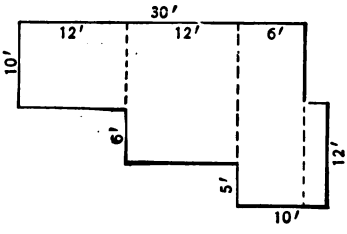
EXERCISE.

1. Determine the area of a floor measuring 25 ft. 4 in. by 16 ft. 8 in.

2. Determine the area of the end section of a metal bar measuring $3\frac{1}{4}$ in. by $1\frac{7}{8}$ in.

3. How many 3-in. squares may be cut from a piece of card board measuring 25 in. by 36 in.?

4. What is the acreage of a field measuring 720 rd. by 96 rd.?



5. A field measuring 88 rd. by 126 rd. produces 280 bu. of grain, an average of —— bu. per acre.

6. Subdivide the field represented by the accompanying diagram into rectangles, as shown by the dotted lines. Name the dimensions of each rectangle. Determine the total area. Check the computation by subdividing into another group of geometric figures and determining area.

7. How many oblongs, 2 in. by $3\frac{1}{2}$ in., can be cut from sheets of composition metal 24 in. by 36 in.? Does it make any difference which way the oblongs are laid off on the sheet?

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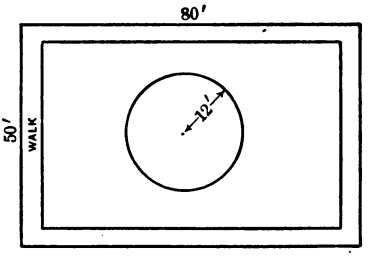
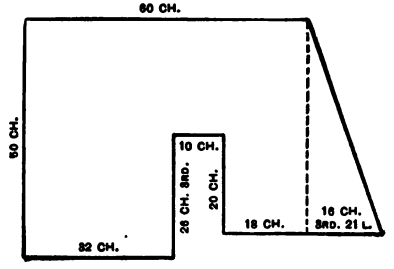
8. The diagram here shown represents a park with a circular flower bed. The walk is 5 ft. wide. Find the area of the flower bed. Find the number of square yards of sodding required for the parking. How is this area determined from the area of the rectangle and the area of the circle?



Compute the cost of constructing the walk, at $1.25 per sq. yd. Compute the cost of curbing for the inner and outer edges of the walk, at 38c per linear foot.

9. Find the acreage of the plot of land here represented. Express dimensions in linear measure. Check computation of acreage by using dimensions just found.

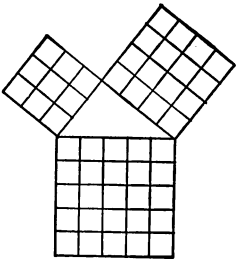


Applications of Square Root.

Square root frequently is employed, in connection with geometric figures, in determining missing dimensions.

1. *Squares.* Since the area of a square is equal to the product of its two equal sides, it is evident that the length of one side is equal to the square root of the area.

2. *Right Triangles.* It has been proved, geometrically, that the square of the *hypotenuse,* or side opposite the right angle, is equal to the sum of the squares of the other two sides. This is illustrated graphically in the drawing. It is evident, then, that the length of the hypotenuse is equal to the square root



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of the sum of the squares of the other two sides; and that the length of either other side is equal to the square root of the difference of the squares of the hypotenuse and the known side.

3. Since the area of a circle equals $\pi \times$ the square of the radius, it is evident that the radius equals the square root of the area divided by 3.1416.

EXERCISE.

1. Determine the side of a square having the same area as a rectangle measuring 54 rd. by 17.3 rd.

2. To have an area of 60,000 sq. ft., a square must have an edge of how many feet?

3. Find the edge of a square that has the same area as a circle of 50 ft. radius.

4. Determine the radius of a circle that has the same area as a square of 80 ft. side.

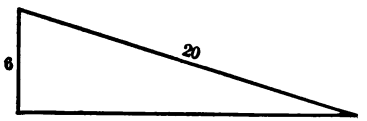
5. How many linear feet of fencing are required for a rectangular chicken run, 80 ft. by 120 ft.? What is the difference in amount of fencing required for a square run of the same area?

6. Compare the perimeters of a square, a rectangle of 20 ft. altitude, and a circle, if all have equal areas of 600 sq. ft.

7. If the two sides of a right triangle are 8 ft. and 16 ft., what is the length of the hypotenuse?

Determine the missing values in the figures shown below:

8.



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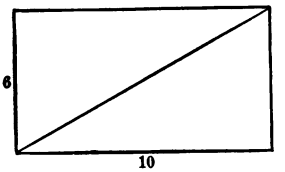

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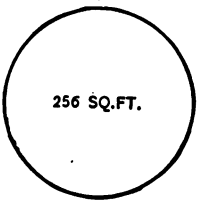

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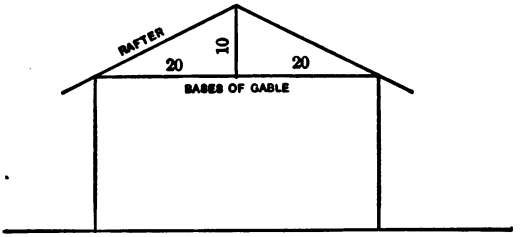
GABLES.

The *pitch* of a roof is the amount of rise of a rafter for each foot in the base of the gable.

ILLUSTRATION. In the cut, the rafter rises 10 ft. for a base of 20 ft. This is a pitch of 10/20, or 1/2. The familiar Gothic pitch is 5/8.



EXERCISE.

Find the missing values:

	Base Width.	Height.	Pitch.	Length of Rafter.	
1.	30 ft.	18 ft.	?	?	(Allow 1 ft. Overhang.)
2.	40 ft.	?	Gothic	?	" " "
3.	45 ft.	20 ft.	?	?	" 1½ ft. "
4.	?	16 ft.	1/4	?	" " "
5.	36 ft.	?	1/2	?	" " "

PRACTICAL APPLICATIONS OF SQUARE MEASURE.

The following applications are selected to illustrate the use of square measure as modified by business custom.

1. FLOORING.

Considerable structural work, such as flooring, roofing, tiling, etc., is measured frequently by the *square*, or 100 sq. ft. More rarely the measurement is expressed directly in sq. ft., or in units of 1000 sq. ft.

In properly matching and fitting tongue and groove flooring, there is considerable waste of material. Thus, for every foot

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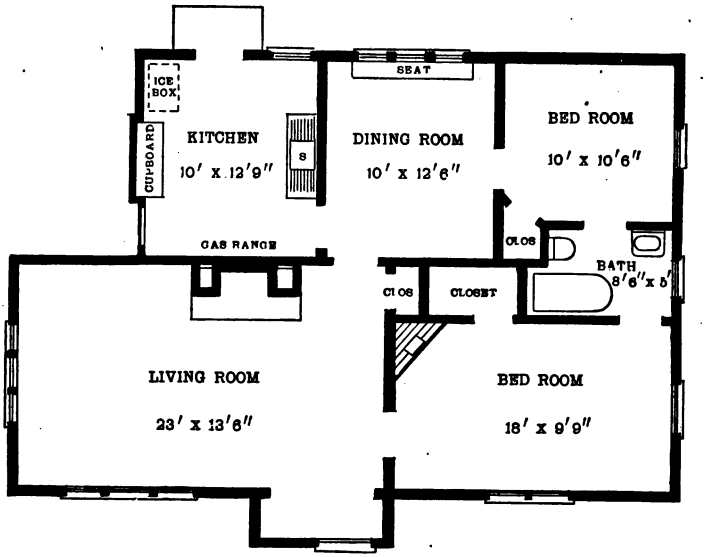
of flooring in place, $1\frac{1}{6}$ to $1\frac{1}{8}$ ft. of flooring material may be required.

EXERCISE.

1. Compute the cost of flooring a hall measuring 80′ by 35′, at $2.95 per square.

2. Compute the cost of flooring with hard wood, the two front rooms shown in this floor plan of a bungalow, at $40 per M, allowing 1/3 waste, and $10.50 for extras.



3. How many tile, 6″ square, are required for the right rear room, allowing 1/20 extra for breakage?

4. A gable roof, 42′ long, and having rafters 21′ long, is to be covered with roofing tile. At the rate of 300 per square, how many tile are required? How many shingles are required for the same purpose, at the rate of 800 per square? How many bundles of shingles, 250 each?

NOTE. Carpenters and builders have tables showing the numbers of tile, shingles, etc., of different sizes, required per square. These are often used as a basis of computation in practical work.

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2. Plastering.

Plastering is commonly measured by the square yard. Allowances for openings are made in various ways, such as (1) by actual measurement, (2) by arbitrary allowance, etc. If working plans for a building are not given, dimensions of rooms are usually stated in this order: Length—width—height.

Example: $21' \times 16' \times 9'\ 6''$.

EXERCISE.

1. Compute the cost of hard finishing the walls and ceiling of the two front rooms of the building, page 152, at 43c per sq. yd. Height of room 9'. Allow 20 sq. ft. for door openings and 18 sq. ft. for windows.

2. Compute the cost of rough finishing the bedroom and dining room at 26c. per sq. yd., allowing by actual measurement for the doors, which are 7' by 4', and for windows, which are 6' by $4\frac{1}{2}$'.

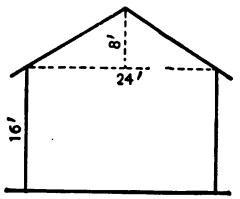
3. Painting.

Painting is usually measured by the square yard, no allowance being made for windows or openings of a similar nature.

EXERCISE.

1. Compute the cost of painting a fence, 6' high, around the lot shown on page 148, both sides, at a cost of 24c per sq. yd.

2. Compute the cost of painting with two coats the walls of a building 48' long, and having this end section. Price for double coat, 45c per sq. yd.



4. Papering.

Although dimensions vary, the width of wall paper is usually assumed to be 18 inches. The length is 8 yards for single rolls and 16 yards for double rolls. Complete rolls must be bought.

Allowance is made for openings in various ways, such as

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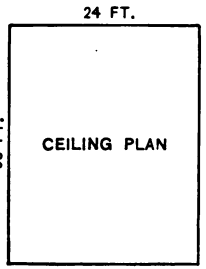
by (1) exact measurement; (2) uniform allowance for each opening; (3) allowance for width of opening and full height of room, the assumption being that remnants of rolls, after cutting full strips, will be sufficient to paper the wall over and under openings; and (4) an arbitrary allowance according to area, as "one roll to 24 sq. ft. regardless of openings."

ORAL EXERCISE.

1. If paper is laid on the ceiling represented in the accompanying illustration parallel to the shorter dimension, what is the length of each strip in yards? How many strips can be cut from each double roll? How many strips are needed? How many rolls? What is the cost, at 45c per roll?



2. The walls of the room, the ceiling of which is shown, are also to be papered. What does the border cost, at 15c per yd? The strips of papering on the walls must be 8 ft. long. How many strips are required? How many strips can be cut from each double roll? How many rolls are needed? What is the cost, at 50c per roll?

It is evident that the simplest method of approximation is to divide the perimeter of the room, less the width of all openings, by the width of a strip, and to divide the number of strips by the number of complete strips in one roll, in order to determine the number of rolls.

EXERCISE.

1. A room 30′ by 28′ has five openings, each 2½′ wide, and one opening 4′ wide. It is 9′ from baseboard to ceiling. Find the cost of papering sides and ceiling, without border, at 55c per roll, allowing 6″ per strip for matching design.

2. Compute the quantity of ceiling, border and wall paper required for your class room.

3. Find the quantity of wall, ceiling and border paper required for each of the rooms shown in the diagram on page 152, assuming the wall height, allowing for border, as 9′, and allowing 6″ per strip for matching design. Assume width of doors 3′; of windows, 2½′.

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5. Carpeting.

Carpeting is measured by the *yard per length of strip*, regardless of width. Oil cloth, linoleum and carpet linings are sometimes measured by the square yard.

In order to compute the quantity of carpet for a room, it is necessary to determine the number of strips and their length, somewhat after the manner of the computations for papering. The number of strips depends on the direction in which the strips are laid. Allowance must be made, also, for matching design.

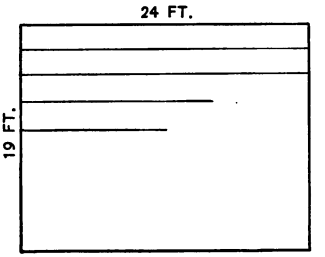

Illustration. Compute the quantity of carpet required for the room shown, if laid lengthwise, the width of the carpet being three feet.

Solution. $19 \div 3 = 6\frac{1}{3}$, no. of strips. Therefore, 7 strips are required. $24' =$ length of one strip $= 8$ yd. $7 \times 8 = 72$, no. of yd. required.

Note. Since each strip is 3′ wide, the total width of 19′ divided by 3 will give the number of strips, a full strip being purchased for each fraction of width.

EXERCISE.

1. Find the quantity of carpet required for the room shown above if laid across the shorter dimension?

2. Compute the cost of Brussels carpet, 27″ wide, for the above room, laid as shown, at $1.50 per sq. yd., allowing 6″ per strip, after the first strip, for matching. Why need no allowance be made for matching the first strip?

3. The above room is to be carpeted with Ingrain carpet, 1 yd. wide, laid lengthwise, and with a border around the room 2′ wide. Determine the quantity of border and carpet, allowing 1/20 for matching. Compute the number of square yards of lining required.

4. Find the cost of carpeting the dining room (page 152) with linoleum, $1\frac{1}{3}$ yd. wide, at $1.45 per sq. yd., allowing 2 sq. yd. for matching design.

5. Compute the cost of carpeting each of the rooms with Wilton carpet, 3/4 yd. wide, at $1.25 per yd. Allow 9″ for matching design, and allow for an 18″ border, at $1 35 per yd. Lay the carpet in the direction that will be more economical.

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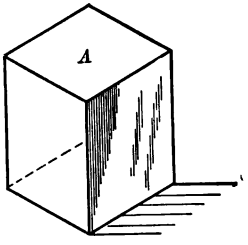
Geometric Conceptions—Solids.

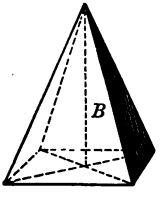
A *solid* is that which has three dimensions, length, width and thickness. It is bounded by surfaces.

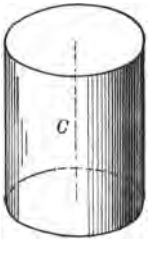


A *polyhedron* is a solid bounded by plane surfaces called *faces*. A polyhedron having two parallel and equal bases, and three or more sides which are parallelograms, is a *prism*.

A prism is a *rectangular solid* (*A*) if it is bounded by six rectangular surfaces. It is a *cube* if these surfaces are all squares.

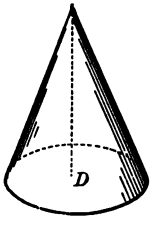
A *pyramid* (*B*) is a polyhedron whose faces, with one exception, meet in a common point called the vertex. The faces of a pyramid are therefore triangles.





A *cylinder* (*C*) is a solid bounded by two equal parallel circles and a uniformly curved surface.

A *conical surface* is a surface generated by the motion of a straight line that continually intersects a fixed curve, frequently a circle, and passed through a fixed point. A *cone* (*D*) is a solid bounded by a closed conical surface and a plane surface.



A *spherical* surface is a curved surface every

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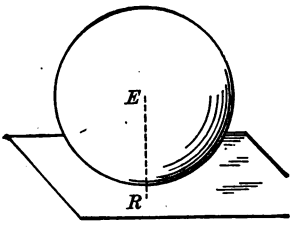
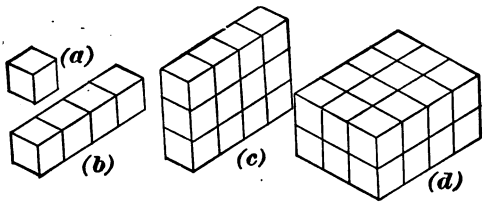
point of which is equidistant from a point called the *center*. A *sphere* (*E*) is a solid bounded by a spherical surface. A *hemisphere* is one-half of a sphere cut by a plane passing through the center.



INTRODUCTORY EXERCISE.

A cubic foot is a solid 1 foot long, 1 foot wide and 1 foot thick. How many cubic feet in figure (*b*) if each edge is 1 foot long? How many cubic feet in each horizontal row of (*c*)? How many rows? How many cubic feet in all? How many rows and units per row in one vertical slice of (*d*)? How many sections? How many units in all? What is the area of the end face? Of the top? What is the total area of all the faces?



It is evident that the volume of a rectangular solid is equal to the product of its three dimensions. These dimensions must ordinarily be expressed in units of the same denomination. The total surface area is equal, evidently, to the sum of the areas of the individual faces.

ORAL EXERCISE.

Compute the cubic contents, or volume, of:

1. A cube of 6″ edge.
2. A rectangular solid measuring 6′ by 5′ by 8′.
3. A rectangular solid measuring 8′ by 4½′ by 10′.
4. An excavation, 10′ by 20′ by 30′.
5. A piece of metal 6″ wide, 12″ thick, and 10′ long.

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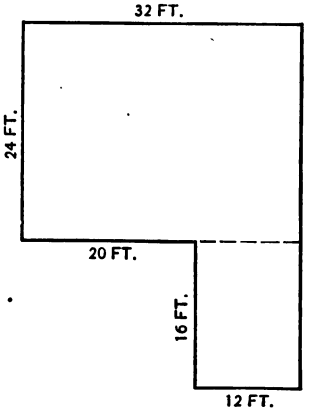
EXERCISE.

1. Compute the number of cubic yards of earth that must be excavated to sink this cellar to a depth of 8′.



NOTE. Find area and multiply by depth. Check, by dividing into separate prisms as shown by dotted lines. Why do these processes bring the same result?

2. Determine the number of cubic feet in a stone column 4½′ by 2′ by 18′.

3. Compute the cost of a masonry wall, 40 long, 6′ high and 2′ wide, at $4.36 per cu. yd.

4. Determine the cubic contents of a room 60′ by 25′ by 10′. Determine the surface area of walls, ceiling and floor.

5. A school room, 36′ by 30′ by 10′, seats 42 people. How many cubic feet of air space is allowed each person?

6. At the rate of 22 brick per cu. ft., compute the cost at $2 per M. of the bricks necessary for a wall 36′ long, 20′ high and 24″ wide for half the height and 18″ wide for the remaining height.

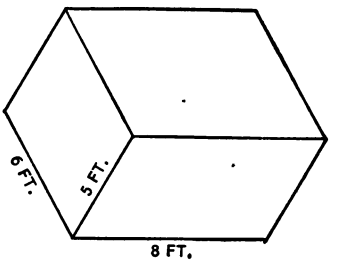
ORAL EXERCISE.

1. Find the cubic contents of this rectangular solid? What is the effect on the volume of doubling any one dimension?

2. If the volume of a rectangular solid is 180 cu. in., and the product of two dimensions is 30 sq. in., what is the third dimension?

3. If two dimensions are 8′ and 12′, what is the third dimension of a solid having a volume of 960 cu. ft.?

NOTE. It is evident that the third dimension may be determined by dividing the volume by the product of the two known dimensions.



EXERCISE.

1. 8′ × 4.5′ × ? = 720 cu. ft.

2. Select two dimensions to be used with a height of 9′, to determine

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a rectangular solid of 120 cu. ft. of volume. How many solutions are possible?

3. Name three dimensions of a solid of a cubic content of 5600 cu. in.

4. Determine the height of a room 30′ long and 20′ 6″ wide, if it is to have a cubic content of 5535 cu. ft.

The Cylinder and Sphere.

If the surface of a paper cylinder is cut along its length, and the paper rolled back, it will be seen to have the form of a rectangle whose height is the altitude of the cylinder and whose base is the length of the circumference of the base circle. Evidently, the *lateral area* is the product of the altitude by the circumference. To this must be added, to determine the entire area, the areas of the two base circles.

ORAL EXERCISE.

1. If the area of the base of a rectangular solid is 14 sq. ft., what is the volume per foot of height?

2. If the area of the circular base of a cylinder is 30 sq. in., what is the volume per foot of height? What is the volume if the height is 12 ft.?

As seems evident from the above, the volume of a cylinder equals the product of the altitude and the area of the base.

EXERCISE.

1. Allowing for 1½″ overlap, how much sheet metal is required for the lateral surface of a hollow cylinder of 16′ altitude and 3′ radius of base?

2. Determine the cubic contents of this cylinder.

3. What must be the altitude of a cylinder of 4′ diameter to have a cubic content of 600 cu. ft.?

4. How many cu. ft. of water will a circular cistern of 12′ depth and 5′ radius contain?

5. Determine reasonable dimensions for a cistern to contain 5000 cu. ft.

6. Metal pieces for the lateral surfaces of cans of 2″ radius and 6″ height are to be cut from sheet metal 24″ by 32″, no allowance being made for overlap? How many can be cut from each sheet?

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The surface of a sphere is equal to $4 \times \pi \times$ *the square of the radius;* and the volume of the sphere is equal to $4/3 \times \pi \times$ *the cube of the radius.*

EXERCISE.

1. Find the surfaces of spheres of radii (*a*) 2″; (*b*) 6″; (*c*) 21″.
2. Find the volume of spheres of radii (*a*) 5″; (*b*) 15″; (*c*) 26′ 6″.
3. Find the volume of a hemisphere of 4′ 6″ radius.
4. Find the radius of a sphere of 750 sq. in. of surface.
5. How many square inches of gilding must be done to cover a sphere of 16″ radius?

SPECIAL APPLICATIONS.

Cord Measure.

Fuel wood, tan bark, sometimes stone, etc., are measured by the *cord,* a unit pile 8′ long, 4′ wide and 4′ high, containing 128 cu. ft. In many localities, a cord of fuel is a pile 8′ long and 4′ high, the price varying with the width—as half sawed, quartered sawed, etc. It is evident that if a pile is 4′ high and 4′ wide, the number of cords may be determined from the length, as measured by the unit length of 8′. Thus a 36′ pile contains $4\frac{1}{2}$ cords. In the case of irregular dimensions, the measure is the number of cubic feet in a cord.

EXERCISE.

1. A wood lot produces piles of wood of regulation 4′ length (width), of dimensions as follows: 10′ long, 3′ high; 12′ long, 5′ high; 16′ long, 4′ high; 20′ long, 4′ high. How many cords are produced?
2. A pile of hemlock bark measures 80′ by 4′ by 21′ and contains how many cords?
3. Sixteen acres of woodland produce a pile of cord wood 12′ high and 47′ long. What is the product in cords per acre?
4. Five cords of wood, regulation size, are to be piled in a shed along a wall measuring 15′. How high should the pile be?

Lumber.

In the measurement of boards of less than 1″ in thickness, square measure of surface is used. In the measurement of

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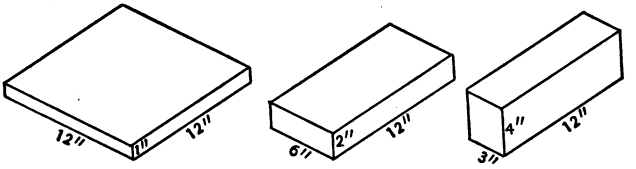
boards of 1″ thickness, or over, such as heavy boarding, planks, joists, scantling, and heavy dressed timber, the unit measure is the *board foot. The board foot is a board 1′ long, 12″ wide and 1″ thick, or its equivalent.* Since the end area is 12 sq. in., the board foot may be said to have a length of 1′ and an end area of 12 sq. in.



Board Feet. In measuring width, except in the more valuable woods, the next lower quarter inch of width is sometimes taken. Thus a $6\frac{7}{8}''$ width is reckoned $6\frac{3}{4}''$ and a $9\frac{1}{4}''$ width, 9″, etc.

In stating quantity and dimensions of an order, or estimate, the arrangement is: Number of pieces—thickness in inches—width in inches—length in feet, thus 30 pc., 6″ × 8″ —16′. The price is generally stated per M feet, board measure.

In computation, both on paper and orally, the method of cancellation is valuable.

ILLUSTRATION. Compute the number of feet, board measure, in 36 pc., $2\frac{1}{2}''$ × 10″ —16′.

Solution. $\frac{3}{\frac{2\frac{1}{2}\times 10\times 16\times \cancel{36}}{\cancel{12}}} = 1200$, no. bd. ft.
$\quad 1$

$\frac{2\frac{1}{2}\times 10}{12}$ = no. bd. ft. per ft. of length.
This no. × length × no. pieces = no. bd. ft.

Evidently the number of board feet *is the product of the width in inches, by the thickness in inches, by the length in feet, by the number of pieces, divided by 12.* The quantity to cancel is always 12. Notice that this is one of the rare cases where dimensions are not expressed in the same unit.

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ORAL EXERCISE.

Determine the number of board feet per foot of length in timbers of the following dimensions:

1.	13″×4″.	**4.**	2″×9″.	**7.**	3″×6″.	**10.**	3″×9″.
2.	8″×16″.	**5.**	2″×8″.	**8.**	2½″×6″.	**11.**	2″×3″.
3.	4″×12″.	**6.**	4″×6″.	**9.**	6″×9″.	**12.**	2″×2″.

Determine the number of board feet in these timbers:

13. 3″×4″—16′.
14. 4″×6″—12′.
15. 8″×10″—6′.
16. 4″×8″—12′.
17. 2″×9″—16′.
18. 3″×6″—12′.

Determine the board feet in these items:

19. 10 pc. 2″×6″—10′.
20. 20 pc., 3″×4″—12′.

EXERCISE.

Determine the number of feet, board measure, in:

1. 162 pc. hemlock, 8″×10″—32′.
2. 40 pc. white pine, 2″×10″—20′.
3. 360 pc. maple, 1½″×3″—18′.
4. 456 pc. oak, 3″×6″—18′.

At $26 per M., compute the cost of:

5. 150 joists, 2″×8″—16′.
6. 42 beams, 6″×9″—24′.
7. 78 scantling, 2″×4″—16′.

8. How much timber is required for a plain board fence around a lot 180′ by 246′, if it consists of two strips of 2″×4″ scantling on which is nailed 1″ boarding 6′ high. This does not include posts. Allow 1/8 for waste in cutting.

Capacity of Bins, Tanks, Etc.

In measuring the capacity of bins and tanks, it is often necessary to express the common dry and liquid measures in terms of cubic feet.

Illustrations. (1) Determine the capacity in gallons of a rectangular iron tank measuring 4′ by 8′ by 3′.

Solution. (Abstract.)

4′×8′×3′=96 cu. ft., capacity.

96×1728=capacity in cu. in.

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$\frac{96\times1728}{231}$ = no. of gal. (since 1 gal. = 231 cu. in.) = 718+ gal., capacity.

(2) What is the weight of the water the tank will hold?

Solution.

96 cu. ft. = contents.

1 cu. ft. of water weighs 62½ lb.

96 × 62½ lb. = 6,000 lb., the weight of water.

EXERCISE.

1. Determine the capacity in bushels of a rectangular bin measuring 8′ by 4′ by 4′. Determine the capacity, also, by approximate measure (1 bu. = 1¼ cu. ft.).

2. A bin in a certain barn can be allowed a floor space of only 4′ by 9′ 6″. What must be its height if it is to have a capacity of 250 bu.? (Approximate measure.)

3. Determine reasonable dimensions for a bin to contain 400 bu. of potatoes. Use 1 cu. ft. as equivalent to .63 of a heaped measure bushel.

4. In order to have a capacity of 1,000,000 gal., a reservoir must have a capacity of how many cu. ft.?

5. Compute the capacity, in gallons, of:

(*a*) A rectangular tank, 4′ × 12′ × 3′ 6″.

(*b*) A cylindrical tank of 6′ diameter and 8′ high.

(*c*) A cylindrical cistern of 2′ 6″ radius and 16′ deep.

(*d*) A vat 6′ square and 4′ 6″ high.

6. A metal tank, known to have a capacity of 60,000 gal., will hold for temporary storage how many bushels?

7. Construct a table to give the number of standard gallons in volumes from 1 to 1000 cubic feet.

SPECIAL INDIVIDUAL PROBLEMS.

1. Estimate the quantity and cost of materials for a cheap board fence to enclose a vacant lot in your neighborhood. Prepare a plot of the land and a diagram showing character of fence.

2. Prepare plans and estimates for a modern earth croquet court (or tennis court) to be constructed on a vacant lot in your neighborhood.

3. Report on the total number of square feet of lighting surface in the school building. Tabulate results (1) by rooms, (2) by floors, (3) by compass.

4. Report on the number of cubic feet of air space in the building, by rooms, corridors and floors.

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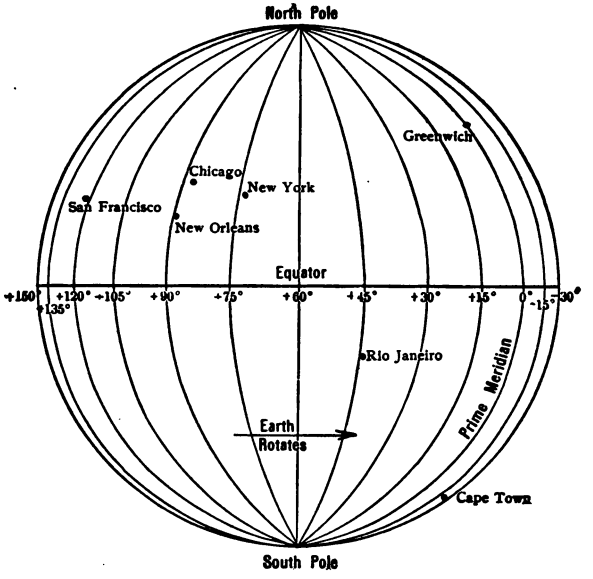
CHAPTER XXIV.

MEASUREMENT OF TIME.

1. Longitude and Time.

Longitude and time are matters of common interest. They are also fundamental to navigation, astronomy and geography.



A *meridian* is an imaginary line on the surface of the earth, extending due north and south and terminating in the poles. It is therefore a semi-circumference of the earth. Any meridian used as a reference line, from which to locate places on

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the earth's surface by determining their distances east or west of it, is a *prime meridian.* It is now the common custom to use the meridian through the Royal Observatory at Greenwich, England, as a prime meridian.

Distances east or west of the prime meridian are termed respectively *east longitude* and *west longitude.* They are measured by degrees, minutes and seconds of circular measure, as shown on page 164, where the meridians intersect the equator. Each place has its own meridian, but two places may have the same meridian even when thousands of miles apart. Longitude west is designated by the symbol '+,' and longitude east by the symbol '−.' Thus the longitude of New York, which is west of Greenwich is +73° 58′ 25.5″, while the longitude of Brussels, east of Greenwich, is −4° 22′ 9″.

From a study of the diagram, it is evident that the *difference in longitude of* two places may be obtained by subtraction, if the signs of the individual longitudes are the same; and by addition if they are different.

Since the earth revolves on its axis once in twenty-four hours, every point on the surface revolves in a circle, or passes through 360° of circular measure, in that time. Hence there is a close relation between *time* and *degrees of circular measure.*

Since 360° correspond to 24 hours.
1° is equivalent to 1/360 of 24 hr., or 4 min.
1′ is equivalent to 1/60 of 4 min., or 4 sec.
1″ is equivalent to 1/60 of 4 sec., or 1/15 sec.

Since 24 hours correspond to 360°,
1 hr. is equivalent to 1/24 of 360°, or 15°.
1 min. is equivalent to 1/60 of 15°, or 15′.
1 sec. is equivalent to 1/60 of 15′, or 15″.

Regarding the sun as stationary with reference to the earth, the earth, revolving from west to east, carries the meridians under the sun in succession, the latter seeming to move toward the west. Since all meridians pass under the sun in succession each twenty-four hours, they really mark, by differ-

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ence in longitude, the *difference in time* of places on the earth's surface. In science, difference of longitude is often expressed in time. Local time is determined at each place by calling the instant the sun crosses its meridian *12:00 noon.*

ILLUSTRATION. If the difference in longitude of two places is 84° 30′, what is the difference in time?

Solution.

84° = 84×1/15 hr.	Since 1° corresponds to 1/15 hr.	
= 5⅗ hr.	= 5 hr. 36 min.	
30′ = 30×4 sec.	= 2	Since 1′ corresponds to 4 sec.
84° 30′	= 5 hr. 38 sec.,	difference in time.

EXERCISE.

1. Do points in the east or west of a country receive sunlight first? Which have earlier local time?

2. If two places have different west longitudes, which has earlier time? Which of two places having east longitude has the earlier local time?

Evidently the more western points have the later time; the more eastern the earlier. Since time bears an intimate relation to degrees of longitude, it is a simple matter to compute one from the other.

ILLUSTRATIONS. (1) The longitude of Albany is 73° 44′ 48″, and of Ann Arbor, Mich., 83° 43′ 48″. Determine their difference in time.

Solution.

83° 43′ 48″, long. of Ann Arbor,
73° 44′ 48″, long. of Albany,
9° 59′ = difference in longitude.
9×1/15 hr. = 3/5 hr., or 36 min.
59×4 sec. = 236 sec., or 3 min. 56 sec.
9° 59′ = 39 min. 56 sec., diff. in time.

(2) The difference in the time of two places is 6 hr. 18 min. 12 sec. Determine the difference in longitude.

Solution.

6 hr. 18 min. 12 sec. = difference in time.
6 hr. = 6×15° = 90°
18 min. = 18×15′ = 4° 30′
12 sec. = 12×15″ = 3′
6 hr. 18 min. 12 sec. = 94° 33′, difference in longitude.

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NOTE. Both these solutions are based directly on the tables of comparative values given on page 165.

LONGITUDE OF WELL-KNOWN PLACES.

Athens,	−23° 43′ 55.5″.	Philadelphia,	+75° 9′ 45.0″.
Berlin,	−13° 23′ 43.5″.	Rome,	−12° 27′ 14.0″.
Chicago,	+87° 36′ 42.0″.	San Francisco,	+122° 25′ 40.8″.
New York,	+74° 0′ 3.0″.	Tokyo,	−139° 42′ 30″.
Paris,	− 2° 20′ 15.0″.		

EXERCISE.

1. If the difference in longitude of two places is 71° 18′ 0″, what is the difference in time?

2. Compute the difference in local time between (*a*) Berlin and Philadelphia; (*b*) Chicago and San Francisco; (*c*) Rome and Paris, and (*d*) Chicago and Tokyo.

3. A man coming to Chicago found that his watch, set by local time at his starting point, was 48 minutes slow by local Chicago time. Did he come from east or west? From what longitude?

4. When it is 5:00 P. M. at Chicago, it is what time at (*a*) San Francisco, (*b*) Berlin, (*c*) Rome and (*d*) New York?

5. A ship sailing in longitude 183° 40′ 9″ is how far in difference of longitude from the port of New York. Near what great body of land is the ship?

The International Date Line has been selected as the 180th meridian, with slight changes at certain points. This meridian in the Pacific Ocean passes through no important land areas. Going westward ships set their calendars forward one day at this point; going east they set them back one day.

Standard Time. If every place used its strict local time, it is evident that endless confusion would result in travel, especially from difficulties in constructing time tables. On this account the railways of the United States and Canada agreed on four time belts for the two countries. The belts are approximately 15° wide, and each belt takes the time of its center meridian. Thus the guide meridians are 15°, or one hour apart. The belts are the Eastern, Central, Mountain and Pacific, the meridians being +75°, +90°, +105° and

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+120°. Thus, when it is 10:00 A.M. in any town in the Eastern time belt, it is 9:00 A.M. in the Central belt, 8:00 A.M. in the Mountain belt and 7:00 A.M. in the Pacific belt.

Foreign countries are adopting similar belts, based largely on meridians at 15° intervals from Greenwich.

2. Commercial Time.

The accurate computation of time between dates, and of dates of maturity, is of vital importance in business. All interest charges depend on time. Many business and commercial papers run for fixed times to dates of maturity. Considerable merchandise is sold on limited time. Failure to correctly compute a payment date may cause a business man to lose standing or credit, by being caught unprepared to meet just demands.

Many seemingly minor details affect time and date computation. Time may be computed either approximately or by exact days, and dates of legal maturity are affected by varying state laws affecting holidays and Sundays (see page 312).

I. Time Between Dates.

Since the common business time is six months, or less, it is possible to make most time computations without use of paper. Where paper is used, the method is often different.

(*a*) *Approximate Time.* (*Paper.*)

Illustration. Find the time between June 21, 1908, and Jan. 17, 1910.

Yr.	Mo.	Da.
1910	1	17
1908	6	21
1 yr.	6 mo.	26 da.

Solution. Follow the method of denominate numbers. In the subtrahend, 1 mo. is reduced to days, making 30+17, or 47 da.; and 1 yr. is reduced to 12 mo.

Note. In reduction, one month is considered 30 days, but if a given date contains the thirty-first day it is retained.

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EXERCISE.

Find the time between:

1. Jan. 27, 1906, and Aug. 29, 1909.
2. Oct. 21, 1907, and Dec. 15, 1912.
3. Jan. 31, 1908, and Sept. 12, 1909.
4. 3/ 3/ '07 and 6/ 1/ '10.
5. 7/ 5/ '06 and 10/ 17/ '06.
6. 3/ 21/ '07 and 5/ 18/ '09.
7. Money borrowed Apr. 16, 1907, and due to-day, has drawn interest for how long?
8. I am given until Oct. 1 to pay a debt due Feb. 11. For how long is time of payment extended?

(*b*) *Approximate Time.* (*Mental.*)

ILLUSTRATIONS. (1) Find the time between Mar. 12, 1906, and May 17, 1908.

Solution. Reason as follows: From Mar. 12, 1906, to Mar. 12, 1908, 2 years; from Mar. 12, 1908, to May 12, 2 months; from May 12 to May 17, 5 days. Total time, 2 yr. 2 mo. 5 da.

NOTE. The process involves *adding* to the earlier date, on the principle used in making change.

(2) Find the time between Aug. 28, 1913, and July 12, 1914.

Solution. Aug. 28 (8th mo.) to June 28 (6th mo.) = 10 mo. June 28 to July 12 = 2 + 12 da. = 14 da. Total time, 10 mo. 14 da.

EXERCISE.

Compute the time between:

1. Feb. 11 and Apr. 29.
2. June 16 and Dec. 31.
3. May 31 and Nov. 24.
4. Mar. 17 and Oct. 9.
5. Mar. 16, 1912, and Dec. 15, 1913.
6. Feb. 28, 1912, and Nov. 29, 1912.
7. Aug. 13 and to-day.
8. May 19 and to-day.

EXACT TIME. In computing by exact time, the true number of days in each month is taken into account. Usually only one of the extreme days is included, although some bankers include both. Exact time is computed for periods less than a year. Longer terms are reckoned in calendar years and exact days.

(*c*) *Exact Time.* (*By Table.*)

The following table shows the calendar year and the *day of the year* of each calendar day. Thus Feb. 17 is the 48th day of the year. Other forms of tables are very common.

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TABLE FOR THE CALCULATION OF TIME.

Jan.	1	1	Feb.	1	32	Mar.	1	60	Apr.	1	91	May	1	121	June	1	152
	2	2		2	33		2	61		2	92		2	122		2	153
	3	3		3	34		3	62		3	93		3	123		3	154
	4	4		4	35		4	63		4	94		4	124		4	155
	5	5		5	36		5	64		5	95		5	125		5	156
	6	6		6	37		6	65		6	96		6	126		6	157
	7	7		7	38		7	66		7	97		7	127		7	158
	8	8		8	39		8	67		8	98		8	128		8	159
	9	9		9	40		9	68		9	99		9	129		9	160
	10	10		10	41		10	69		10	100		10	130		10	161
	11	11		11	42		11	70		11	101		11	131		11	162
	12	12		12	43		12	71		12	102		12	132		12	163
	13	13		13	44		13	72		13	103		13	133		13	164
	14	14		14	45		14	73		14	104		14	134		14	165
	15	15		15	46		15	74		15	105		15	135		15	166
	16	16		16	47		16	75		16	106		16	136		16	167
	17	17		17	48		17	76		17	107		17	137		17	168
	18	18		18	49		18	77		18	108		18	138		18	169
	19	19		19	50		19	78		19	109		19	139		19	170
	20	20		20	51		20	79		20	110		20	140		20	171
	21	21		21	52		21	80		21	111		21	141		21	172
	22	22		22	53		22	81		22	112		22	142		22	173
	23	23		23	54		23	82		23	113		23	143		23	174
	24	24		24	55		24	83		24	114		24	144		24	175
	25	25		25	56		25	84		25	115		25	145		25	176
	26	26		26	57		26	85		26	116		26	146		26	177
	27	27		27	58		27	86		27	117		27	147		27	178
	28	28		28	59		28	87		28	118		28	148		28	179
	29	29		—	—		29	88		29	119		29	149		29	180
	30	30		—	—		30	89		30	120		30	150		30	181
	31	31		—	—		31	90		—	—		31	151		—	—

ILLUSTRATION. (1) The time from Aug. 17 to Dec. 21 is ? days.

Solution. From table. Dec. 21 = 355th da.

Aug. 17 = 229th da.

The difference equals elapsed time = 126 da.

(2) Nov. 11 to Jan. 26 = ? da.

Solution (*a*). Nov. 15 is 315th day. 365 da. − 315 da. = 50 da.

Jan. 26 is 26th day of next year 26

Total time 76 da.

Solution (*b*). The complementary time between Nov. 11 and Jan. 26 is the time between Jan. 26 and Nov. 11.

Nov. 11 = 315th da.

Jan. 26 = 26th da.

Complementary elapsed time = 289 da.

True elapsed time = 365 da. − 289 da. = 76 da.

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TABLE FOR THE CALCULATION OF TIME.

July	1	182	Aug.	1	213	Sept.	1	244	Oct.	1	274	Nov.	1	305	Dec.	1	335
	2	183		2	214		2	245		2	275		2	306		2	336
	3	184		3	215		3	246		3	276		3	307		3	337
	4	185		4	216		4	247		4	277		4	308		4	338
	5	186		5	217		5	248		5	278		5	309		5	339
	6	187		6	218		6	249		6	279		6	310		6	340
	7	188		7	219		7	250		7	280		7	311		7	341
	8	189		8	220		8	251		8	281		8	312		8	342
	9	190		9	221		9	252		9	282		9	313		9	343
	10	191		10	222		10	253		10	283		10	314		10	344
	11	192		11	223		11	254		11	284		11	315		11	345
	12	193		12	224		12	255		12	285		12	316		12	346
	13	194		13	225		13	256		13	286		13	317		13	347
	14	195		14	226		14	257		14	287		14	318		14	348
	15	196		15	227		15	258		15	288		15	319		15	349
	16	197		16	228		16	259		16	289		16	320		16	350
	17	198		17	229		17	260		17	290		17	321		17	351
	18	199		18	230		18	261		18	291		18	322		18	352
	19	200		19	231		19	262		19	292		19	323		19	353
	20	201		20	232		20	263		20	293		20	324		20	354
	21	202		21	233		21	264		21	294		21	325		21	355
	22	203		22	234		22	265		22	295		22	326		22	356
	23	204		23	235		23	266		23	296		23	327		23	357
	24	205		24	236		24	267		24	297		24	328		24	358
	25	206		25	237		25	268		25	298		25	329		25	359
	26	207		26	238		26	269		26	299		26	330		26	360
	27	208		27	239		27	270		27	300		27	331		27	361
	28	209		28	240		28	271		28	301		28	332		28	362
	29	210		29	241		29	272		29	302		29	333		29	363
	30	211		30	242		30	273		30	303		30	334		30	364
	31	212		31	243		—	—		31	304		—	—		31	365

EXERCISE.

Compute the time between:

1. Feb. 11 and Oct. 16.
2. Mar. 19 and Nov. 10.
3. Jan. 15 and Oct. 29.
4. Apr. 24 and Dec. 27.
5. Feb. 28 and Aug. 7.
6. July 15 and Oct. 1.
7. Mar. 27, 1910, and Oct. 16, 1911.
8. Apr. 14, 1909, and Nov. 22, 1911.
9. May 29, 1910, and Dec. 19, 1911.
10. Nov. 16, 1912, and Apr. 26, 1914.
11. Show how to use the table in leap years.

(*d*) *Exact Time.* (*Without Tables.*)

ILLUSTRATIONS. Find the time between Aug. 18 and Oct. 27.

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Solution (*a*). Addition method. Aug. 18 to 31 = 13 da.
Sept. 1 to 30 = 30
Oct. 1 to 7 = 27
Total time = 70 da.

NOTE. Mentally, each group is added to the previous total, as found. One would say 13, 43, 70.

Solution (*b*). From approximate time.

Reason as follows: Aug. 18 to Oct. 27, 2 mo. 9 da., or 69 da. But this period includes a 31st day in August. Therefore the exact time is 69 da. + 1 da., or 70 da.

NOTE. Since February has 28 days, this method involves subtraction when the period includes the last of this month.

EXERCISE.

Find the exact number of days from:

1. Aug. 28 to Dec. 29.
2. Sept. 19, 1907, to Jan. 25, 1908.
3. Oct. 1 to Jan. 13.
4. Aug. 12 to Dec. 3.
5. Mar. 28 to Apr. 14.
6. Jan. 5 to Mar. 8.
7. Nov. 11 to date.
8. Oct. 16 to date.
9. Feb. 21 to June 9.
10. Mar. 3 to Aug. 2.

II. DATES OF MATURITY.

In computing final dates and dates of maturity, it is necessary to know the initial date and the "term" or period of time. If the term is stated in months and days, approximate methods are used; if stated in days, as "90 days," exact methods are used. "Day" terms are seldom over 90 days.

(*a*) *Approximate Method.*

ILLUSTRATION. A payment due 1 yr. 7 mo. 19 da. from Aug. 11, 1912, is due on what date?

Solution. (Paper.) Write in denominate form. 1912 8 11
(Add and reduce upward.) 1 7 19
1914 3 30
Date due, Mar. 30, 1914.

Solution. (Mental.)

Reason as follows: Aug. 11, 1912, + 1 yr. = Aug. 11, 1913; Aug. 11, 1913, + 7 mo. = Mar. 11, 1914; Mar. 11 + 19 da. = Mar. 30, 1914.

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EXERCISE.

Find the final date.

	Initial Date.	Term.
1.	Jan. 27, 1907	1 yr. 3 mo. 18 da.
2.	Feb. 16, 1913	2 yr. 5 mo. 11 da.
3.	Mar. 9, 1908	1 yr. 11 mo. 3 da.
4.	Apr. 14, 1908	2 yr. 5 mo. 29 da.
5.	May 30, 1908	1 yr. 2 mo. 11 da.
6.	June 14, 1912	9 mo. 19 da.
7.	Aug. 21, 1908	1 yr. 5 mo. 25 da.
8.	Sept. 4, 1913	2 yr. 5 mo. 6 da.

ORAL EXERCISE.

1. What date is 1 mo. from Jan. 28? From Jan. 29? Jan. 30? Feb. 1? Mar. 30? Apr. 6? June 20? Aug. 31?

NOTE. If there is no corresponding date, the last date in the month is taken. Thus, 1 mo. from Oct. 31 is Nov. 30.

2. Mar. 6+1 mo. = ?
3. Apr. 20+3 mo. = ?
4. June 30+4 mo. = ?
5. July 31+4 mo. = ?
6. Nov. 30+3 mo. = ?
7. Dec. 31+2 mo. = ?
8. Dec. 31+6 mo. = ?
9. July 14+8 mo. = ?

(*b*) *Exact Method.* (*By Table, pp. 170, 171.*)

ORAL EXERCISE.

1. What day of the year is June 14? What day of the year is 60 days later? What date?

2. Find a date 90 days in advance of June 29.

(*Solution.* June 29 = 180th day. 180 da. + 90 da. = 270 da.
By table, the 270th day is Sept. 27.)

Find the final dates.

	Initial Date.	Term.		Initial Date.	Term.
3.	June 26	90 da.	7.	Oct. 14	125 da.
4.	Aug. 31	90 da.	8.	June 11	45 da.
5.	Jan. 30	30 da.	9.	July 29	40 da.
6.	Feb. 26	60 da.	10.	Aug. 27	165 da.

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(c) *Exact (Periodic) Method.*

ORAL EXERCISE.

1. What date comes 30 days after Jan. 31? After Mar. 31? After April 20? After July 16?

2. When, for periods of 30 days, is the final date the corresponding date of the proper month? When is it earlier? When later?

3. Find the date of maturity of a paper running 90 days and dated Jan. 26.

Solution. Jan. 26+30 da. = Feb. 25, since Jan. has 31 da.; Feb. 25 +30 da. = Mar. 27. (Why?) Mar. 27+30 da. = Apr. 26. Therefore, the paper is due Apr. 26.

Find the dates of maturity.

	Initial Date.	Term.		Initial Date.	Term.
4.	Jan. 29	60 da.	**8.**	June 27	90 da.
5.	Feb. 17	90 da.	**9.**	Aug. 14	75 da.
6.	Apr. 30	60 da.	**10.**	Sept. 19	60 da.
7.	May 19	40 da.	**11.**	Nov. 30	50 da.

INDIVIDUAL ORIGINAL WORK.

1. Report on "The Arithmetic of Time Table Construction." Give illustrations. Give some idea of the character of information obtainable from time tables.

2. Prepare a time table and car schedule for a double track street railway. Estimate the number of motors, trailers, and the working force necessary for efficient service. Length of trip 40 minutes from 6 A.M. to 9 P.M.; 35 minutes from 9 P.M. to 6 A.M. 3 minute schedule, 7 to 10 A.M.; 3 to 7 P.M.; 5 minute schedule, 6 to 7 A.M.; 10 A.M. to 3 P.M.; 7 to 10 P.M. 8 minute schedule, 10 to 12 P.M. 30 minute schedule, 12 P.M. to 6 A.M.

3. You are to leave Washington on April 30 for a tour of the following places to look after business interests for me: Boston, Portland, Me., Albany, Buffalo, New York, Cincinnati, Memphis, St. Louis, New Orleans, Charleston, S. C., Charlestown, W. Va., and Minneapolis. Examine railway time tables and submit detailed report showing routes selected, provisional time table, etc. If possible make an estimate of expense. Plan for at least ten business hours in Boston, New York, Cincinnati, and St. Louis, eight hours in Minneapolis, and at least four hours in each of the other places.

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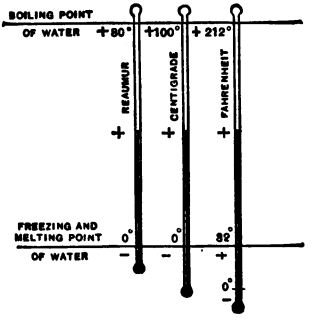
CHAPTER XXV.

PRACTICAL MEASUREMENTS—*Continued.*

1. Measure of Temperature.

Ordinary temperatures are measured by thermometers, graduated on one of three scales: (1) the *Fahrenheit*, commonly used in business and private life; (2) the *Centigrade*, used in scientific work, and in countries employing the metric system and (3) the *Reaumur*, used on the continent of Europe, especially in Germany.



The common thermometer, consisting of its column of mercury in a glass tube, cannot be used for extreme high temperatures. For such temperatures, especially those over 500°, instruments of many varied types are constructed. Several, for example, measure temperature by the relative expansion of two metals, such as copper and iron.

The level at which the mercury stands, when the thermometer is in contact with melting ice is marked 0° on the Centigrade and Reaumur scales, and +32° on the Fahrenheit scale. When in contact with water commencing to boil, the mercury level is marked +80°, Reaumur; +100°, Centigrade; and +212°, Fahrenheit. These limits being determined, the degrees of the scale are uniformly marked off. Degrees below 0° are marked "−," those above "+."

ORAL EXERCISE.

1. Name the number of degrees between freezing and boiling point on each scale.

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2. Rank the individual degrees in order of size.
3. 1° R. is what part of 1° F.? Of 1° C.?
4. 1° F. is what part of 1° C.? Of 1° R.?
5. 1° C. is what part of 1° R.? Of 1° F.?

Scale Reduction.

Owing to the lack of uniformity on the use of scales, the problem of reduction from one to another is very common.

Illustrations. (1) 40° F. = ?° C.

Solution.

∵ 180° F. = 100° C., between limits.

$$1° \text{ F.} = \frac{100°}{180} \text{ C., or } 5/9° \text{ C.}$$

40° F. = 40° − 32° above zero, or 8° above zero.

8 × 5/9° = 4.44 + ° C.

Solution.

∵ 100° C. = 180° F.

1° C. = 9/5° F.

25° C. = 25 × 9/5° F.;

= 45° F., above freezing.

But 0° C. = 32° F.

∴ 25° C. = 45° F. + 32° F. = 77° F.

(2) 12° F. = ?° C.

Solution.

32° − 12° = 20° F., distance below 0° C.

20° F. = −20 × 5/9° C.

= −11.11° C.

(4) −5° C. = ?° F.

Solution.

5° C. = 5 × 9/5° F. = 9° F.

But 0° C. = 32° F.

∴ −5° C. = 32° − 9° F. = 23° F.

EXERCISE.

Express each of the following temperature reading on two other scales:

1.	+196° F.	**5.**	+18.6° F.	**9.**	+182° C.	**13.**	+67.2° R.
2.	+108° F.	**6.**	−18° C.	**10.**	+46.5° C.	**14.**	−50° R.
3.	+64° F.	**7.**	+64° C.	**11.**	+18° R.	**15.**	+112° R.
4.	−13° F.	**8.**	−30° F.	**12.**	−66° C.	**16.**	+100°R.

17. Express the following table in Centigrade scale:

Approved Cold Storage Temperatures.

Degrees Fahrenheit.

Beef	36 to 40.	Poultry, frozen	25 to 30.
Pork	29 to 32.	Poultry, to freeze	5 to 10.
Lamb, mutton	32 to 36.	Eggs	32 to 34.
Fish	25 to 28.	Butter	32 to 38.
Oysters	33 to 45.	Vegetables	34 to 40.
Fruits	32 to 36.	Canned goods	38 to 40.

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18. Find the missing temperatures in these tables:

(*a*) *Boiling Points.*

Substance.	C.	F.
Benzine	——	176°
Chloroform	——	140°
Mercury		676°
Sulphur	——	570°

(*b*) *Melting Points.*

Substance.	C.	F.
Copper	——	2012°
Platinum	1775°	——
Sulphur	115°	
Zinc	——	779°

19. Construct a conversion table, Centigrade-Fahrenheit, for 0° to 50° Centigrade.

2. Composite Units of Measure.

It sometimes happens that an accurate measure of values cannot be made by a simple standard unit, because of some special influencing condition. Thus the freight traffic of a railway cannot be measured accurately by tons, because some tons are carried only short distances, while others are carried great distances; while if measured by freight earnings, the factor of quantity is lost sight of. To meet the difficulty, a composite unit, the *ton-mile,* was adopted. This unit is *one ton carried one mile.* If the ton is carried ten miles the traffic amounts to 10 ton-miles; if 5 tons are carried 8 miles, the traffic amounts to 5×8, or 40 ton-miles. Measured by such a unit the full effect of both quantity and distance is taken into account. Moreover, it makes possible a comparison of the traffic of different branches, or of independent roads. Composite measures of a similar order are increasingly used in science and in statistics.

A Few Other Composite Unit Measures.

Unit of work. A foot-pound—the work done in raising one pound one foot.

Unit of power. The horse-power. It is equal to 33,000 foot-pounds per minute, or 550 foot-pounds per second.

The irrigation unit. The acre-foot. The quantity of water required to cover one acre to a depth of one foot.

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The unit of freight traffic. The ton-mile. The equivalent of one ton of freight carried one mile.

The unit of passenger traffic. The passenger-mile. One passenger carried one mile.

The unit of train traffic. The train-mile. One train run one mile.

ILLUSTRATION. What horse-power in an engine will raise 4,950,000 lb. 10 ft. in 5 min.?

Solution.

$4{,}950{,}000 \times 10 = 49{,}500{,}000$, no. of ft. p.
$49{,}500{,}000 \div 5 = 9{,}900{,}000$, ft. p. per min.
$9{,}900{,}000 \div 33{,}000 = 300$, no. of horse power.

ORAL EXERCISE.

1. How many foot-pounds of work are performed in raising 48 lb. 15 ft.?

2. How many acre-feet of water are required to supply a $2\frac{1}{2}$ ft. depth to an area of $4\frac{1}{2}$A.?

3. What is the ton-mileage of a car load of 60 T., carried a distance of 56 mi.?

4. Compute the passenger mileage of 32 people, carried an average distance of 15 mi.

EXERCISE.

1. Compute the horse-power employed in raising 30,000 lb. an average distance of 48 ft. per min.

2. Compute the horse-power performed in raising 5000 gal. of water per sec., a distance of 10 ft.

NOTE. Express the water in terms of weight.

3. The owner of the tract of land shown on page 149 contracts for a supply of water of $2\frac{1}{2}$ ft. per annum, for irrigation. What will it cost him, at $46 per acre-foot?

4. A storage reservoir is to be built with a capacity of 50,000 acre-feet. What must be its capacity in gallons?

5. A metal casting weighing 48 T. 150 lb. is carried a distance of 169.3 mi. Compute the ton-mileage.

6. In a recent year, the railways of the country carried 1,309,899,165 T. of freight an average distance of 133.23 mi. Compute the ton-mileage.

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7. In the year above mentioned, 715,419,682 passengers were carried a passenger mileage of 21,923,213,536. What was the average distance each passenger was carried?

8. In the year above referrred to the passenger train mileage was 440,464,866 train miles. What was the passenger mileage per train-mile?

3. The Formula.

In estimating of every kind, it frequently occurs that a general rule for the solution of problems of a certain type is reduced in statement to a simple form of an equation, employing words, or more frequently, symbols.

Illustration. The fact that the area of a triangle is equal to one-half the product of base and altitude may be expressed:

$$\text{Area of a triangle} = \frac{\text{Altitude} \times \text{base}}{2}.$$

We may need, however, to compute areas of many different figures. In such a case, we may let "A" stand for the area of any figure, "a" for altitude, "b" for base, "c" for circumference and "R" for radius, the Greek letter "π" for the ratio of diameter to circumference. Using symbols, certain common rules for area reduce to the following:

1. For area of a triangle, $A = \frac{a \times b}{2}$, or simply $\frac{ab}{2}$.
2. For area of a rectangle, $A = ab$.
3. For area of a circle, $A = \pi R^2$.

Frequently the determination of the formula is a matter of summing up many experiments (in arbitrary rules—called empirical rules), or of complicated computations by higher mathematics. But once formulated, computations may often be made by simple arithmetic if in any problem numerical values are known for all except one symbol.

Illustrations: (1) Find the area of a circle of 6″ radius.

Solution.

Formula, $A = \pi R^2$.

But $\pi = 3.1416$, $R = 6$, $R^2 = 36$.

Substituting in the formula,

$A = 3.1416 \times 36$.

$= 113.0976$ sq. in.

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(2) Find the base of a rectangle whose area is 135 sq. in. and whose altitude is 9″.

Solution.

Formula, $A = ab$.

But $A = 135$ and $b = 9$.

Substituting in the formula,

$135 = 9a$.

$15 = a$, the altitude. (Dividing by 9.)

NOTE. Care should be taken that denominations are correct. Thus, in above case, if area had been stated in square feet while other factors were in inches, both factors should have been reduced to a common denomination of feet or inches.

EXERCISE.

1. Express in symbols the value of the circumference of a circle, in terms of the radius and constant ratio.

2. Express in symbolic form (*a*) the area of a parallelogram, (*b*) the surface of a cube, (*c*) the volume of a rectangular solid, (*d*) the surface area of such a solid, (*e*) the perimeter of a parallelogram.

3. The safe weight of a floor for a certain type of bridge is 80 lb. per sq. ft. Express this fact in formula for any sized bridge of the type.

4. Find by the bridge formula, the safe load of a bridge 90 ft. long and 17 ft. 6 in. wide.

5. If $x = 12$, $b = 6$ and $c = 5$, find A, in the formula $A = \frac{xb}{c}$.

6. Find S, in the formula $S = \frac{4}{3}\pi R^3$, if $R = 17$.

7. The number of board feet in a rectangular timber = length in feet × breadth in feet × thickness in inches.

a. Express this formula in symbols.

b. Use the formula to determine the number of board feet in a timber 12′ long, 9″ wide and 2″ thick.

8. The pressure of wind blowing directly against an object is sometimes measured by the formula $P = .0023V^2$, in which P = pressure in pounds per sq. ft., and V = velocity in feet per second. Find the pressure per square foot under a wind blowing 40 miles per hour.

9. A roof is so inclined that the wind pressure is .45 of what it would be if the wind struck it perpendicularly. What is the total wind pressure on the roof, which measures 18′ by 60′, when the wind is blowing 32 miles per hour?

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10. The volume of boards that may be sawed from a round log may be computed by the formula

$$V = \frac{L(d-4)d/2}{8},$$

in which d = diameter in inches, L = length in feet, and V = volume in feet, board measure. Compute the board measure in a 60 ft. log of 15 in. diameter.

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CHAPTER XXVI.

THE METRIC SYSTEM.

The metric system is a decimal system of measures adopted by the French government about 1800. It is used, scientifically, the world over, and commercially in many civilized countries. It is authorized in the United States and Great Britain, but is used only in a part of their foreign trade.

The system is based on the *meter*, a unit of length of approximately 39.37 in., which was assumed to be exactly 1/10,000,000 of the distance from equator to pole. Later measurements, however, have proven this computation incorrect. The unit of capacity, the *liter*, is a cube of $\frac{1}{10}$ m. edge. The unit of weight, the *gram*, is the weight of a cube of water of 1/100 m. edge. The tables contain multiples and decimals of units. Multiples are known by the Greek prefixes: myria (10,000), kilo (1000), hekto (100) and deka (10); decimals by Latin prefixes: deci (1/10), centi (1/100), milli (1/1000). Owing to the decimal system, few measures are commonly used. Those used are shown in black-faced type.

NOTE. Literature on the Metric System is published by The Bureau of Standards, Washington, D. C.

LINEAR MEASURE.

1 myriameter	=10 kilometers	=10,000	meters.
1 kilometer (Km.)	=10 hectometers	= 1,000	"
1 hektometer	=10 decameters	= 100	"
1 dekameter	=10 meters	10	"
1 meter (m.)	=10 decimeters	= 1	"
1 decimeter (dm.)	=10 centimeters	= .1	"
1 centimeter (cm.)	=10 millimeters	= .01	"
1 millimeter (mm.)	=	.001	"

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Square Measure.

1 sq. myriameter	= 100 sq. kilometers	= 100,000,000	sq. meters.
1 sq. kilometer (Km.²)	= 100 sq. hektometers	= 1,000,000	"
1 sq. hektometer	= 100 sq. dekameters	= 10,000	"
1 sq. dekameter	= 100 sq. meters	= 100	"
1 sq. meter (m.²)	= 100 sq. decimeters	= 1	"
1 sq. decimeter (dm.²)	= 100 sq. centimeters	= .01	"
1 sq. centimeter	= 100 sq. millimeters	= .0001	"
1 sq. millimeter	=	= .000001	"

Land Measure.

1 hektare (ha)	= 100 ares	= 10,000 sq. meters.
1 are	= 100 centares	= 100 " "
1 centare	=	1 " "

Cubic Measure.

1 cu. myriameter	= 1000 cu. kilameters	= 1,000,000,000,000	cu. meters
1 cu. kilometer	= 1000 cu. hektometers	= 1,000,000,000	" "
1 cu. hektometer	= 1000 cu. dekameters	= 1,000,000	" "
1 cu. dekameter	= 1000 cu. meters	= 1,000	" "
1 cu. meter	= 1000 cu. decimeters	= 1	" "
1 cu. decimeter	= 1000 cu. centimeters	= .001	" "
1 cu. centimeter	= 1000 cu. millimeters	= .0000001	" "
1 cu. millimeter	=	= .000000001	" "

Table of Weight.

1 metric ton (t.) (tonneau)	= 10 quintals	= 1,000,000	grams.
1 quintal (q.)	= 10 myriagrams	= 100,000	"
1 myriagram	= 10 kilograms	= 10,000	"
1 kilogram (Kg.)	= 10 hektograms	= 1,000	"
1 hektogram	= 10 dekagrams	= 100	"
1 dekagram	= 10 grams	= 10	"
1 gram (g.)	= 10 decigrams	= 1	"
1 decigram	= 10 centigrams	= .1	"
1 centigram (cg.)	= 10 milligrams	= .01	"
1 milligram (mg.)	=	.001	"

The metric ton or tonneau = weight of 1 m. of water; the kilogram = weight of 1 liter of water.

Table of Capacity.

1 hektoliter (Hl.)	= 10 dekaliters	= 100	liters.
1 dekaliter	= 10 liters	= 10	"
1 liter (l.)	= 10 deciliters	= 1	"
1 deciliter	= 10 centiliters	= .1	"
1 centiliter (cl.)	= 10 milliliters	= .01	"
1 milliliter (ml.)	=	.001	"

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Table of Equivalents.

Note. Importers and customs officials have to perform a relatively slight amount of reduction to and from the metric system. Common equivalents follow.

Measure.	For fairly accurate work.	For approximation.
Meter	39.37 in.	3¼ ft. or 1.1 yd.
Liter	1.0567 l. qt. / .9081 d. qt.	1 qt.
Gram	15.432 gr.	15½ gr.
Kilometer	.6214 mi.	.6 mi.
Hectare	2.47 A.	2½ A.
Sq. meter	1.196 sq. yd.	1⅛ sq. yd.
Cu. meter	1.308 cu. yd.	1⅓ cu. yd.
Metric ton	1.1023 T.	1.1 T.
Kilogram	2.2046 lb.	2.2 lb.

Mile	1.6093 km.	Quart (liquid)	.9436 l.
Yard	.9144 m.	Quart (dry)	1.101 l.
A.	40.47 a.	Pound	.4536 kg.
Sq. yd.	.8361 m^2	Ton	.9072 t.
Cu. yd.	.7645 m^3		

ORAL EXERCISE.

1. Read as meters:
   a. 7 km., 5 dekameters and 3 cm.
   b. 9 km., 4 cm., 8 mm.
2. Read as square meters:
   a. 9 km^2, 2 m^2, 3 dm^2.
   b. 5 hm^2, 8 m^2, 9 cm^2.
3. Read as grams: 4 kg., 8 dkg., 7 cg.
4. Read as liters: 9 hl., 7 l., 4 cl., 8 ml.
5. Use decimal values:

100 m.	= ? in.	1000 m.	= ? cu. yd.
1000 l.	= ? l. qt.	200 t.	= ? T.
2 km.	= ? mi.	10,000 m^2	= ? sq. yd.

6. Use approximate equivalents:

20 m.	= ? ft.	20 t.	= ? T.
68.5 l.	= ? qt.	200 ha.	= ? A.
80 km.	= ? mi.	600 m^2	= ? sq. yd.
40 kg.	= ? lb.	200 g.	= ? gr.

7. Express in metric system: 100 l. qt.; 1000 lb.; 100 mi.; 1000 d. qt.; 10 T.; 200 gr.; 100 cu. yd.; 10 A.; 200 sq. yd.

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8. A "hundred meter" dash is equivalent to a race of ? yd.

9. A man who orders 1000 m. of dress goods should receive ? yd.

EXERCISE.

1. If a merchant desires 1200 yd. of French silk, he must order ? meters.

2. A consgt. of 50 l. olive oil will fill how many pint bottles?

3. A measurement on a foreign map is found by scale to be equivalent to 87.56 km. The equivalent distance is ? miles.

4. An importer is required to pay a tax of 15c per gal. on 120 l. of oil. The tax amounts to $?.

5. How does a speed of 71 km. per hour for a German express train compare with a record of 71 miles in 69 min. made by a train in this country?

6. Granite is 2.7 times as heavy as water. Estimate the weight of a piece measuring 3.1 m. by 6 m. by 4.55 m.

7. The United States requires foreign letters weighing 15 g. or less to pay 5c postage. What is the limit of weight in our standards?

8. What is the mileage rate in cents for first class passage from Rouen to Paris, 136 m., if the fare is 15.20 fr. (1 fr. = $.193).

9. A man who weighs 172 lb. weighs ? kg.

10. Determine the weight of an aluminum bar 1 m. long and 30 cm. cross-section (specific gravity, 2.71)?

11. A tank measuring $5' \times 4' \times 8'$ has a capacity of ? m^3.

12. Determine reasonable dimensions for a rectangular tank to have a capacity of 500 l.

13. How many tons of coal will fill an order for 5000 t.?

14. How many pieces 8.1 cm. long, may be cut from 70 m. of wire?

15. A 400 meter running track is ? yd. long.

Individual Original Work.

1. Prepare a brief on the metric system contrasting it with our own system. Illustrate by parallel computations.

2. Construct a six-place table for converting Linear Measure to Metric Measure.

3. Show how an increasing error is caused in the construction of a working table, by a minor error at the start.

4. Compare computation by table and by process, giving illustrations to show the value of tables.

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CHAPTER XXVII.

RATIO AND PROPORTION.

INTRODUCTORY EXERCISE

1. Find the difference between 150 and 30.

2. 30 is what part of 150?

3. 150 is how many times 30?

4. $3000 worth of a novelty were sold in one year, and $12,000 worth the next year. Find the increase in sales. The second year's sales were how many times those of the first year?

It is evident that division offers one effective means of comparing numbers. By it one may determine *how many times as great,* not *how much greater* one number is than another. The numbers so compared must be like numbers, and consequently the quotient is always abstract. This quotient, whether represented or computed, is termed a *ratio.*

The ratio may be *expressed* by any of the signs of division, or by the colon. The numbers involved are called *terms,* the first term being the *antecedent* and the second the *consequent.*

ILLUSTRATION. The ratio of a payment of $500 to one of $2500 may be expressed (*a*) $500/$2500, (*b*) $500 ÷ $2500, or (*c*) $500 : $2500.

The *value* of the ratio is determined by performing the represented division. In the above illustration, the value is 1/5. If the second payment had been referred to the first, the ratio would have been expressed $2500 : $500, and its value would have been 5. The ratio of the first quantity to the second is termed the *direct ratio;* that of the second to the first the inverse ratio.

ORAL EXERCISE.

Express and determine the ratios of:

1. 6 to 42.

2. 2 yd. to 2 ft.

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3. 5 qt. to 6 gal.
4. 2 sq. ft. to 1 sq. yd.
5. $720 to $900.
6. 75c to $2.
7. 1″ to 1′.
8. 1′ to 1/4′.
9. 2 lb. to 4 oz.
10. 900 to 1500.
11. Measure a rod by a yard.
12. Measure a bushel by three pints.

13. Name groups of two numbers having the ratios 12, 4, 1/2, $1\frac{1}{4}$.

14. Add 1 to the antecedent of the ratio 2 : 5. What is the effect on the value of the ratio?

15. By experiment, find the effect on the value of the ratio, from:
(1) Increasing its antecedent.
(2) Decreasing its antecedent.
(3) Increasing its consequent.
(4) Decreasing its consequent.
(5) Multiplying or dividing the antecedent.
(6) Multiplying or dividing the consequent.
(7) Multiplying or dividing the consequent and antecedent by the same value.

16. Compare the above cases with the corresponding cases of simple fractions.

Since $\frac{\text{Antecedent (dividend)}}{\text{Consequent (divisor)}} = \text{Ratio (quotient)}$, it is evident that the antecedent equals the product of ratio value and consequent, and that the consequent equals the quotient of the antecedent divided by the ratio. It is therefore a simple matter to find a single missing term.

ILLUSTRATION. A manufacturer issues one design of oblong calendars in assorted sizes, but with the dimensions of length and breadth in a constant ratio of $2\frac{1}{2}$. (*a*) If the length of one is 15″ what is the width? (*b*) If the width of one is 8″ what is the length?

Solution (*a*). $\frac{\text{Length}}{\text{Width}} = \text{ratio} = 2\frac{1}{2}$.

$\text{Width} = \text{Length} \div 2\frac{1}{2} = 15'' \times 2/5 = 6''$.

Solution (*b*). $\text{Length} = 2\frac{1}{2} \times \text{the width} = 2\frac{1}{2} \times 8'' = 20''$.

ORAL EXERCISE.

Determine the missing values in each of these ratios:

1. 24 : 4 = ?
2. ?/18 = 3.
3. 36/? = 2.

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4. ? : 4 = 8. 6. 3/4 : 2/3 = ? 8. 1/4 yd. : ? = 1/8.

5. 1/2 : 1/4 = ? 7. ? : 3 mi. = 1/4. 9. 2½ ft. : 1½ ft. = ?

NOTE. If two variable (changing) quantities or dimensions have an unchanging or *constant* ratio, one is said to *vary as the other*. Thus the ratio of the circumference of a circle to its diameter is always 3.1416, no matter what the size of the diameter. Quantities are said to *vary inversely* if one increases as the other decreases, in a constant ratio. In common speech, we refer to approximate changes in the same way.

10. What do we mean by saying that summer travel from the big cities varies with the heat?

11. What do we mean when we say that prices vary with demand?

12. What do we mean when we say that the cost of production of a certain article varies inversely as the quantity produced?

The ratio method is often used in geometry and its applications, in physics and in the solution of many business principles. It is really a method of fractional analysis and is used, without the name, throughout this book.

EXERCISE.

1. The specific gravity of copper is 8.9. What is the weight of three cubic feet? See page 134.

2. Water is composed of hydrogen and oxygen in the ratio of 2 : 1. How many parts of each gas in 15,000 parts of water?

3. If a 3″ line in a drawing is used to represent a real length of 4′ 9″, what is the ratio of real to represented length?

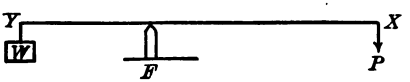
4. A piece of ore weighs 1.3 kg., while the weight of an equal volume of water is .2 kg. Determine the specific gravity of the ore.

5. If the circumference of a wheel is to be 16 ft. what must be its diameter?

6. One of two cubes is 3/4 of the height of the other. Determine the ratio of surfaces and volumes.

7. Show by ratio, the effect on volume of reducing all dimensions of a bin measuring 16′ × 8′ × 4′, by one-third.

THE LEVER.



If a bar is supported on a fulcrum, F, a weight, W, attached at the point Y

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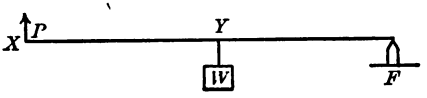
may be balanced by the application of a sufficient pressure at the point X. If the weight is between the pressure point and the fulcrum, pressure must be applied upward. Otherwise, it is applied downward. If the distances x and y are expressed in units of measure, and W and P in units of weight, it is a law of physics that the ratio of the product of the weight by its arm to the product of the pressure by its arm is equal to unity, when the lever balances. In other words,



$$\frac{\text{Weight } (W) \times \text{weight arm } (y)}{\text{Pressure } (P) \times \text{pressure arm } (x)} = 1.$$

ILLUSTRATION. If the fulcrum in the first figure is 4 ft. from Y and 6 ft. from X, what pressure supports a weight of 12 lb.?

Solution.

By the law of levers, $\frac{12 \times 4}{6 \times P} = 1.$

$$48 = 6P.$$

$$8 = P.$$

$\therefore$ the pressure must be 8 lb.

EXERCISE.

1. The fulcrum being at one end of a 6 ft. bar, what pressure balances a weight of 80 lb., 1 ft. 6 in. from the end?

2. A pressure of 20 lb., applied 9 ft. from the fulcrum, supports what weight 2 ft. beyond the fulcrum?

3. Illustrate how the principle of the lever is applied in the use of a crowbar. Write and solve an illustrative example.

4. Find, by investigation, six common applications of the lever principle, and prepare and solve simple problems illustrating them.

5. Where must one place a support under a 12 ft. bar, in order that it may balance weights of 16 and 40 lb.?

6. A brake chain is connected with a brake lever 1 ft. below the supporting pin. The brake handle is 6 ft. above this support. If the brake lever is pulled with a force of 40 lb., what pressure does the brake exert? What pressure per square inch, if the brake shoe measured 8 in. by 3 in.?

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Proportion.

An equality of ratios is termed a *proportion.* Thus 6 lb./21 lb. = \$12/\$42 is a proportion because the abstract value of the ratios is the same. An equality of several ratios, as 3/4 = \$9/\$12 = 18 lb./24 lb., is termed a *continued proportion.* An equality of products of ratios, as $1/12 \times 5/4 = 5/6 \times 1/8$, is termed a *compound proportion.* Continued and compound proportion are seldom used in practical arithmetic.

In any proportion, as 1 : 2 = \$8 : \$16, the first and fourth terms (1, \$16) are the *extremes,* and the second and third (2, \$8) are the *means.* The proportion is read "1 is to 2, *as* \$8 is to \$16." Since the two ratios are equal in numerical value, and since both the extremes and the means include a numerator of one ratio and a denominator of the other, it is evident that the *product of the means equals the product of the extremes.* Thus $1 \times \$16 = \$16$, and $2 \times \$8 = \$16$.

It is evident, also, that any term of a proportion may be found if the other terms are known. *The first or third terms are always the products of the opposite ratio by the second or fourth term respectively. The second term equals the product of the first term by the opposite inverse ratio.* To avoid multiplying by concrete numbers, each term of either ratio may be made abstract.

Illustrations. (1) In the proportion $\frac{x}{6} = \frac{15}{45}$ find x.

Solution.

$x = \frac{6 \times 15}{45} = 2$. Evidently 1/6 of $x = \frac{15}{45}$, and $x = 6 \times \frac{15}{45}$.

(2) In the proportion $\frac{\$480}{x} = \frac{24}{6}$ find x.

Solution.

$x = \$480 \times \frac{6}{24} = \$120$. Evidently $\$480 = x \times \frac{24}{6}$,

and $x = \$480 \div \frac{24}{6} = \$480 \times \frac{6}{24} = \$120$.

Note. Since the product of the means equals the product of the extremes, it is evident, also, that any mean may be found by dividing the

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product of the extremes by the other mean; and any extreme may be determined by dividing the product of the means by the other extreme. Thus, in illustration (1), writing the proportion in the form $x : 6 = 15 : 45$, it is seen that one extreme is missing and that its value is equal to 6×15, the product of the means, divided by 45, the other extreme. This principle affords a simple method of solution.

EXERCISE.

Find the missing terms:

1. $x : 12 = \$72 : \$18$.
2. $5 : 8 :: 14 : ?$
3. 275 lb. : 125 lb. = \$11 : \$$x$.
4. 3 sq. ft. : 16 sq. ft. : : 48c : ? c.
5. $1/4 : 1/2 :: ? : 3/8$.
6. $x : 12 :: 7$ yd. : 480 yd.

Two quantities are *directly proportional* if one varies directly as another. They are *inversely proportional* if one varies inversely as the other.

ILLUSTRATIONS. (1) If one is traveling on mileage, the cost of his railway passage varies directly as the distance, or is proportional to distance.

(2) Temperature being constant, the volume of a gas varies inversely as the pressure, that is, grows less at the same rate that the pressure increases. Thus doubling the pressure halves the volume.

The common applications of proportion are those already mentioned under simple ratio. In solutions by proportion, it will be found more convenient to place the unknown quantity in the first term, although any term may be used. Solutions may be checked by unitary analysis, and by fractional methods.

ILLUSTRATION. A contractor is to receive \$4872 for the construction of 2140 ft. of concrete pipe. What has he earned after he has completed 535 ft.?

Solution.

Let x equal the sum earned.

x : \$4872 = the ratio of amount earned to contract price.

535 : 2140 = the ratio of quantity done to total quantity. Since the amount earned depends on the work done, the ratios must be equal.

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$\therefore x : \$4872 = 535 : 2140.$

$x \times 2140 = \$4872 \times 535.$ (Products of means and extremes.)

$$x = \frac{\$4872 \times 535}{2140} = \$1218.$$

Check Solution.

The part of work done is 535/2140, or 1/4.

$\therefore$ the contractor is entitled to 1/4 of \$4872, or \$1218.

EXERCISE.

1. The weight arm of a lever is 16 ft.; the pressure arm is 9 ft.; the weight is 4000 lb. Find the pressure or power to balance it.

2. A farmer found that he required 7 lb. of a certain seed for a 1/4 acre lot. What proportional amount of seed is required for a $3\frac{1}{2}$ acre lot?

NOTE. In electricity, the resistance of a wire to the flow of a current varies directly as its length, or inversely as the square of its cross section.

3. Determine the resistance of $1\frac{3}{4}$ mi. of a certain copper wire, if the resistance of 100 yd. is 5 ohms. (The ohm is an electricity unit of resistance.)

4. If the resistance of iron is seven times that of copper, find the resistance in 1000 ft. of iron wire, if 20 yd. of copper wire of same diameter has a resistance of 2.6 ohms.

5. If a metal sphere of 8 in. diameter weighs 214 lb., what is the weight of a sphere of the same metal having a diameter of 12 in.?

6. It is a principle of mechanics that the power acting parallel to an inclined plane will support a weight proportional to the ratio of the length of the incline to the distance the weight is raised. If the height is 5/6 of the length, what power will support a weight of 840 lb.? If the height is 1/4 of the length?

NOTE. For the principle of partitive proportion, see page 387.

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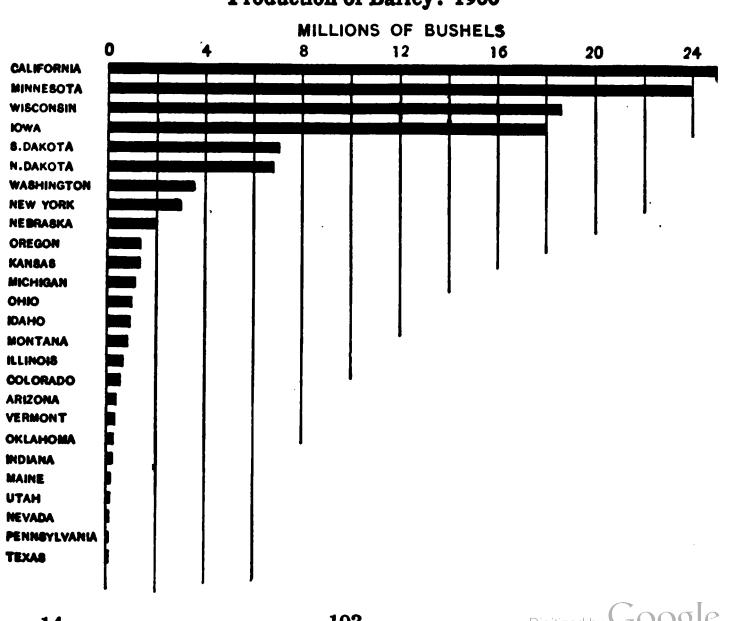
CHAPTER XXVIII.

GRAPHIC ARITHMETIC.

A simple drawing often does away with the necessity for a complicated numerical statement. A line diagram, drawn to scale, will often convey its meaning instantaneously. A number statement, on the other hand, may convey little meaning, or require time for interpretation, or be less likely to attract attention. Diagrams or graphs, therefore, have become a common form of number expression, or language, especially in the presentation of statistical matter. Thus the accom-



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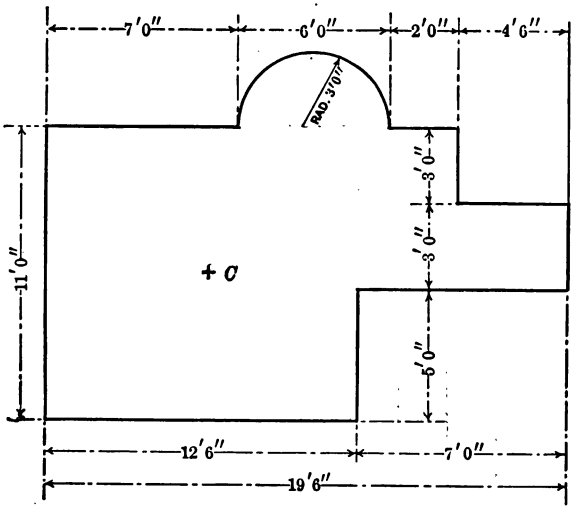
panying graph from a Census Report gives the average reader a far clearer impression of the relative production of barley, in different states, than would a column of figures stating the production in bushels. Moreover, if desired, the graph may be read to approximate numerical value by means of its scale.

General diagrams, such as building plans, plots of land, maps, etc., commonly are used for reference. It is far easier and surer to say "Find the area of the floor shown in the accompanying diagram" than, without a diagram, to describe the shape and dimensions in such a way as to enable one to compute its area.



EXPERIMENT. Try to describe the plan here shown, in words so that one could compute its area accurately

I. SCALES AND PLOTTING.

Ability to read and construct explanatory drawings and statistical graphs depends on a thorough knowledge of the arithmetic of scales, and of the fundamentals of plotting.

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A *linear scale* is either the ratio of the plotted length to the true length or distance, or the length that is to represent on paper a fixed quantity of any denomination. If every line in a drawing is one-fourth of the corresponding true length of the object represented, the diagram is said to be drawn on "1/4 scale." If a one-inch line is used to represent a quantity of 10,000 bushels, the scale is said to be "1 inch = 10,000 bushels."

Linear scales are represented, commonly, in one of three ways:

(1) As a simple fraction. Illustrations: 1/4, 3/8, 5/4.

(2) By corresponding dimensions. Illustration: 1″ = 1′. This means that one inch on paper represents one foot of the object.

(3) By diagram. Illustration:

From zero to the right the scale shows diagram lengths corresponding to true lengths of 10 ft., 20 ft., 30 ft., etc. A true distance of 22 ft. is represented by a paper distance from "20" (on the right of zero) to the second sub-division on the left. (Why?) Notice that the paper scale reads to true lengths. In diagram scales decimal sub-divisions are common.

Linear scales of distance may be expressed in any of the three forms. Thus a scale of 1/3 may be written 1″ = 3″, or 1 ft. = 3 ft.; or $\frac{\text{1 INCH}}{\text{3 INCHES}}$.

QUESTIONS FOR DISCUSSION.

1. What is the advantage in having the diagram scale read to true lengths?

2. Why is a *linear* scale used on a drawing that represents an area?

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3. How does the "line" scale, if carefully constructed, save computation, in making or reading drawings?

4. Name a common scale used by architects.

5. How does the graduated edge of a ruler correspond to a scale?

EXERCISE.

(Answer orally, if possible.)

Express each of these scales in two other forms:

1. $\frac{1}{5}$. **3.** $\frac{1}{20}$. **5.** $\frac{1}{240}$. **7.** $\frac{1}{720}$. **9.** $1''=4'$.

2. $\frac{3}{8}$. **4.** $\frac{1}{1200}$. **6.** $\frac{600}{1}$. **8.** $\frac{5}{3}$. **10.** $1'' = 10$ yd.

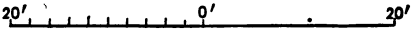
11. $1'' = 100'$. **12.** $2' = 1\frac{1}{2}''$.

13. Which of the above scales could be used to make a diagram larger than the original?

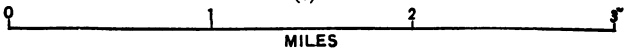
14. Express the following scales in numerical form. (Use ruler to find ratio.)

(*a*)

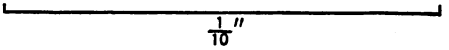
(*b*)



(*c*)



(*d*)



15. Which of these scales will make the largest diagram:

$\frac{1}{16}$; $1'' = 4'$;

16. An engineer uses a ruler divided into units, tenths and hundredths of feet. If he uses the smallest subdivision to represent two feet, find his scale. Express it in two ways.

17. If the smallest subdivision on your ruler is used to represent one foot, what is the scale?

18. An architect uses a ruler having inches divided into sixteenths. If he uses one of these subdivisions to represent a foot, what is his scale?

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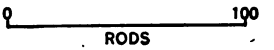
19. Two towns, known to be twelve miles apart, are represented on the map at an interval of three inches. What is the map scale?

EXERCISE.

1. Construct the scale 1/20 in diagram form.

2. Determine a fractional scale for this line scale.



3. Construct a line scale for the scale 1″ = 1200 bbl.

4. Construct a line scale for the scale 1″ = 2500 lb., such that each division shall represent 1000 lb.

5. Find and illustrate four methods of representing line scales.

Reduction to Plotted Lengths or Distances. Since a true linear scale shows, in fractional form, the part that the scale length is of the length to be plotted, computations in scale reduction are simple fractional reductions. A scale of one-sixth means that the plotted lengths are one-sixth of the real lengths. Real lengths, in this case, are reduced to plotted lengths by dividing by six. When scales are expressed in other forms, they may be used directly, or in difficult cases, reduced to fractional form. In the scale shown five feet of true length are represented by the given line. Twenty-five feet would be represented, evidently, by a line five times as long. A scale of 1″ = 4′ reduces an 8′ true dimension to a plotted dimension of 2″. (Why?) On a scale of 1″ = $600, the amount of $1200 is represented by a 2″ line. (Why?)

EXERCISE.

(Answer orally if possible.)

Find the scale lengths for true dimensions of

1. 4′ 6″, on a scale of 1/2.

2. 3′ 9″, on a scale of 1″ = 1′.

3. 100 yd., on a scale of 1/600.

4. 35 ft. on a scale of 1″ = 10 ft.

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5. $2000, on a scale of 1″ = $400.

What length lines should be used in graphs to represent these quantities:

6. 4500 bu. on a scale of 1″ = 250 bu.
7. 4 tons, on a scale of 1/2″ = 500 lb.
8. 1600 cans, on a scale of 1/4″ = 200 cans.
9. 2,400,000 bales, on a scale of 1″ = 60,000 bales.

EXERCISE.

Find the missing values:

No.	True dimension.	Scale.	Scale Dimension.
1.	4′ 6″	2 FEET	?
2.	2′ 3″	1/12	?
3.	14′ 9″	1/4″ = 1′	?
4.	270′ 0″	1″ = 50′.	?
5.	4 mi. 80 rd.	1 MILE	?
6.	8″	5/3	?
7.	2 1/2″	1″ = 1/3″	?

8. Which of the above scales will make enlargements?

PRACTICE EXERCISE.

1. List twenty true dimensions taken in the class room. In a parallel column, write the corresponding scale lengths. Use the smallest subdivision on your ruler to represent *one inch*.

2. Make a freehand sketch of the outline of the class room floor. Take measurements and determine plotted lengths on a scale of 1/16.

Dimensioning is the process of noting on a drawing the true dimensions of the parts represented (page 194). The symbols (′) and (″) are commonly used in place of the abbreviations (ft.) and (in.). If the true length contains no fraction of a foot, zero inches is written. For example, 5 ft. is written 5′ 0″. Light *dimension lines* are drawn parallel to the lines of the drawing to which the dimensions refer, the dimension figures being entered in a break in the center. Dimension

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lines are limited by arrow points that just touch short perpendiculars erected at the extremes of the line to be dimensioned. Dimensions are generally written *up and to the right*, in order that all may be read from one position. A circle or arc is dimensioned by locating its center and giving its radius. "Over all" or "total" dimensions are written as a check on a series of intermediate dimensions.

EXERCISE.

1. Point out the different principles of dimensioning illustrated on page 194.

2. What is the real value of an "over all" dimension?

3. What is the advantage in writing dimensions outside, rather than inside a small drawing?

4. Collect from newspapers, periodicals, or other sources, six specimens of dimensioned drawings. Make a list of any unusual details of dimensioning.

Plotting, as related to drawings, is the process of locating the main points of a proposed diagram preliminary to drawing in the details. With the actual mechanical construction of drawings we have little to do, but certain mathematical preliminaries should be noticed. Before plotting, it is necessary to choose a paper large enough to contain the drawings of the object at the scale required. The size of paper and the dimensions of the object to be represented will necessarily affect the scale to be chosen.

EXERCISE.

1. The greatest dimensions of a tract of land are 1800 ft. by 1450 ft. If plotted on a scale of 1/450, how large a sheet is required, allowing for a two-inch margin all around?

2. What is the largest scale that could be used in reproducing, on this page, the map of a piece of land measuring 4 miles by 3 miles 40 rods?

3. What is the largest scale on which the land diagram on page 149 could have been drawn?

4. What is the largest scale on which a diagram of your class room could be drawn, on a paper measuring 17 in. by 24 in.?

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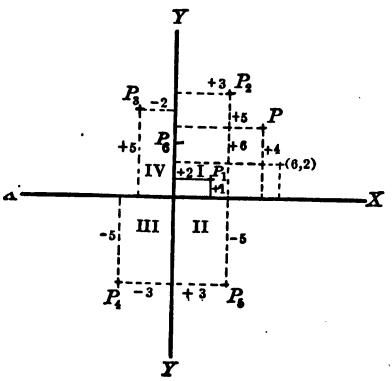
5. A rectangle measures 16 ft. by 5 ft. 2 in. How large a sheet of paper is required for a drawing of it, on a scale of 1/4″ = 1′, if a 1½″ margin is to be left between the drawing and the edges of the paper?

Coördinates. While the general principles of scientific projection and plotting, as used in projection drawing, have no place in this discussion, the system of plotting by coördinates has a direct use in the graphic phases of applied arithmetic. If we assume two fixed lines, Y and X, at right angles, any point, P, may be exactly located if its perpendicular distances from X and Y are known, together with the particular angle, I, II, III, or IV, in which it is situated. If P is in angle (quadrant) I, 4 units from X and 5 from Y, it is located by measuring on X from Y 5 units; then up from X, parallel to X, 4 units. The *coördinates*, or locating distances, are always taken parallel to the *axes of coördinates* (X and Y), so that it makes no fundamental difference whether the axes are at right angles or at a slant. In writing (not plotting), the position of a point with respect to the distance parallel to X is stated first, then the distance parallel to Y. Thus the point P is the point (5, 4); the point P_1 is (2, 1); the point P_2 is (?, ?); the point P_6 is (0, 3). If the point is located in other quadrants, its position is located in the same way, except that the distances parallel to X and to the left of Y; or parallel to Y and below X are marked *minus*. Thus P_3 is the point (−2, 5); P_4 is the point (−3, −5), and P_5 the point (3, −5). In the simpler applications, with which we have to deal, it is possible



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so to locate the axes as to have all our points in the same angle, and thus disregard the plus and minus signs.

EXERCISE.

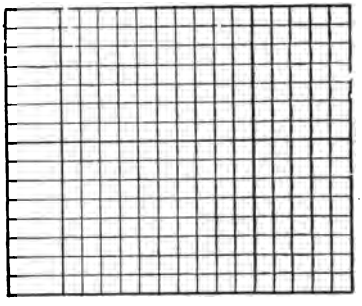
Draw the axes and locate these points:

1. 2 in. from X and 3 in. from Y, first quadrant.
2. 4 in. from Y and 3 in. from X, first quadrant.
3. On X 2 in. from Y.

4. (3″, 3″).	7. (2″, −4″).	10. (0′, 0′).
5. (4″, 2″).	8. (0″, $-1\frac{1}{2}$″).	11. (3/4″, −2″).
6. (2 $\frac{1}{2}$″, 5″).	9. (5″, 0″).	12. (1″, 2$\frac{1}{2}$″).

13. (8″, 5″), on 1/2 scale.
14. (2 mi., 7 mi.) on a scale of ——— = 2 miles.

If one locates the corners of a field with respect to axes, he may plot the points and, by connecting them, obtain the outline. In taking measurements of an irregular room, one may find the distances of each corner from any side or sides, taken as axes, and then prepare a plot. Points on a curve may be taken at such close intervals as, after plotting, to outline the curve on paper. As a help in plotting, cross-section paper, similar to the illustration, may be used, any two lines at right angles being taken as axes. The sides of the sub-squares may then be taken as units of scale, and diagrams plotted speedily. The large squares are usually divided decimally.



EXERCISE.

1. Plot the points (4, 3), (7, 3) and (6, 6). Connect them. What is the resulting figure?

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2. Plot to scale the points (2, 2), (3, 4), (6, 2) and (7, 4). What is the resulting figure?

3. Make a diagram on a scale of 1″ = 6′, of a room whose corners, taken in order, are (0′, 0′), (0′, 11′), (8′, 11′), (8′, 13′), (13′, 13′) and (13′, 0′). Which sides are taken as axes.

4. Make a diagram of your class room by the same process.

5. Plot and find the area of a field whose corners, taken in order, are the points (50 rd., 70 rd.), (200 rd., 70 rd.), (250 rd., 200 rd.) and (50 rd., 200 rd.).

6. Plot one side of a curve of a driveway from the points (0′, 10′) (1, 8), (2½, 6), (5, 4), (6, 3), (8, 2), (10, 1).

Reduction of Plotted Lengths to Real Lengths. Few drawings or maps, except draftsmen's construction drawings, are so fully dimensioned as to supply all dimensions desired. It is often necessary to find the real length corresponding to a plotted length. If a plotted length is drawn on a one-fifth scale, the true length is naturally five times that plotted. If the drawing scale is 1″ = 4′, a 2″ line on paper represents 8′ of real length. (Why?) If a scale is 2 FEET, a line six and one-half times as long represents a true length of 13 feet. (Why?)

EXERCISE.

(Oral and written.)

No.	Length on Drawing.	Scale.	True Length.
1.	3¼″	1″ = 1000′.	?
2.	5.3″	1/1200	?
3.	2.7″	1/63360	?
4.	1′, 3.4″	2 INCHES.	?
5.	9.2″	1″ = 2 mi.	?
6.	1′, 3½″	1/16	?
7.	11.45″	1/400	?
8.	2¼″	1″ = $5250	?
9.	4⅛″	1/2″ = 10,000 bu.	?
10.	———	1″ = 10′.	?

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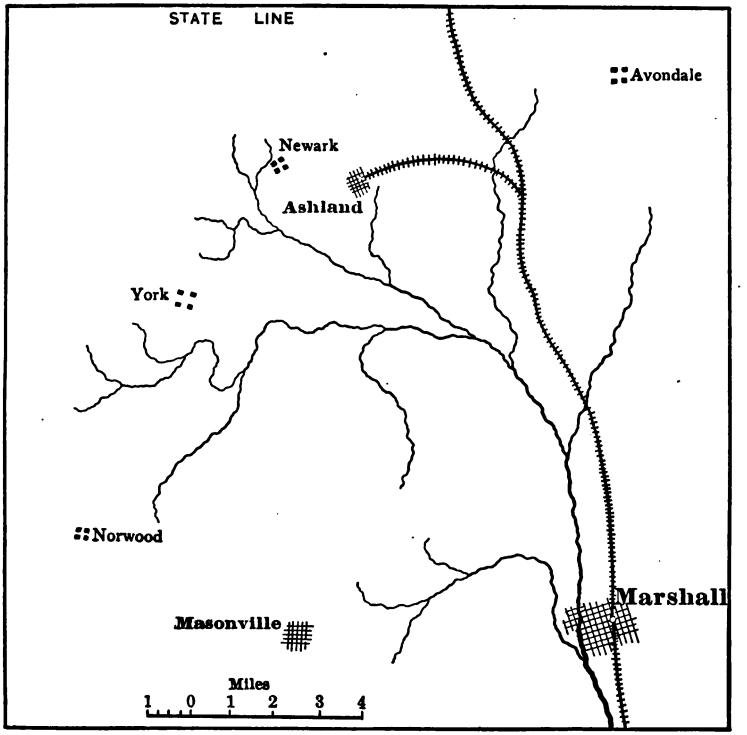
11. Determine the scale of the figure on p. 155. Find the true distance diagonally across the room; and the distances of any point C from each corner, and perpendicularly, from three sides.



12. This map is drawn at a scale of 1″ = ? mi., or what fractional scale? Which towns are in a radius of eight miles of the center of Marshall? By airline, Masonville is ? miles from Avondale; Norwood is ? miles from the nearest point on a railway; Masonville is ? miles from the state line, and Norwood is ? miles from the nearest railway junction. An airline electric road from Ashland to Marshall would be ? miles long, approximately ? miles shorter than the nearest railway connection.

Are distances computed by scale from this map absolutely accurate? What factors tend to affect the results?

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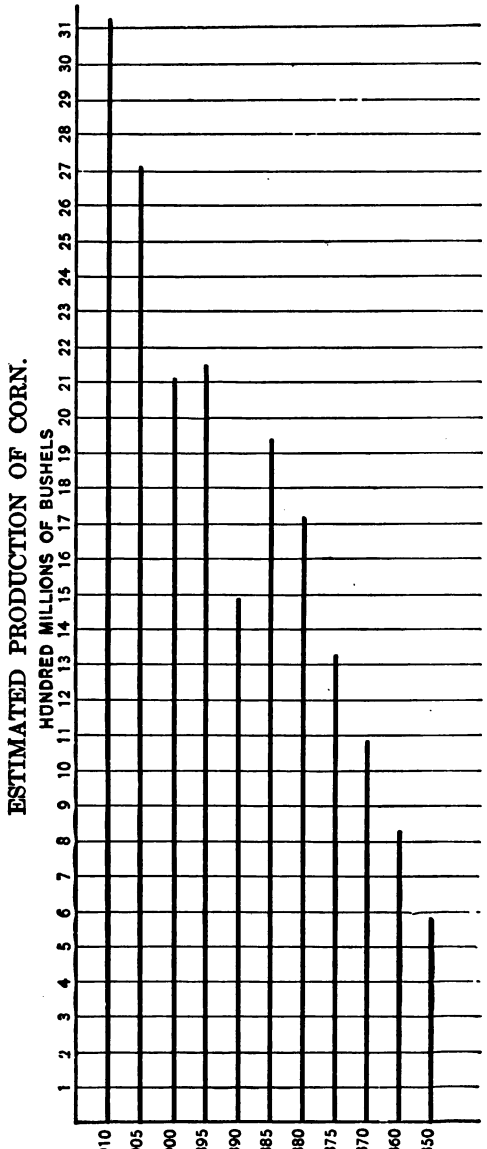

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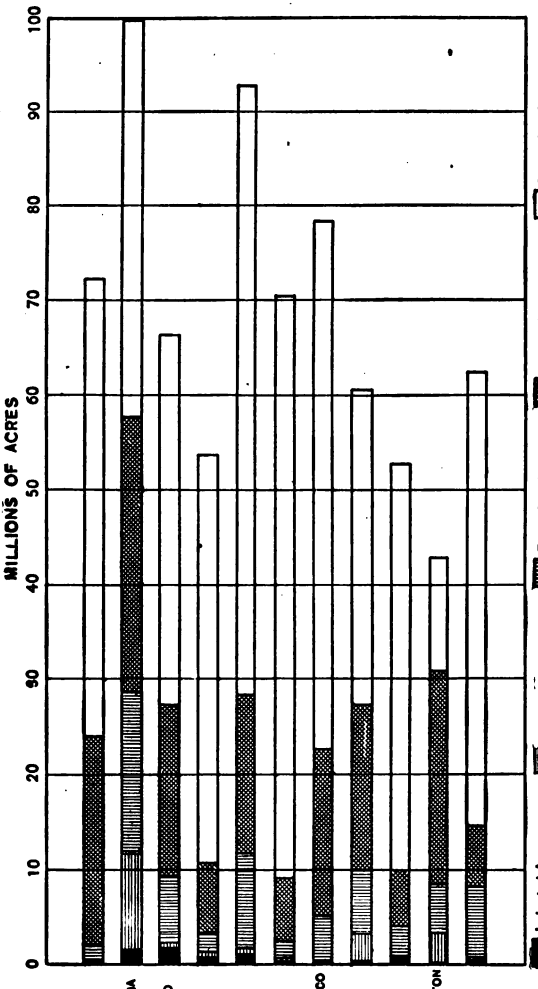

Diagram showing the relative size of eleven arid states with area in private ownership, farm area, improved and irrigated acreage.

From the Report of the Twelfth Census of the United States.

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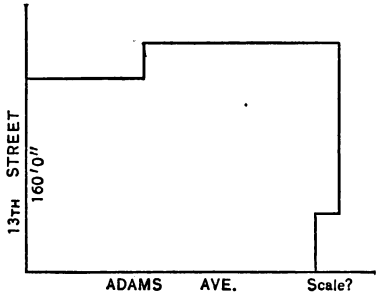
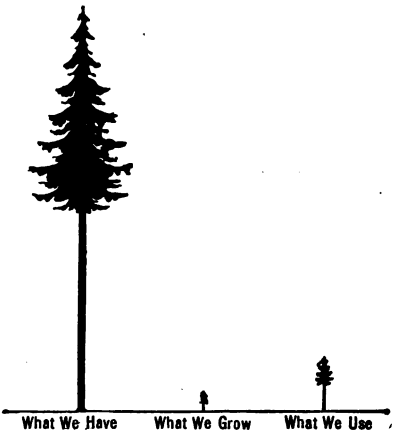
13. The owners of the plot here shown desire to buy additional land, fronting 60′ on 13th Street and 80′ adjoining on Adams Ave., and running back in each case so as to make the entire lot a rectangle measuring ?′ by ?′. Plot the figure and show the additions. Locate on the plot a building with a frontage parallel to Adams Ave., of 80′, set back 20′ from the street line, and running back, parallel to 13th street, to a depth of 60′. The building is 15′ from the 13th Street line. The area of the original plot is ? sq. yd.; of the addition, ? sq. yd.; and of the improvement ? sq. ft.



14. Sixth Street, north, meets Ave. A at right angles. On a scale of 1/1200, make a plot of the land on the northwest corner, from this description: From the corner, the boundary runs north 180′; thence west, 40′; thence north, 60′; thence west, 80′; thence diagonally to a point on Ave. A 100′ from corner. The area is — sq. ft., and the valuation, at $1.95 per sq. ft. is $——.

II. Graphs.

Reference has been made to the graph as a means of conveying general impressions of relative value. It deserves, however, more extended notice, now that the principles of scales are understood.



OUR TIMBER SUPPLY AND ITS DEPLETION.

The large tree represents the amount of timber that we have, the second tree the relative amount of the annual growth, and the third tree the relative amount of annual use.

From *The World's Work.*

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The graph may be used in the form of: (1) lines drawn to scale in lengths proportional to the quantities of bushels, dollars, acres, etc., which they represent; (2) proportional areas of any shape; (3) lines based on the principle of coördinate; or (4) figures, varying in one or more dimensions. An illustration of (4) is shown on page 206.

The *Parallel Line Graph* is often drawn superposed on a reading scale that enables one to read approximate numerical values at sight. In the graph of the production of corn (p. 204) in the United States, based on Government figures, each scale division represents one hundred million bushels. Thus the crop for 1910 was approximately 3,125,000,000 bushels and for 1850 it was just under 600,000,000 bushels. Approximate values are all the general public desires, hence for large values round numbers are sufficient. Such graphs also show at a glance the general increase of crop from period to period and any period of unusual crop or crop shortage.

By using rectangles of a uniform height so that areas vary with widths, the principle of the line graph is maintained, but changes in the *filling* of the different sections are made to add details of information. Thus in the figure on p. 205 each entire space represents the full arid area of a state, but, by a change of shading, five facts concerning this area are made clear.

Area graphs are of endless variety but they usually conform to geometrical figures. In the circular graphs (p. 208), areas vary with represented values. Both were constructed to scale, although none is stated.

The temperature and precipitation chart shown on p. 209 illustrate a coördinate graph. The subdivisions of the scale background vertically read represent (1) temperatures in degrees above or below the normal for the periods shown, and (2) the precipitation in tenths of inches. The solid lines show

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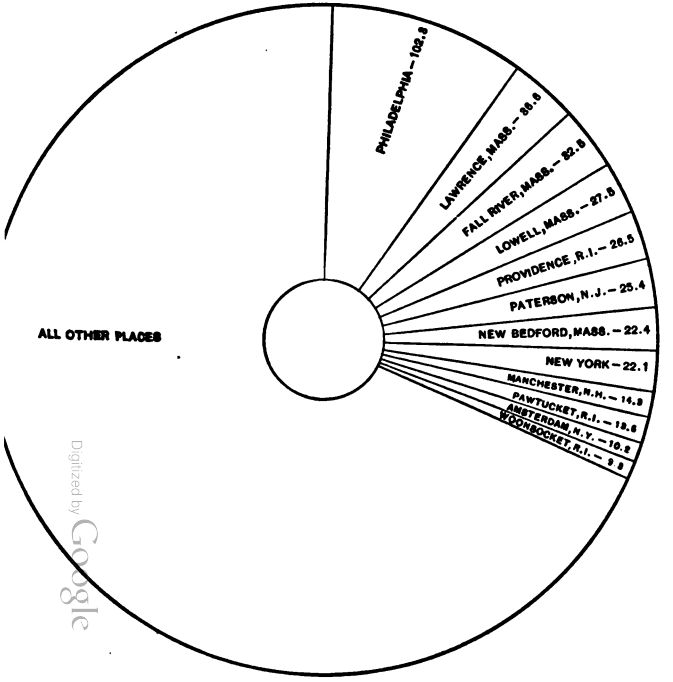
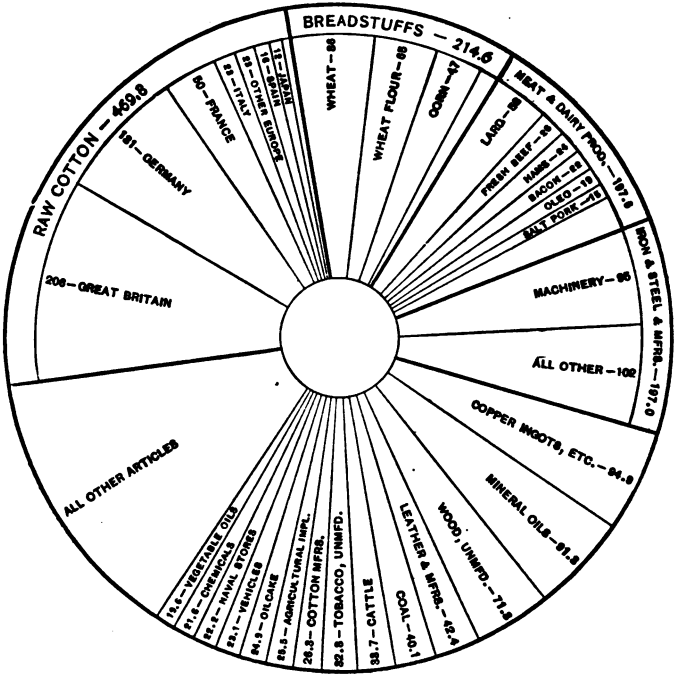
UNITED STATES MANUFACTURERS
BY CITIES—1905
LEADING TEXTILES Total value, $1,089,842,058
Value by cities given in millions of dollars.



EXPORTS FROM THE UNITED STATES—1907
BY CLASSES OF GOODS
Total value, $1,923,395,267



From "Foreign Trade of the United States for 1907," published by the Philadelphia Commercial Museum.

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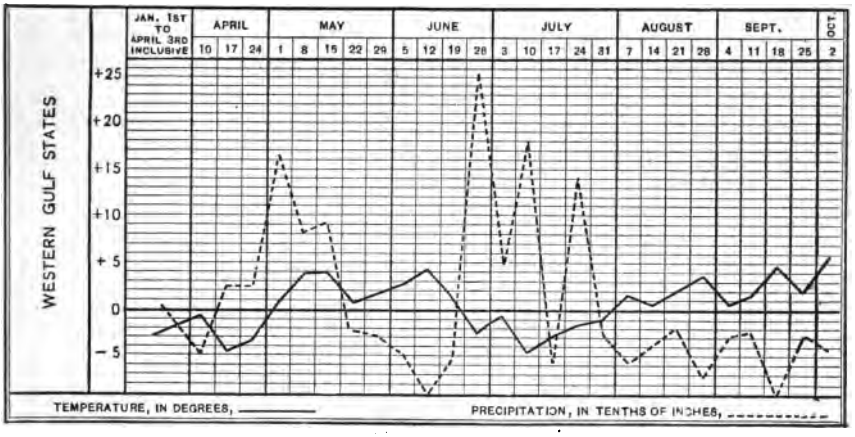
the variation in average temperature from week to week, from the normal as shown by the horizontal line. The dotted line shows the amount and variation (departure) in rainfall.



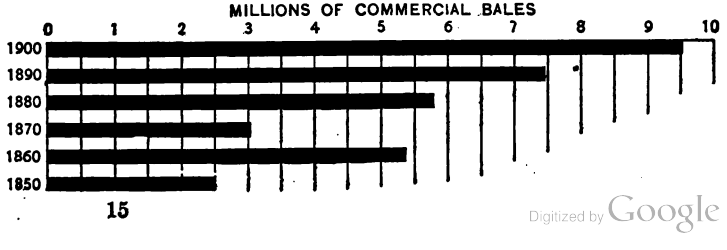
Temperature (degrees Fahrenheit) and precipitation (inches) departures for the season of 1905 from the normal of many years for the Middle and South Atlantic States and Gulf States.

From the Year Book of the U. S. Department of Agriculture for the year, 1905.

EXERCISE.

1. **Read at sight the figure for the production of corn, page 205.**
2. **Reduce to a number table the graph shown on page 206.**
3. **Read the following graph at sight.**

PRODUCTION OF COTTON: 1850–1900.



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4. The corresponding 1910 figure for the above cotton graph may be taken as 10,386,000 bales. Determine the length of the proper line on the graph.

5. A man, receiving a wage of $25.00 per week, estimates his expenses as follows: Rent, $25 per month; food, $360 per year; fuel and light $80 per year; clothing $100 per year; amusements, $2 per month; charity $2 per month; miscellaneous expense, $6 per month—the balance remaining representing a savings and emergency fund. Draw an outline to represent his $20 bill or weekly wage, and show graphically how it is apportioned to meet his expenses.

6. The following have been estimated to be the approximate expenditures of anthracite coal in New York City in a year:

Anthracite.	Tons.
Domestic, private houses and small stores	2,500,000
Flats and apartment houses	2,850,000
Hotels, clubs and theaters	1,250,000
Gas and electric plants	1,300,000
Elevated and surface roads	650,000
Harbor shipping	400,000
Department stores and office buildings	650,000
Municipal departments	400,000
Total	10,000,000

Represent these figures graphically in some other way than by a line graph.

7. By means of properly proportioned outline, to suggest "skyscrapers," represent the relative amount of new building operations in different cities, in 1911 based on the following figures: Baltimore, $9,325,000; Buffalo, $10,365,000, Chicago, $105,270,000; Cincinnati, $13,485,000; Cleveland, $17,000,000; Detroit, $19,000,000; New York, $173,500,000; Pittsburgh, $11,700,000.

8. Represent in some other graphic form the graph on page 211.

9. Using round numbers, to the nearest "ten thousand," prepare a graph to illustrate these facts concerning immigration:

Immigration from 1830 to 1906.

1831-1840	599,125
1841-1850	1,713,251
1851-1860	2,598,214
1861-1870	2,314,824
1871-1880	2,812,191
1881-1890	5,246,613
1891-1900	3,844,420
1901-1910	8,795,386

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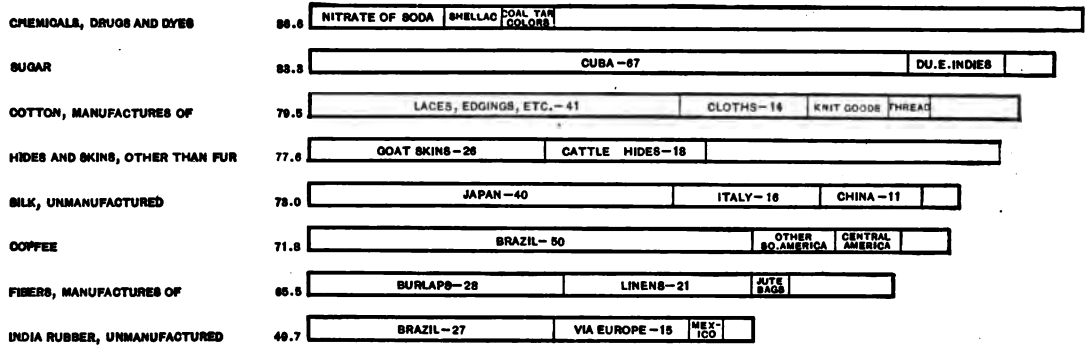
IMPORTS INTO THE UNITED STATES

BY CLASSES OF GOODS



From "Foreign Trade of the United States" for 1907, published by the Philadelphia Commercial Museum.

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10. Design a graph to illustrate this statement: "The llama will carry from 50 to 200 pounds; a man from 75 to 150 pounds; the donkey, 100 to 200 pounds; an ox, 150 to 200 pounds; a horse from 200 to 250 pounds; the camel from 350 to 500 pounds; the elephant, from 1800 to 2500 pounds."

11. Construct, on a coordinate or cross-section plan, a graph to represent the recapitulation of daily sales by departments, shown on page 13. Design a form that may be extended day by day.

Individual Original Work.

Prepare a set of statistics of some class of the high school, compiled from office records by direction of Principal. Show same facts graphically.

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CHAPTER XXIX.

PERCENTAGE.

In the expression of numerical relations, 100 is a common basis of comparison. The process of *percentage*, or of computing by hundreds, is one of the most important of all the working processes of arithmetic and has most varied applications.

INTRODUCTORY EXERCISE.

1. Find 1/100 of 200, 2472, 36, 592 bu., ½, .006.

2. Find 3/100 of 400, 70 lb., 9000 tons.

3. Find (*a*) 4/100 of 720; (*b*) 25/100 of 800 sheep.

Per cent. is a standard term for *hundredths*, derived from the Latin *per centum*, meaning "by the hundred." The term is represented by the symbol "%."

ILLUSTRATION. 4 per cent. of a number = 4% of it = 4/100 of it = .04 of it.

ORAL EXERCISE.

1. What per cent. of a quantity is one-hundredth of it? 3/100 of it? 246 hundredths of it? .03 of it? .92 of it? 3/4 of it?

2. Express as rates per cent.: 36/100, 270/100, .40, .02, 1.25, 3.205, .6.

NOTE. The *rate per cent.* is the number of hundredths taken.

3. Express as hundredths: 8%, 12%, 120%, ½%, 33⅓%, 600%, 12½%.

4. ¼ = ? hundredths. ¼ of a quantity = ? % of it.

5. Express as per cents.: 1/3, 3/4, 1/4, 5/8, 3/20, 1/40, 3/5.

6. 20% of a quantity equal how many hundredths of it? What fractional part of it?

7. Name simple fractions corresponding to these per cents.: 50%, 5%, 25%, 40%, 36%, 133⅓%, 100%, 106%, 96%.

8. Express as decimals: 1/4, 1/3, 5/4, 3/25, 4.3%, 125%, 20%, 12½%, 92%.

9. Express as common fractions .24, .4, 3.2, .0125, 90%, 120%, 43%.

10. Express each of these rates per cent. in at least two other forms: .26, .8, 1 1/4%, .004, 9/5, 3/4, 1/2%, .50, 48%, 75%.

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11. Express these statements in other arithmetical forms:

(*a*) I pay 33⅓% of a debt.

(*b*) The use of wood pulp paper in this country has increased 49.9% in five years.

(*c*) 94% of the men returned to work yesterday.

(*d*) He saves 25% of his salary.

(*e*) Thompson's plurality was cut down 37½% at the last election.

12. 1/2 = how many per cent.? What is the difference in per cent. between 1/2 and 1/2% of a quantity?

13. Find the difference in per cent. between:

1/4 and 1/4%. 1/4 and 1/3%. 33⅓% and 3/8. .002 and 1/4%.

EXERCISE.

Find the missing values in this table:

TABLE OF COMPARATIVE VALUE.

Decimal.	Per Cent.	Hundredths (Fraction).	Common Fraction.
.01	——	——	——
.0125	——	——	——
.02	2%	2/100	——
.05	——	——	1/20
.06⅔	——	——	——
.10	——	——	——
——	12%	——	——
——	12½%	——	——
——	15%	——	——
——	16⅔%	——	——
——	20%	——	——
——	25%	——	——
——	——	33⅓/100	——
——	——	37½/100	——
——	——	——	1/2
——	——	——	11/20
——	——	——	3/5
——	——	——	5/8
——	——	——	2/3
.80%	——	——	——
——	——	——	7/8
——	90%	——	——
1.75	——	——	——
——	——	——	9/4

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To Find the Percentage.

The *percentage* of a quantity is the value of the hundredths taken. The *base* is the number on which the percentage is reckoned.

Illustrations. (1) I collect 45% of a debt of $720, or what sum?

Solution. (Decimal method.) 45% = .45

$720
.45
3600
2880
$324.00 the am't collected.

(2) What profit is made by selling a $42 article at 3% advance on cost?

Solution. (1% method.) 1% of $42 = $.42
3% of $42 = 1.26 (3×1% of $42)
The profit = 3 × 1% of $42, or $1.26.

(3) On a $420 purchase, I am allowed 25% discount, or $?.

Solution. (Aliquot method.)
The discount is 25% or 1/4 of $420. 1/4 of $420 = $105, discount.

Note. The general tendency is toward the use of the decimal method.

It is evident that the percentage equals the product of the base by the rate.

FOR DISCUSSION.

1. What rates per cent. in the preceding table are aliquot parts?
2. Is the base concrete or abstract?
3. Is the percentage concrete or abstract?

ORAL DRILL EXERCISE.

1. Find 1% of 24, 36, 500, 4.65, 1/5, 72.5 yd., 246 lb.
2. By the 1% method, find: 3% of 2400; 8% of 20 bbl.; 12% of $300; 4% of 1/3; 15% of .2.

Note. 100% of a number = 100/100 of it, or the number itself. 200% of a number is how many times the number?

3. Find: 200% of 2640; 300% of $48; 400% of 1/3; 250% of 2.4.
4. Find: 1/4% of 5600; 3½% of 1400; 2/3% of 1/2; 1/4% of 1/4; 3/4% of .4.
5. Find, by aliquot parts: 25% of $3640; 83⅓% of .006; 33⅓% of 1/4; 12½% of 624 yd.; 90% of $7.20.
6. Compare 72% of $50 with 50% of $72. Conclusion?
7. Find: 18% of $250; 84% of 75 T.; 19% of 66⅔ A.

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EXERCISE.

Find:

1. 25% of 27,240 lb.
2. 1½% of $7654.20.
3. 18% of .00624.
4. 333⅓% of 645 T.
5. 4½% of 5/36.
6. 75% of 15.02″.
7. 3/8% of $95,240.
8. 62½% of 11,200 shares.

EXERCISE.

Each problem illustrates a different use of percentage. State use in each case. Re-word each problem in question form. Find missing values.

1. The directors of the Randall Mfg. Co. pass a resolution to increase all salaries and wages 15%. My own salary of $1200 is thereby raised to $——. The advance will cause an increase of $—— in a monthly pay roll of $24,600.

2. I decide to set apart 12½% of my monthly income of $96, or $——, for minor personal expenses, and to deposit in the bank, 25%, or $——.

3. On a purchase of 40 O. C. stoves, listed at $4.80, a customer is allowed 40% off the price, because of the size of his order. The discount is $—— and the customer pays $—— each.

4. The purchaser of the stoves marks them at an advance of 25% on what they cost him. His selling price is $——, of which $—— is profit.

5. A. C. Crane and Thomas Drew form a partnership with a capital of $12,600, Crane supplying 33⅓%, or $——, and Drew ——%, or $——.

6. C. B. Brown fails in business and compromises with his creditors, agreeing to pay at once 66⅔% of his debts, and a month later 15% more. His debts total $46,240.20. His payments are $—— and $——.

7. It is reckoned that the machinery equipment in a factory decreases 5% of its original value in each six months. If worth $26,400 originally, it is worth $——, 12 months later.

8. An architect charges as a fee for the design of a building, 5% of its cost. His fee on a building costing $11,380 is $——.

9. Of 700 pupils in a school, 90%, or —— pupils, are present on January 11. Of the number present, 40% are boys and —— are girls. In an arithmetic test, 73% or —— pupils make a passing average.

10. "The newspapers predict that Black's majority of 42,400 two years ago will be cut down 75% at the coming election. If this is true he will still win by —— majority."

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ORAL DRILL EXERCISE.

(The teacher should prepare similar exercises.)

1. Base, 240. Find 1%, 3%, 25%, 33⅓%, 10%, 11%, 16⅔%, 87½%, 1/4%, 20%, 160%, 500%.

2. Base .6. Find 1/3%, 3/4 of it, 2/3, 2/3%, 150%, 3%, 400%, 2¼%.

3. Base, 3/4. Find x% of it, 3%, 20%, 200%, 8%, 12½%, 400%.

4. 60 decreased 33⅓% of itself = ? (Called the *difference*.)
320 increased 25% of itself = ? (Called the *amount*.)

5. Define *amount* and *difference*. Which implies subtraction and which addition? If the rate is 46%, name the *difference per cent.* and the *amount per cent.*

6. Decrease. 240 bu. 33⅓%; 1/4 by 1/3 of itself; 3/8, 33⅓%; 2460, 10%; 800 by 1/4 of itself; .6 by 40%; 4.2, 14⅔%.

7. Increase: 12, 33⅓%; 900, 2/3%; 2172 lb., 1%; .04, 125%; 1/4, 25%; 3.6 T., 16⅔%

In the following, use each intermediate answer as the base for the next step.

8. Increase 20 by 50% of itself; increase that result 33⅓%; that result, 25%; that result 20%; that result, 33⅓%; decrease that result, 50%; decrease that 10%; increase 10%. Answer?

9. 4 increased 50%; that result increased 33⅓%; increased 12½%; decreased 66⅔%; increased 100%; decreased 50%; multiplied by 4; decreased 25%; multiplied by 1/3. The final result is ?.

FINDING THE RATE PER CENT.

INTRODUCTORY EXERCISE.

1. If 20 = 5% of a given number, 1% of it equals ? and 30 = ?%.

2. 16 = what part of 20? What per cent. of 20?

3. 1% of 400 = ? 36 = ? % of 400. 924 = ? %.

ILLUSTRATIONS. (1) What per cent. is 536 of 1200?

Solution. (Decimal.) Since it is desired to know how many hundredths of 1200 is 536, divide 536 by 1200

$$1200\overline{)536.}\quad .44\tfrac{4}{6} = 44\tfrac{4}{6}\%.$$

(2) By selling for $2.27 an article costing $2.00, what is gained in per cent?

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Solution. (*1% Method.*)
A gain of 1% = $.02.
$.27 (total gain) divided by $.02 equals number of per cent. gained.

$2.27 Sell pr.
2.00 Cost
$.02)$ .27 Gain.
13½, or per cent. of gain.

(3) The Charleston basket ball team wins 18 out of 24 games or —— % of its gains.

Solution. (*Fractional.*)
The question is: 18 is how many hundredths of 24?

$$\frac{18}{24} = \frac{3}{4} = \frac{75}{100} = 75\%, \text{ Ans.}$$

It is evident that the *rate equals the quotient of the percentage divided by the base.*

ORAL DRILL EXERCISE.

Find missing values:

1. 1/4 = ——% of 1/2, of 1, of 3/4, of 1/8, of 2, of 1/16, of 3/8.

2. 4.8 = ——% of 2.4, of 4, of 1.2, of 9.6, of 48, of 6.

3. What per cent. of

2400 is 16?	1.6 is .4?	4 is .8?	2/3 is 1/2?
300 is 29?	.2 is .06?	7½ is 1?	4 is 12?

4. $2.40 are ——% of $60. 6 qt. are ——% of 20 qt.
760 lb are ——% of 300 lb. 90 sq. yd. are ——% of 2000 sq. yd.

5. What per cent. of

3 is 4?	25 is 30?	2.5 is 5?	1/4 is 1/3?
4 is 3?	25 is 5?	.25 is .5?	3/8 is 5/8?

6. 4 is what per cent. of 5? 5 of 3? 3 of 4? 4 of 6? 6 of 4? 6 of 8? 8 of 6? 8 of 10? 10 of 15? 15 of 20? 20, increased 20 %, of 30? 30, decreased 1/5, of 40?

GENERAL EXERCISE.

1. What per cent of the cost of a $160 horse is lost by selling it for $120?

2. I invest $6000 in a partnership capitalized at $27,000, under an agreement to share in profits according to investment. What per cent. of any profits should I receive

3. If $64 is the cost of the premium on a $4000 fire insurance policy, what is the rate of premium?

4. What per cent. has a city's population increased in a year, if it now has 240,000 inhabitants, and a year ago had 220,000?

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5. How does a profit of \$4 on a \$20 investment compare in rate with a profit of \$16 on a \$60 investment?

6. What per cent. is made in a class test if 943 credits are obtained out of a possible 1140?

7. A new printing press has a maximum output of 35,000 copies per hour, compared with an output, for the machine it replaced, of 27,500. What is the per cent. of increase?

NOTE. Percentage is frequently employed in technical writing and statistics, to show proportions, rates of increase and decrease and for comparisons of values.

8. Find the missing rates in this table to three places decimally:

WHERE EXPORTS FIND MARKETS.

Countries.	Total Imports.	Share from U. S.	Per Cent.
United Kingdom,	\$2,958,289,000.	\$638,006,000.	?
Germany,	1,696,660,000.	236,082,000.	?
Italy,	398,463,000.	45,956,000.	?
China,	349,913,000.	36,304,000.	?
Canada,	290,361,000.	175,862,000.	?
Australia,	217,676,000.	22,549,000.	?
Japan,	208,554,000.	34,834,000.	?
Spain,	204,401,000.	23,006,000.	?
Argentina,	197,974,000.	27,908,000.	?
Denmark,	166,837,000.	26,832,000.	?
Brazil,	161,587,000.	18,518,000.	?
Mexico,	109,884,000.	72,509,000.	?

ORIGINAL EXERCISE.

Write and solve original problems illustrating the use of rates per cent.:

1. In measuring the increase of sales of a business during one year.

2. In measuring the loss of a firm through bad debts.

3. In comparing the expense of repairs on a machine with its original cost.

4. In comparing the votes of successful and defeated candidates for office, in a political election.

EXERCISE.

Compute missing values in this article.

BASEBALL AVERAGES.

Baseball records furnish an excellent illustration of the use of rates per cent. in showing relative standing. Thus, the standing of any team is

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determined by the per cent. of games won out of games played. This rate is computed to three places decimally. For example, the standing of the American League on September 25, of a recent year was:

AMERICAN LEAGUE STANDING.

Team.	Won.	Lost.	P. C.	Team.	Won.	Lost.	P. C.
Philadelphia	84	51	.622	Detroit	70	69	.504
Chicago	82	54	?	New York	66	66	?
Boston	69	66	?	Washington	56	80	?
Cleveland	70	69	?	St. Louis	48	90	?

Owing to bad weather, or other causes, some scheduled games may be postponed, so that on any particular date, the number of "games played" by each team may vary. (Is this true above?) It is evident that per cent. rates offer a better means of comparison than common fractions. (Show why.)

Of course, averages change every day when teams are playing. On Sept. 26, Washington added a game to its "won" column by defeating Chicago, whose —— column was increased by 1. Detroit lost to Philadelphia; New York defeated St. Louis; and Cleveland beat Boston. At the close of September 26, the standing would appear as follows:

AMERICAN LEAGUE STANDING.

Team.	Won.	Lost.	P. C.	Team.	Won.	Lost.	P. C.
Philadelphia	85	51	?	.....	—	—	—
.....	—	—	—	.....	—	—	—
.....	—	—	—	.....	—	—	—
.....	—	—	—	.....	—	—	—

Towards the close of the season possible results are often forecasted. On Sept. 26, for example, these possibilities might be considered:

1. Chicago and Philadelphia have each eight more games to play. If Chicago wins six and loses two, while Philadelphia wins four and loses four, will Chicago be in the lead?

2. Philadelphia plays the first four of these games in Chicago. If Chicago should win three would Philadelphia lose the lead?

3. A Philadelphia enthusiast asks "How many of these last eight games *must* Philadelphia win to be sure of the championship?"

In a similar way the records of pitchers are determined. Thus a pitcher winning 25 games and losing 18 would have an average of games won of .—.

In batting, the player who makes 93 safe hits in 240 times at bat is said to have a batting average of .—.

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The average of the players "in the field" are based on three factors: (1) the number of opposing players they "put out"; (2) the number of plays in which they "assist" in putting out a player; (3) any attempted plays in which the player fails to complete a play within his power (errors). The players' fielding averages are expressed decimally as the per cent. that (1) plus (2) is of the sum of (1), (2) and (3).

FIELDING AVERAGES.

Player.	Outs.	Assists.	Errors	Per Cent.
C. Baker (Outfielder)	217	109	21	?
J. Town (1st base)	824	103	37	?
S. Crane (2d base)	162	219	33	?
T. Parker (Shortstop)	124	137	41	?

FINDING THE BASE.

INTRODUCTORY EXERCISE.

1. If 14 = 2% of a number, 1% = ?; 100% or the number = ?
2. Find 100%, if 2% = 4, 32, 420, 6.
3. Find the numbers of which 33⅓% = 35, 1240, 520, 9, 1/4.
4. Find 100%, if 300% = 700, 45, 6.

ILLUSTRATIONS. (1) What is the cost of an article that sells for $618, at 3% profit?

Analysis and Solution. Let the cost equal 100%.

The selling price equals 103% of the cost and the cost equals 100/103 of $618.

$600, cost.
1.03)$618

(2) 40 pounds of Mocha coffee will form 66⅔% of a mixture of how many pounds of Mocha and Java?

Solution. 40 lb. = 66⅔% or 2/3 of the mixture.

40 = 2/3 of mixture.
20 = 1/3 of mixture.
60 = 3/3 of mixture, or the total quantity.

It is evident that the base equals the percentage divided by the rate.

ORAL DRILL EXERCISE.

1. When is the "1% method" superior to the aliquot or fractional method? Which is better in the following?

40 = 10% of ?; 27 = 9% of ?; 62 = 50% of ?; 28 = 4% of ?; 75 = 33⅓ of ?

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2. 24 = 4% of ?; 25% of ?; .4% of ?; 1/3% of ?; 150% of ?
3. 420 = 6% of ?; 250% of ?; 40% of ?; 25% of ?; 1/4% of ?
4. 2/3 = 25% of ?; 3% of ?; 10% of ?; 400% of ?; 1/2% of ?
5. 426 = 25% of ?; 6750 = 50% of ?; .06 = 75% of ?; 1/4 = 800% of ?
6. 16 = 1/4% of ?; 480 = 2/3% of ?; 12,000 = 1½% of ? .006 = 4/3% of ?

EXERCISE.

Find missing values:

1. 297.6 = 3/5% of —.
2. 97,485 = 20% of —.
3. 2.0406 = 1/8% of —.
4. 45.297 = 7% of —.
5. 1605 = 120% of —.
6. 29,623 = 47.2% of —.

EXERCISE.

(Each problem illustrates a practical use of this phase of percentage.) Find missing values.

1. What sum must I invest at a profit of 7½% per year to earn a profit of $900 yearly?

2. A certain cloth shrinks 6½% in dyeing. In order to have 640 yd. of finished material, one must dye — yd.

3. "Thus the price has fallen 16% in two months, and the money that would buy 100 shares two months ago, will now buy — shares."

4. A merchant sends a manufacturer an order for 600 articles, enclosing a draft in payment, and writing, "In case you grant me a discount from your last quotation, send all the goods the money will buy." The manufacturer allows a discount of 20% from his last quotation. What number of articles should he ship his customer?

(NOTE: "grant a discount" means "lower the price.")

5. For a certain contract a carpenter requires 72,000 ft. of flooring. As he knows the waste in cutting will average 25%, he should order — ft.

6. If a farmer offers to sell his land "at 75% of its real value" for $8250—in order to dispose of it at once—he places what valuation on it?

ORAL DRILL EXERCISE.

ALL CASES.

Find missing values:

1. 16 = 4% of —; that result = 20% of —; that result = 200% of —; that = 500% of —; that = 20 times —; that = 25% of —; that result = 400% of —; that = 1/2% of —.

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2. A percentage study of 36.

(*a*)		(*b*)		(*c*)
1%	of 36 = —; 36 =	4% of —	36 = — % of	1200
5%		100% of —		200
20%		80% of —		3000
150%		7% of —		30
400%		3% of —		6
33⅓%		33⅓% of —		1/2
1/2%		2/3 of —		3.6
2 3%		2½% of —		.9
1¼%		1½% of —		40000

EXERCISE.

1. 12½% of 372 = 1¼% of —.

2. 8% of 33⅓% of 960 = —% of 1200.

3. 175% of 36 = 1/3 of 24% of —.

EXERCISE.

AN ARGUMENT FOR THE PANAMA CANAL.

Trip.

New York to San Francisco via Panama, 5278 M.; via Cape Horn, 13,340 M.
San Francisco to Liverpool via Panama, 7907 M.; via Cape Horn, 13,678 M.
New York to Manila
via Panama 11,412 M.; via Cape Good Hope, 13,555 M.
New York to Yokohama
via Panama 9692 M.; via Cape Good Hope, 15,178 M.
New York to Sydney
via Panama, 9560 M.; via Cape Good Hope, 12,218 M.

Present these figures in tabular form. Express also, the distance via Panama, (*a*) as a per cent. of other routes; (*b*) via the other routes as a per cent. of the Panama route.

ANALYSIS DRILL.

MIXTURES AND COMPOUNDS.

ORAL.

1. 20 lb. of one grade of tea are blended with 30 lb. of a second grade, forming a mixture of ? lb., of which ?% is first grade. The quantity of first grade is ? % of that of second grade.

2. 30 lb. are blended in the ratio of 5 : 4 with ? lb. of a second grade. The second grade forms ? % of the mixture and equals in quantity ? % of the first grade.

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3. How much witch hazel is required for 8 gal. of a 12% solution?

4. ? lb. Mocha and ? lb. Java are required for a mixture of 180 lb., if the quantity of Mocha exceeds that of Java 25%.

WRITTEN.

1. What quantity of each ingredient is required for 25 gal. of a mixture of hard oil diluted 40% with turpentine?

2. A mixture of tallow and rosin is to contain 15% tallow. How much tallow is required for 40 lb. of rosin?

3. 22 lb. of standard bullion, 80% pure, will make — lb. of standard purity (90%).

4. The products of the distillation of 100 lb. of average gas coal of a certain mine approximate: 64 lb. coke, 6½ lb. tar, 12 lb. ammonia liquor, 14 lb. purified gas, and — lb. minor products and waste. Express composition in per cents. What amount of coal (2240 lb. to the ton) must be distilled to manufacture 1,500,000 cu. ft. of gas, if gas measures 10,540 cu. ft. to the ton?

5. The composition of Babbitt's metal is 3.7% copper, 89% zinc, and 7.3% antimony. Determine the quantity of each metal required for an order of 16 T.

6. The analysis of a metallic paint, per 5000 parts, is: 3943 parts iron peroxide, 165 parts alumina, 598 parts silica, 254 parts water, 16 parts lime, 14 parts manganese, — parts minor elements. Express the paint formula in per cent. form.

7. A common asphalt varnish contains 3, 4 and 18 parts respectively of asphaltum, boiled oil and turpentine. Give formula in percentage form. What quantity of asphaltum is required to combine with 600 gal. of the proper mixture of oil and turpentine?

8. A mixture of three grades of material prepared in proportions of 20%, 30% and 50%, is found to be markedly improved by reversing the proportions of first and second grades. It is decided to change 120 lb. of the old mixture to the new by adding whatever quantities are necessary. What quantities should be added? What is the quantity of the new mixture?

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CHAPTER XXX.

ELEMENTARY PROFIT AND LOSS.

INTRODUCTORY.

1. What do I gain by selling an article that costs me \$400 at a price 25% higher?

2. If I sell for \$40 a desk that cost me \$60 what do I lose? What per cent. of the cost do I lose?

3. Compare these terms with the corresponding terms of percentage: Cost, selling price at a loss, selling price at a profit, rate of gain or loss.

Practically all transactions in business are carried on directly or indirectly for profit, or to prevent or reduce loss, or to decrease expenditure. A business man continually faces the question, "Does it pay?" The question may concern the selling price of an article, the results of some business policy, the comparison of investments, or endless other conditions. In most cases arithmetic is used to determine the answer. Percentage is very commonly used to measure results where profit may be expressed in money.

ILLUSTRATIVE EXERCISE.

Find missing values:

1. Tables bought at \$8 are sold at \$10, at a profit of —% on cost.

2. I sell apples costing \$2 per barrel at \$3 and dispose of 6 bbl. per day. By selling at \$2.50 I find my sales average 20 bbl. per day. Lowering my selling price increases my profits \$— or —% per day.

3. By the installation of new machinery, a manufacturer reduces the force required for the same output from 500 to 420, thus affecting a reduction of — %.

4. A manufacturer, by a change of process reduces the cost of manufacture of a rug from \$4.00 to \$3.80. If his selling price remains \$4.80 his profits increase — %.

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NOTE. The principle of *profit* as a *saving* influences our daily acts as individuals.

5. I can buy potatoes at an up-town store for $3.00 per bbl., and at a down-town market for $2.65. By buying at the latter, I save — c.

6. I can buy a working bench for $16.00 or I can buy material for $4.00 and make it myself, at a saving of $ —, or — %. If, however, my time is worth $1.50 an hour for the four hours required, I save only $—.

Ordinarily, profit or loss is reckoned as a percentage of cost. The cost, however, is an indefinite quantity. In addition to what he pays the manufacturer for goods, the merchant pays freight, insurance and storage, the expense of sale, and the cost of keeping bookkeeping records, etc. Thus the question, often arises as to what incidental charges shall be added to first cost, or subtracted from selling price to determine a base. Partly to avoid this difficulty, some dealers reckon individual profits as a per cent. of sales.

ILLUSTRATIONS. (1) I buy 6 desks @ $31 each, paying $6 freight. Find the selling price each at 25% profit.

Solution. Net cost as base.

15% or 1/4 of $31 = $7.75 profit. $31 + $7.75 = $38.75 selling price
+ $1.00 freight = $39.75, gross selling price.

Solution. Gross cost as base.

The freight is $1.00 per desk.
$31 + $1 = $32, gross cost per desk.
5/4 of $32 = $40, sell. price.

(2) One desk is sold for $36 or ? % profit.

Solution. Net cost as base. The profit, $36 − $31 = $5.
The rate, 5/31, or 16.1%.

Gross cost as base. The profit, $36 − $32 = $4.
The rate, 4/32, or 12.5%.

Sell. price as base. The profit, $36 − $32 = $4.
The rate, 4/36, or 11.1%.

NOTE. *In this book profit will be reckoned as a per cent. of net cost unless otherwise stated. Read problems carefully.*

The department store offers a good illustration of profit reckoning. Here the selling price is fixed by the merchandise manager, or by the department buyer. Computations are on a percentage basis. The rate of profit, however, varies with

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the article or department. Rapid selling articles yield a lower rate than slow selling goods. The latter may sell at 30% profit, where the rapid seller brings 5%. The department, however, that does $60,000 worth of business annually on $10,000 capital can deal on a smaller margin of profit than one that does $80,000 business on $15,000 capital. By the more rapid "turning over" of capital, goods sold at low profits may bring a department as high an average as others sold at a high rate. Suppose an article sold a month after purchase at 5% profit, the original cost being reinvested in the same goods, and the process continued for a year. The profit on original capital is then 5×12, or 60% for the year. The tendency is to make all departments yield the same rate annually by means of higher marking of slow selling goods.

To the price at which the goods are billed at the store, are added the fixed charges for rent, bookkeeping, selling, delivery, etc. These have been reduced to an average per cent. of cost for each store or department—ranging from 15% to 25%. Many of the stores reckon the profit as a per cent. of sales. It is then a simple matter to approximate any day's profits from the amount of sales.

I. Gains, Losses and Selling Prices.

EXERCISE.

Illustrating simple phases of profit and loss.

Answer orally if possible.

1. Find the loss on desks costing $25 if sold at 10% loss; at 20% loss; at 12½% loss; at 5% loss; at 40% loss.

2. Find the gain on toys costing $1.60 and sold at 25% gain; at 20% gain; at 5% gain; at 15% gain.

3. Find the gain or loss if goods cost $1.20 a doz. and are sold at 15 c each; at 20 c each; at $2.00 per doz.; at 5% profit; at 20% loss; at $1.05 per doz.

4. From what known values may the following be computed: (*a*) Selling price at gain, (*b*) gain, (*c*) selling price at loss, (*d*) loss?

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5. Find the missing values by inspection.

	Cost.	Rate Gain or Loss.	Selling Price.	Gain or Loss.
a.	$56.	20% gain	$——	$——
b.	9.60	16⅔ % loss.	$——	$——
c.	4260.00	25% gain	$——	$——
d.	3.88 doz.	——	50 c each	$——
e.	1.80	——	$——	39 c

EXERCISE.

1. What profit is realized by buying gloves at $9.20 per doz. and selling them at $1.15 per pair?

2. The goods in the crockery department of a store are marked at 20% profit and 18% store expense. What is the selling price of plates costing $9.60 per doz.?

3. The book buyer of a department store pays $7.50 for ten copies of a popular novel which he disposes of at $1.08. As each lot is disposed of he re-invests the original sum in a new lot, until he has disposed of 120 copies. He has "turned over" his capital — times, and gained a total of $—.

4. Apples are bought @ $2.40 per bbl. (3 bu.) and are retailed at 90 c. per bu. Determine the final loss or gain, on a lot of 60 bbl., the average loss by decay being 15%.

5. A dealer retails at $6.50 a grade of flour that costs him $5.00 per bbl. The wholesale price is raised 10%, and the dealer immediately raises his price to $7.50, but his sales fall off 10% in amount. How does this affect his net profits?

II. Profit as a Rate of Selling Price.

(Omit if desired.)

ORAL EXERCISE.

1. If 30% of selling price is profit, how many per cent. is cost?

2. If $14 is the cost, in ex. 1, find the selling price and profit.

3. The goods in a grocery department are sold at a profit of 15% on selling price. What are the profits on sales of $6400?

4. Find the selling price of a table costing $6.00 to gain 25%. To gain 33⅓%; to lose 20%; to lose $1.40; to gain 20%.

EXERCISE.

1. Find the selling price of desks costing $16.00, to allow for 20% store burden (based on cost) and 25% profit.

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2. Find the profit on an invoice of $2400 worth of goods, sold at 20% profit, the expense of sale being $98.60.

3. Find the selling price of silks costing $1.80 per yd., if store burden is 20%, profit allowance 20%, waste in cutting 5%.

4. Which base for profit, cost or selling price, yields the higher profit at any given rate? Illustrate.

III. Per Cents of Gain or Loss.

ORAL EXERCISE.

1. By selling a $2.00 engraving for $2.40 a profit of — c or —% of the cost is realized.

2. A profit of 75 c on a $5.00 article is a profit of —%.

3. A carriage-maker buys a cart for $40.00, expends $15 in repairs and sells it for $69.00, making —% profit.

4. Does it pay a dealer to reduce his profit from 15% to 9%, if his sales are thereby doubled?

5. Compute the rate of gain or loss.

	Cost.	Selling Price.		Cost.	Gain.	Loss.
(*a*)	$3.20	$3.60	(*d*)	$600	$ .90	
(*b*)	4.00	3.98	(*e*)	15.00	1.20	
(*c*)	8.40	9.60	(*f*)	4000.00		$180.00

	Selling Price	Gain.		Selling Price.	Loss.
(*g*)	$8.00	$2.00	(*i*)	$ 12.00	$ 3.00
(*h*)	15.00	5.00	(*k*)	150.00	30.00

7. Find the rates in *g*–*k*, ex. 5, if based on selling price.

EXERCISE.

1. Skates purchased at $11.40 per doz. are retailed at $1.60 per pair, yielding a profit of —%.

2. A real estate dealer bought a house at auction for $3600. He expended $160.00 in repairs, $95.00 in painting, $5.00 for advertising and then sold the property for $4250. Compute his gain per cent. on investment.

3. A dealer purchased 60 doz. straw hats @ $9.00 a doz. He sold 42 doz. @ 40% profit, and 6 doz. @ $1.25 each. If he disposes of the balance at cost, what is his net rate of gain or loss?

4. A novelty, manufactured at a cost of 40c, is to be retailed at a price to yield manufacturer, jobber, wholesaler and retailer each a profit of 10c. What rate of profit is realized by each?

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5. A dealer sells Mocha coffee, costing 25c, @ 32c; and Java, costing 28c, @ 36c. A mixture of the two grades containing one-third Mocha, readily sells @ 40c. Compare rates of gain on straight grades and blends.

6. If a dealer sells at 30% below his regular price of 30% profit, he loses what per cent. of profit?

IV. The Cost.

(Answer orally if possible.)

1. By selling a coat for $6.00 a dealer gains 20% on cost. What is the cost? Suggestion: $6.00 = 120% of cost.

2. If a profit of 45c each is realized on hats sold for $1.65 what do they cost per dozen?

3. $40.00 represents 5% profit on what sum? 25% loss on what sum?

4. Find the *cost* from the values given:

	Selling Price.	Gain.		Selling Price.	Loss.
(*a*)	$ 5.40	$1.40	(*c*)	$ 2.50	$.18
(*b*)	$240	25%	(*d*)	$960.	33 1/3%

	Gain.	Rate.		Loss.	Rate.
(*e*)	$ .84	7%	(*g*)	$ .52	4%
(*f*)	$30.	20%	(*h*)	$84.	25%

5. Markward offers to sell a piece of land at "40% below cost," for $750. He evidently values the land at what price?

6. If the retail price of an article is fixed by competition at $5.40 the cost of manufacture must be kept down to what figure to allow for 15% profit to the manufacturer and 20% to the dealer?

Examples in Savings.

7. A manufacturer, by a change in process, reduces the cost of office desks, which he sells direct, for $30.00, from $21.00 to $18.60. The saving in cost is ? % and the increase in profits ? %.

8. By the installation of new machinery, the time required for certain work was reduced from 17 hours to 14½ hours. If labor costs 48c per hour this represents what saving on each piece of work?

9. By a change in the boiler equipment of a factory, the coal consumption was reduced from 34 tons per day to 31½ tons, or ? %.

GENERAL EXERCISE.

1. Does it pay to buy apples @ $2.40 per bbl. (2½ bu.) and to sell them @ 32c per pk., if 10% is allowed for decay?

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2. What rate of profit is obtained by retailing eight articles at the price they cost per dozen?

3. A blend of 32c and 18c coffee, mixed in the ratio of 3 to 4, is to be put on the market in two-pound packages, and sold at 25% advance on cost. Find the selling price per package.

4. Does it pay to lower the selling price of tea (costing 48c) from 80c to 72c, if the change in price leads to a 10% increase in amount of sales?

5. What is the per cent. of gain on cost if 16% of what is received for an article is gain?

6. What retail selling price, on an article costing $8.00 to manufacture, will yield profits of 25% to the retailer and 20% to the wholesaler and to the manufacturer?

7. At a price yielding 6% profit, the sales of a certain article equal 3600 pkg. per week; at a price yielding 15% profit, sales average 2500 pkg. per week. Which is the more profitable price and why?

8. A 60c grade of pepper is blended with a 30c grade in the ratio of 3 to 5, and the blend is retailed at 50c. By increasing the proportion of the first grade to 50%, sales increase 20%. Does the change of proportion pay, if there is no increase in selling price?

ORAL DRILL EXERCISE.

I.

The cost of a certain make of chair is $6.00.

1. *What is the selling price each* at 10% gain? (Give analysis.) At 5% loss? At 8% gain? At 20% loss? At 15% profit? At a profit of 36c? At a profit of $1.62 for three chairs (analysis)? What is the selling price of two chairs at 20% gain? Of half a dozen at 20% loss?

2. *What is the selling price of one chair* if 20% of the selling price is profit? If 25% of the selling price is loss? If $1.40 is loss? If 87 cents is gain?

3. *What is gained* by selling one chair at 5% profit (analysis)? On three at 10% profit? On one dozen at 20% profit? On three sold for $20.60? *What is lost* on one at 5%? On six sold at 12½% below cost? By selling three for $17.00? What is the profit if 20% of the selling price is profit? (Analysis.)

4. *What per cent.* is gained by selling one chair for $6.50? Three for $24.00? Two for the cost of three? Five for what one-half dozen cost? *What per cent.* is lost by selling at 18c below cost? At $4.73? By selling three for $15.00? By selling three for the cost of two?

What is the rate of gain or loss, *based on selling price*, on one chair

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sold for $7.20. (Give analysis.) On three sold for $21.00? On two sold for $10.00. On four sold at price equal to the cost of three?

5. The selling price of an article is $3.60.

The cost must be limited to $? to gain 20%? To gain 25% based on selling price? To gain 40%. To lose $1.23? What per cent. of profit on cost is realized by buying at $2.40? At $3.00? At $3.20?

II.

Give and solve an original example to illustrate each case.

1. From what known values may *cost* be computed? (Several cases.)

2. From what known values may selling price be computed? Rate of loss? Rate of gain? Gain? Loss?

3. What values can be computed from cost and loss? From gain and rate? From selling price and rate? From cost and selling price?

4. Cost and ? determine rate of gain.

5. Selling price and ? determine rate of profit

FOR DISCUSSION AND ILLUSTRATION.

1. Why may a dealer *lose*, who sells an article for the price he paid for it?

2. Could a dealer lose if he sold an article for *more* than he paid for it?

3. What may increase the *first* or *direct* cost of an article?

4. Why may a dealer have less of a certain lot of mdse. to sell than he bought?

5. How may the following affect the selling price of an article: overstocking; competition; style; fire; depreciation; season of the year; locality?

6. What may lead to the sale of parts of a lot of mdse. at different rates?

7. What is meant by "quick sales and small profits"?

8. May sales at a fixed price increase, and yet profits decrease?

9. Under what circumstances may profits increase, if selling price decreases?

10. Under what circumstances may profits decrease, if the selling price increases?

MARKING GOODS.

In many businesses it is a common custom to mark goods, directly or by means of a tag, with the selling price and, at times, with the cost. The cost, and sometimes the selling

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price, is written in a *key* or private symbol, so that it may be known only to certain employees of the business. While some keys are exceedingly complicated, consisting of arbitrary characters, the usual form consists of a word or phrase containing ten different letters, each of which stands for a particular figure. The letters may be numbered forward, backward, or arbitrarily.

ILLUSTRATION. Key: Now be quick.

(1) N O W B E Q U I C K or (2) N O W B E Q U I C K
1 2 3 4 5 6 7 8 9 0 0 9 8 7 6 5 4 3 2 1

By using different keys, the cost may be concealed from all except marker and manager, and the selling price from all except marker, manager and selling force. There is a tendency, however, to mark the selling price in plain figures.

In writing with a *key*, the last two letters represent cents. If any figure occurs twice in succession, an arbitrary letter, called a *repeater*, is used the second time. Arbitrary letters are sometimes used for fractions, but are seldom needed, as fractions of a cent are uncommon in the marking of such goods as are commonly "tagged."

ILLUSTRATION. Example. Write the marking tag for canes, no. 221, costing \$2.80 and selling for \$3.60, using the key (1), as above.

Solution. Substituting the corresponding letters for the figures, \$3.60 is written "wqk," and \$2.80 "oik." The tag is written

No. 221
wqk
oik

NOTE. If the selling price were \$4.00, and *s* were repeater, the price would be written "bks."

EXERCISE.

1. Interpret the following, based on the key "hypodermic":

$$\frac{\text{peo}}{\text{yic}},\ \frac{\text{hoc}}{\text{hcx}},\ \frac{\text{iy}}{\text{er}},\ \frac{\text{poxc}}{\text{ydcx}},\ \frac{\text{opd}}{\text{pyc}},\ \frac{\text{hmr}}{\text{rm}}.$$

"x" is repeater.

2. Write, using the key "Mayflower," and any repeater:

$$\frac{37}{5},\ \frac{4.92}{3.80},\ \frac{6.75}{5.40},\ \frac{19.20}{11.85},\ \frac{1.10}{65},\ \frac{99}{80},\ \frac{4.75}{2.60},\ \frac{12.32}{9.64}.$$

3. Repeat (2), writing selling price in figures, and the cost in the key "Now be quick."

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EXERCISE.

Write the cost and selling price in the following examples, using these keys:

Cost Key.

Selling Key.

W A S H I N G T U B M
1 2 3 4 5 6 7 8 9 0 R

	First Cost.	Charges.	Gain.
1.	$2.40	$ .30	20%
2.	3.25	.45	25%
3.	.88	12½%	15%
4.	14.56	.60	30%

	First Cost.	Sell. Ex.	Gain or Loss.
5.	$2.40	5% of S. P.	20% of S. P.
6.	3.82	10% of S. P.	25% of S. P.
7.	9.60	10% of S. P.	20%
8.	15.00	$1.00	25%

Using the key "Mayflowers," write cost and selling price per article for the following:

	Cost.	Per.	Gain.		Cost.	Per.	Expense.	Gain.
9.	$9.00	dz.	25%.	12.	$15.00	dz.	$.72	20%
10.	4.80	C.	20%	13.	7.20	gr.	5.00	25%
11.	6.40	cas. (2 dz.)	15%.	14.	1.26	1/2 dz.	.12c	20%

15. Using the key "Washing tub," mark the goods in the following invoice at an advance of 25% on gross cost.

PHILADELPHIA, January 29, 19—.

MESSRS. TOPHAM AND CHASE,
Pittsburgh, Pa.
BOUGHT OF C. R. NEWMAN & Co.
Terms: 60 days.

1829	2½	dz. Neckwear,	3.60				
47	1½	"	2.80				
639	3	"	4.50				
426	4	Outing Shirts	9.60				
358	3	Caps,	3.25				
		Less 10%.					

Expressage, $2.80.
Selling Expense, 10%.

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Individual Original Work.

Design a marking sheet showing first cost of each class of goods in bulk, cost per article, discounts, gains, terms, selling and list prices, etc. Make entries for at least fifteen articles.

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CHAPTER XXXI.

COMMERCIAL DISCOUNT.

One of the most common and important business applications of percentage is the expression and reckoning of trade discounts.

ILLUSTRATIVE EXERCISE.

1. On Jan. 6, J. P. Robertson buys 60 bbl. flour @ $5.00. He agrees to pay in 90 days, but is offered 3% off (discount from) the debt if he pays within ten days. Up to Jan. 16, $? will settle the bill; after that, and until April 6, $?.

NOTE. These terms may be written "90 days net, 3% 10 da." or "Net /90; 3/10." In business, the credit terms are usually 30 da., 60 da., or 90 da. Rates of discount vary with firm and customer. Interest is sometimes charged for *overdue* payments.

2. A Chicago mail order house allows 2% discount on orders of $10; 3% on orders of $20; 5% on $50 orders, etc. What discount should a customer receive on a $20 order? (Why?)

3. The Virginia Brick Co. allows 5% discount on orders for over 10 M. brick. 26 M. @ $12 per M. cost what sum?

4. The catalog price of 5″ metal piping is $1.40 per foot. The market price is 20% less. 1000 ft. at the market price cost what sum?

NOTE. Market prices of some articles vary with demand and with the cost of raw materials used in their manufacture. To avoid constant reprinting of trade catalogs, prices are *listed* high, and customers are notified of current discounts on goods that they desire.

5. The Sandford Pub. Co. issues a nature book to retail at $1.80. The book is sold to dealers at 20% discount, enabling them to realize what per cent. of gain?

NOTE. Publishers and some manufacturers fix the retail price of their products, selling to dealers and jobbers at a discount, and to private customers at 'list.'

Solution. $1.80 Retail price.

20% or 1/5 of $1.80 = $.36, discount.

$1.80 − $ .36 = 1.44, cost to dealer.

$ .36 ÷ $1.44 = .25 or 1/4, or 25%, the rate of gain.

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It is evident that a *commercial discount* is an allowance or subtraction from the *list* or marked price of goods for such business reasons as: (1) payment before due; (2) size of order; (3) reduction of *list* to market price; (4) to meet competition; (5) to allow for profits to middlemen who are expected to sell 'at list,' etc. These discounts commonly are expressed as rates per cent. Many discounts are aliquot rates, thus permitting short methods of computation.

EXERCISE.

(Answer orally if possible.)

1. To what terms in percentage do the following correspond: Rate, of discount, list price, discount, reduced or net price?

2. Find the per cent. of list price paid for goods bought at one of the following discounts: 40% (suggestion, 100% − 40%), 30%, 28½%, 63%, 75%, 1/3%.

3. Determine the net cost of:

200 ft. of piping @ $1.20, less 20%.
200 pr. gloves @ $3.60, less 12½%.
A $500 piano, less 40%.
40 valves @ $.75, less 15%.

4. How do terms of "90 days, 4/30" affect the settlement of a $950 bill of goods, bought February 7 and settled March 1?

5. Check up the Randall Mfg. Co.'s bill of $552, as follows: 6 sewing machines @ $40, less 25%, $180; 4 washing machines, @ $27, less 30%, $72; 3 farm wagons, @ $85, less 20%, $202.

EXERCISE.

A billing clerk handled and checked the following transactions on Oct. 24. Find missing values.

1. Received of S. P. Adams, check for $—— in full settlement for 1200 lb. Japan tea, @ 23c, bought on Oct. 18, on terms of 60 da. net; 3/10.

2. Mailed Waukeska Canning Co. check for $——, for invoice of Oct. 22, 300 bbl. Cerota Flour, 1/4 s. (i. e., 1/4 bbl.) @ $5.80 per bbl. Terms, 30 da.; 1%, 10 da.

3. Checked Parker Bros. invoice of 120 bbl. S. T. Flour @ $4.10; 12 sacks Graham Flour, @ $2.80; 40 doz. C. Tomatoes @ $.92; 144 bx. K.

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Soap, @ $7.00. Discount, 12½%. Terms: 30 day note, $400; cash for balance. Mailed check for $——.

4. Received check for $—— from Franklin Cereal Co., being rebates on September sales of $746.20, at 12½%.

NOTE. Manufacturers of certain lines of groceries sell to wholesalers at prices at which the latter sell to retailers. At intervals the manufacturers return wholesalers a discount or rebate, based on the amount of their monthly sales.

Series of Discounts. Often two or more discounts are allowed on the same item; one, for example, for quantity, and a second for early payment. In such cases, one discount is reckoned on the list price; the second on the list price minus the first discount and so on.

ILLUSTRATION. Successive discounts of 33⅓%, 20%, 5%, are allowed on a shipment of 4000 ft. iron pipe @ $1.50, reducing the cost to $——.

Solution (a).

4000 × $1.50		= $6000 list price.
33⅓% = 1/3	1/3 of 6000	= 2000 first discount.
		$4000 second price.
20% = 1/5	1/5 of 4000	= 800 second discount.
		$3200 third price.
5% = 1/20	1/20 of 3200	= 160 third discount.
		$3040 net cost.

Solution (b).

	100%	= list price.
		100%, list price.
		20 one discount.
		80% second price in per cent.
	5% of 80%	= 4% second discount.
		76%
33⅓% of 76% = ¼ of 76		= 25¼% third discount.
		50¾% net cost in per cent.

50¾% of $6000 = $3040.

NOTE. 100% − 50¾% = 49¼%, which is called the single discount equal to the series.

FOR PROOF OR DISCUSSION.

1. Why does it pay a dealer to allow discounts for the causes mentioned?

2. PROPOSITION. The order in which the discount rates for a series are used does not affect the final result.

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3. Illustrate with the series 33⅓%, 40%, 10%, the possible arithmetical advantage of changing the order of discounts.

4. PROPOSITION. The single discount equal to a series is always less than its sum.

5. PROPOSITION. The single discount equivalent to a series of two discounts, equals the difference between the sum and product of the series.

ORAL EXERCISE.

1. Find the cost, in per cent. of list price, if goods are bought at discount of: (See solution (*b*), illustration 2, page 238.)

40%, 10%, 5%.	5%, 10%.
80%, 3%.	50%, 20%, 10%.
16⅔%, 10%, 25%.	10%, 10%, 10%.
10%, 33⅓%, 40%.	3 20s (*i. e.*, 20%, 20%, 20%.)

2. Find the single discounts equivalent to the following series:

40%, 25%.	20%, 25%.	10%, 25%, 33⅓%.	16⅔%, 20%.
50%, 10%.	50%, 50%.	10%, 15%	20%, 25%, 5%.

3. Required, the net cost:

(*a*) Of 400 lb. castings @ 10c, less 20%, 25%.

(*b*) Of 50 pr. gloves @ $2.00, less 10%, 25%.

4. Compare these offers for goods of the same quality:

(*a*) List price $800. Discounts 25%, 33⅓%, 10%.

(*b*) List price $600. Discounts 16⅔%, 25%.

5. What case of percentage is involved in these examples.

EXERCISE.

1. Find the net receipts from the following sales of merchandise:

$326.50 at discounts of 10%, 20%.

$415.10 at discounts of 30%, 5%.

$300.00 at discounts of 40%, 20%, 10%.

2. Bought, Aug. 30, on terms 90 da., 3/30, 5/10, 4 dz. knives @ $8.00, less 15%; 2 dz. sauce-pans @ $3.40, less 10%, 10%; 3½ dz. wash boilers @ $38.80, less 30%, 5%. Paid, Sept. 16, $——.

3. A library has an opportunity to purchase books at 40%, 5% off. A purchase fund of $420 per year will buy $—— worth of books at list prices.

4. In selling a bill of 80 rockers @ $8.40, less 25%, 10%, a clerk reckons the equivalent discount at 31%. What overcharge may the customer claim?

5. Both Brown & Co. and A. C. Lewis offer me 8000 ft. of choice cypress @ $80 per M., but Brown & Co. quote discounts of 33⅓%, 5%; while

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Lewis quotes 20% and 20%. Which is the better offer? By accepting it, $—— is saved.

Rates of discount. In wholesaling and manufacturing, there is constant necessity to calculate rates of discount—owing to fluctuations in market prices and other causes.

ILLUSTRATION. An article listed at $9.00 drops to a market value of $4.00. 33⅓% discount has already been quoted customers. What further discount rate must be allowed?

Solution.

The list price	=$9.00
33⅓% discount is 1/3 of $9.00	= 3.00
First discount price	6.00
Required price	4.00
Additional discount to be allowed	2.00

$2.00 dis. must be reckoned on $6. It=1/3 of $6, or 33⅓%. Therefore the second discount is also 33⅓%.

ORAL EXERCISE.

1. Discounts of 20% and ? % yield a selling price of 60% of list.

2. Discounts of ? % and ? % enable one to sell at 50% of list.

3. Discounts of $20.25% and ? % lower a list price of $380 to a selling price of $180.

EXERCISE.

1. What discount in addition to one of 15% will lower a list price of $7.20 to a market price of $5.00?

2. The Essex chair manufactured at a cost of $5.00 retails at $8.40. The manufacturer lists at retail price. If he is to gain 20%, he can allow the retailer what rate of discount?

Suggestion. Find the manufacturer's selling price. Compare with list.

3. The same manufacturer lists at $10 an article costing $6.40 to make. If dealers sell at list price, they should buy at what discount in order that they and the manufacturer may make an equal money profit?

EXERCISE.

(Rates of profit and loss.)

Find missing values.

Questions of profits or losses on individual items sometimes arise. Thus a dealer may wish to determine his rate of profit on carpet bought at $2.00 and sold at $4.00 less 20%, 30%. By comparing net selling price

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with cost, he finds the rate to be ——%. Or he discovers that a clerk has lost him $—— by discounting at 45% a $120 sale on a series of 20% and 25%.

Sometimes the difference of measure affects the problem, as in computing the rate of profit on step-ladders bought @ $12.00 per dz. less 50%, 5% and selling at $1.20, finding it to be ——%.

Perhaps a case arises in which part of an invoice has been sold. Say, 20 out of 35 bookcases, bought @ $8.00 less 40%, 10% have been sold at list, and the balance at 20% profit. This, he finds, yields him a profit of ——%.

Questions of comparative sales arise. Goods costing $6.00 may be listed at $16.00. An increase of discount rate from 40% to 50% results in increasing average sales 30%. "Does it pay?" he asks, and finds that ——. Or, a $20.00 chair, costing $8.40 to make is usually discounted 30% to the trade. By experiment over a long period, the dealer finds that his average sales per week increased from 40 to 50 when he allowed 10% additional discount. By percentage he finds that his present rate of profit is ——%, where formerly it was ——%.

Some cases of marking goods arise.

ILLUSTRATION. Mark a desk, costing $15.00 to manufacture, at a price to yield the maker 33⅓% profit, after allowing discounts of 40% and 16⅔% to the trade.

Solution.

Let 100%	=list price.
100% − 40%	=60%, 1st discount price.
16⅔%, or 1/6 of 60%	=10%
60% − 10%	=50%, second discount price, or selling price per cent.
33⅓% of $15.00	=$ 5.00 profit to maker.
$15.00+$5.00	=$20.00 selling price.
∴ 50% of list	=$20.00
100%	=$40.00, list price.

EXERCISE.

1. $8.00 rockers, bought at discounts of 25% and 5%, must be sold at $—— to gain 20%.

2. At what price must articles costing $18.00 be marked to allow for 33⅓% profit and 5% bad debts?

3. A manufacturer's patent extension table cost $8.40 to manufacture. If he is to allow for 20% profit for himself, and for 20% discount to the wholesaler, he must name what list price?

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4. The public does not like to pay more than $7.20 for a certain article. Allowing for a manufacturer's profit of 20%, and a dealer's discount of 20%, the maker must strive to keep the cost of manufacture within $—

EXERCISE.

1. Sold to Robt. M. Drake, Morristown, N. J., via DLW frt. 30 days 2/10; 40 doz. files @ $2.60, less 40%, 10%; 3½ doz. hand saws, No. 82, @ $17.50, less 25%, 10%. Write bill.

2. Compare bids received for the purchase of 100 C machine bolts 3/8″ × 6 1/2, needed for stock.

R. C. Shafer	offers them @ $4.00 less 50%, 10%, 5%.
James Casler	offers them @ $3.90 less 60%, 10.
Newark Bolt Co.	offers them @ $4.10 less 60%, 15%, 5%.

3. Extend this bill.

Pittsburgh, Pa., ————, 191—

THE MORRIS HARDWARE CO.
New York City.

Bought of THE RANDALL IRON CO.
Terms: 60 days; 3/10

*	2	C Machine bolts, 1/2 × 5	5.60	——	—			
	4½	C Machine bolts 7.20 7.52 5/8×4″ 4½″ Less 50%, 15%, 5%		——	—			
				——	—	——	——	
	6	dz. pr. Hinges, No. 382, 8″ Less 60%, 10%	4.20	——	—	——	——	
	2¼	dz. Steel Squares No. 11 @ Less 30%, 10%	42.	——	—	——	——	

*NOTE. 4½ hundred machine bolts 5/8″×4″ @ $7.20 per 100, and the same quantity 5/8″×4½″ @ $7.52.

EXERCISE.

(Give an illustration in each case.)

1. What values must be known to determine: (*a*) list price, (*b*) missing discount rates, (*c*) net cost, (*d*) discount in money, (*e*) rate of profit or loss?

2. What values can be determined from: (*a*) list price and series of discounts; (*b*) series of discounts, two as rate, one also as money; (*c*) cost and series; (*d*) cost, desired profit, and rate of discount; (*e*) two out of three rates, cost and list price?

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It is not at all necessary, in the study of many questions of profits, discounts, and even of questions of business policy involving direct financial results, that the fundamental calculation, or the calculations to determine an advisable course, should be based on money values. Percentage is a valuable means of studying business actions, somewhat abstractly, and of deducing results as rates per cent., that are practically formulas.

ILLUSTRATION. A question like this may arise: If merchandise is bought at 40% discount from list, and sold at list, how much of the selling price may be allowed for discounts and selling expense, and yet realize 25% profit. The cost equals 60% of list. 25% profit on cost equals 15% of list. The desired selling price equals 75% of list. 100% list − 75% = 25%. Therefore selling expense and discounts must be kept within 25%.

EXERCISE.

Each example in this exercise should be solved without assuming any money value—although the resulting answer may be applied as a rate per cent. to an infinite number of prices and money values. Find missing values.

(*a*) QUESTIONS PRIMARILY OF DISCOUNTS.

1. The portion of the list price paid for goods bought at 34 ½% discount is —% .

NOTE. The answer, 65 ½%, is general for the given discount, regardless of price concerned.

Apply this answer to finding the cost of articles listed respectively at \$4.20, \$5.00, 20c, \$340.

2. How do terms of "90 da., 3/60, 5/10" affect the settlement of any bill paid within five days of date of purchase?

3. ——% of the list price is paid for goods bought at discounts of 40% and 10%. *Apply*, to finding the cost of goods listed at \$486, and at \$56.

4. The difference between a series of 10% and 40%, and a single discount equal to their sum is ——%. What is the money difference on a list price of \$840?

5. Which is the better for a seller to offer—any series of discounts or its sum?

6. Discounts of 25% and x% will lower a list price 50%.

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7. Determine a series of two discounts to lower a list price 45%.

Suggestion. Assume *any* rate less than the total discount, for example, 20%. Calculate the other rate. How many solutions are possible?

8. The separate discounts in a series of 3 20s compare how in real value? Illustrate, after obtaining the proportional value, by application to a list price of $8000.

9. Determine a series of two discounts, equivalent to a single discount of 40%, which shall be equal in numerical value.

Suggestion. Divide the single discount into equal parts, and find the equivalent rates. The principle applied in this, and in one or two of the following examples, is applied by producers in allowing for middlemen's profits.

10. Determine a series of three discounts equivalent to a single discount of 48%, and equal in numerical value.

11. A series of two discounts is equal to 45% off list, but the first discount is twice the second in real value. The series is — % and — %.

(*b*) QUESTIONS PRIMARILY OF PROFIT AND LOSS.

12. ——% is realized by buying at 10% and 33⅓% discount from the list price at which the goods are sold?

13. When a publisher sells his net books to dealers at 40% discount, the latter realize ? % profit.

14. Is it better for a retailer to sell at 30% advance on cost, or to sell at a list price from which his wholesaler has allowed him 30% discount?

15. Compare these offers, based on the same list price: (*a*) Discount 20%, 5%, 10%. (*b*) Dis. of 3 20s. (*c*) Dis. of 30%, 10%, 15%.

16. My stock is bought at 40% discount from the list price at which I sell, but the expenses of sale average 15% of the selling price. What net rate of profit is realized?

17. A 50% increase in sales, resulting from allowing 20% discount, has what effect on former profits of 30% made by selling at list?

18. Merchandise is bought at 40% discount and sold at 20% discount from the same list price. What is the effect of interchanging rates?

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CHAPTER XXXII.

AGENCY.

An *agent* is one who does business for another. The person whom he represents is termed the *principal.*

ILLUSTRATIONS. (1) Jones, at your request, buys you a saddle horse. (2) Jefferson buys for Chase 400 shares P. R. R. stock. (3) A farmer ships a carload of potatoes to a city produce dealer who returns the farmer the money proceeds from their sale, less his charge for selling.

EXERCISE.

1. Give other illustrations of agency in everyday affairs.
2. Name the agent and the principal in each illustration.
3. Why are agents required in business?
4. State some of the advantages and disadvantages of dealing through agents.
5. Name common businesses that are agency businesses.

In certain businesses the agent is called a *broker* (stocks, bonds, notes, etc.), or a *factor* (cotton), or a *commission merchant* (produce). Drummers, buyers, attorneys, auctioneers and salesmen are agents. The agent may act without pay, may receive a salary, or may receive a payment, often termed a *commission,* based on the quantity or value of goods bought or sold. He may receive both salary and commission. The pay of a broker is termed *brokerage* and that of a factor, *commission.* The terms *fee, share* and *allowance* are also common. Per cent. commissions are based on the exact amount paid (net cost), or received (gross proceeds) by the agent for the goods—*regardless of charges.*

The principal pays his agent *for a purchase,* the *net cost* plus all charges for freight, drayage, commission, etc. (the

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gross cost). He receives *from a sale* the *gross proceeds* minus all charges (the *net proceeds*).

Guaranty is an additional fee paid the agent for taking the risk of securing payments from customers to whom he sells. *Warranty* is an additional fee paid the agent for ensuring the quality of goods. The agent pays, but charges to his principal, freight, drayage, storage, insurance, inspection charges, etc.

The agent acts for his principal. The principal, as a *consignor*, consigns or ships goods to the agent, or consignee, to be sold. The principal calls the goods a *shipment;* the agent calls them a *consignment.*

EXERCISE.

1. After carefully reading the preceding sections, define agent, broker, commission, guaranty, warranty, principal, gross proceeds, gross cost, the sum earned by the agent, the sum the principal receives from a sale.

2. Compare the following with the terms of percentage: agent's commission, rate of commission, gross proceeds, net proceeds (if commission is only charge).

1. PAYMENT OF AGENTS.

The rate of commission is expressed as a lump sum, as a rate per cent. based on the price at which the agent buys or sells, as a fractional part of this value, or as a rate per article.

ILLUSTRATIONS. I pay Scott $25 for selling my horse for me. My agent sells 200 bbl. of my potatoes at $2, and charges me 5% of the selling price for his services. I offer one-fifth of the sum he obtains to a collector who secures payment of a debt due me.

EXAMPLE. What does an agent earn for buying or selling 200 bbl. of flour @ $4.80, on 5% commission?

Solution. The selling price = 200 × $4.80 = $960.
The commission = 5% of $960, or $48.

It is evident that the agent's per cent. of commission is reckoned on the price *he* pays or receives for the goods.

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ORAL EXERCISE.

Find the commission:

1. On 400 bbl. @ 2c per bbl.
2. On 2000 bu. @ 1/4c.
3. On a $3600 purchase at 2½% commission; at 4%; at 5%; at 25%.
4. On 1200 bbl. at 15c per bbl.
5. On 800 bbl. purchased @ $2 on 5% commission.

What is the agent paid for his services?

6. For the collection of a $1200 debt on 5% commission?
7. For auctioning off $1800 worth of goods on 3% commission?

EXERCISE.

1. Compare a 5% rate of commission with a rate of 1/2c per bu. on merchandise costing 30c per bu.

Suggestion. Reduce the second rate to a rate per cent.

2. Compare a rate of 3% with a rate of 15c per bx. on goods selling at $2.40 per bx Is it possible to make a comparison in the above case if no money value is given?

3.

C. JOHNSON.

$ 40.00	$102.10
$400.00	
$206.40	

An agent succeeds in collecting three-fourths of this debt for me. If his commission is 6%, he earns $ —.

2. SALES.

INTRODUCTORY EXERCISE.

1. You sell for me 10 T. timothy on 5% commission. The customer pays you what sum? You charge $?, sending me what sum?

NOTE. The principal receives the selling price (gross proceeds) — charges, or the net proceeds.

2. An agent secures a purchaser for a house and lot who pays $12,600. The agent pays $30 for survey, $10 for transfer, and charges 5% commission. What does the agent receive and what sum does he pay to other parties?

3. Whose money does the agent use to pay for charges? Explain.

The standpoint of each party to an agency transaction is different, and the calculations in which he is directly inter-

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ested vary. Recognition of "viewpoint" is often essential to accurate analysis and solution of agency problems.

ILLUSTRATION. A. B. Newton ships to Robert Kent, commission merchant, 200 bbl. apples which the latter sells at $2.40. He pays freight of 15c per bbl., and charges 5% commission. He mails to Newton a check for $?, net proceeds.

From the standpoint of Newton the problem is: "I ship Robert Kent 200 bbl. apples which he sells at $2.40, on 5% commission, paying 15c freight per bbl. I should receive $? "

From the standpoint of the freight agent: "What is the freight on 200 bbl. at 15c?"

From the standpoint of the agent (To customer): "What must I receive for 200 bbl. at $2.40?" (To freight agent): "What freight is due on 200 bbl. at 15c?" (For himself): "What is my commission on this sale at 5%?" (To principal): "Considering these previous transactions, what do I owe Newton?"

From the standpoint of the customer: "Two hundred bbl. apples at $2.40 will cost me $?"

Solution. 200 × $2.40 = $480 gross proceeds, or sum paid by customer.
200 × 15c = $ 30 freight paid to freight agent by agent.
5% of $480 = $ 24 commission.
$ 54 total charges.

Gross proceeds − charges = net proceeds, due principal.

$480 − $54 = $426, net proceeds.

ORAL EXERCISE.

Memoranda of Sales.

(*a*) 400 bu. oats, @ 40c, commission 1/4c per bu.

(*b*) 5 copies Marston's Business Encyclopedia, $14, on 5% commission.

(*c*) A horse, $240; wagon $90; and harness $30. Commission 3%; misc. expense $4.

(*d*) 20 crates eggs, 30 doz. each, 30c per doz. Commission 5%; express, 50c per crate.

1. What do agent and principal receive in each case?
2. Any increase in general charges affects what parties? Illustrate.

EXERCISE.

Before solving, re-word the transaction from each party's standpoint, and state necessary and unnecessary values for each computation.

1. The consignor should receive a check for $——, as proceeds of 20 crates eggs, 30 doz. each, @ 18c. Commission 5%.

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2. Find the sum due the principal on a shipment of 60 T. timothy hay @ 90c per C. What sum does the agent earn at 4% commission?

3. S. P. Naylor sells for James Carter 12,000 bu. corn at 50 1/4c, commission 1/4c per bu. What sum does Naylor receive?

NOTE. In the following, name the new factors, and discuss their effect, and find missing values.

4. I shipped M. C. Parker 360 bbl. apples which he sells at the market quotation of $2.90; on 4% commission and 1% warranty.

The customer pays $——; Parker receives $——; he earns $——; and he pays me $——.

5. J. C. Jones receives from his agent a check for $60.40 as net proceeds from a sale of three hundred pounds of butter at 27c, commission 5½%, miscellaneous charges $4.75. Check up the returns. The customer paid $——.

6. 250 bbl. apples, shipped to be sold at the market quotation of $3.20, arrive on a falling market and are sold at $2.90, drayage, 10c per bbl., commission 5%. What effect has the fall in price on the net proceeds?

7. The consignor should receive $—— net proceeds from a shipment of 16,000 bu. oats, disposed of at 41c. Freight, 2c per bu.; insurance 1/2c; storage, 1/2c; commission, 1/4c.

8. Extend

ACCOUNT SALES.

From JOHN C. CARTER,

Baltimore, Md., Jan. 17, 1913.

To ROBERT G. MASON,
Hyattsville, Maryland.

For Merchandise Received Jan. 2, 1913, via B. & O.
Consignment No. 4726 Folio 88 R. M. Green Clerk.

		Sales			
Jan.	4	100 lb. Dressed chicken	.16		
	10	450 lb. Dressed turkey	.18		
	16	620 lb. Dressed chicken	.17		
		Charges.			
		Expressage $2.45 Drayage $1.10 Storage $3.15			
		Inspection $ Insurance $ Commission $ (4½%)			
		Other Charges $	Total Charges		
			Net Proceeds		

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3. PURCHASES.

INTRODUCTORY EXERCISE.

1. You buy through an agent, on 1/8c per bu. commission, 6000 bu. wheat at 90c. The agent pays $— for the grain, charges $—— commission, and you pay $——. What effect have charges on the cost to the principal?

2. An agent buys for his principal 80 bbl. winesap apples @ $2.50 on 5% com., and 50c per bbl. for drayage and storage. The net, or prime cost of the good is $——; the total charges are $——. The principal must pay the agent in full settlement, $——.

3. If buyer and seller had dealt direct, in example 2, who would have saved money? How much?

ILLUSTRATION. C. D. Rand sends Robert Gates a draft for $3000 with orders to invest in Greening apples, market quotation $2.40. Com. 5%, drayage 20c bbl., storage 10c. Find all values.

Solution.

	$2.40 net cost per bbl.	
(5% of $2.40)	.12 com.	The number of bbl. = $\frac{\text{gross cost}}{\text{gross cost per bbl.}}$
	.20 drayage	
	.10 storage	1063 = no. bbl.
	$2.82 gross cost per bbl.	2.82)3000
1063 ×	$2.40 = $2551.20 prime cost	282
1063 ×	.12 = 127.56 com.	1800
1063 ×	.20 = 212.60 drayage	1692
1063 ×	.10 = 106.30 storage	1080
	$2997.66 gross cost	846
$3000 − $2.34 (surplus) = $2997.66, gross cost.		2.340

The principal receives 1063 bbl. and $2.34 surplus.

The agent receives $3000, returns $2.34, keeps $127.56 com. and pays $2551.20 for the apples, $212.60 for drayage, and $106.30 storage.

ORAL EXERCISE.

(Memoranda of Purchases.)

(*a*) 8000 bu. oats @ 40c, commission 1/8c.

(*b*) 200 sh. stock @ $80, brokerage $1/8 per sh.

(*c*) 2000 lb. lard @ 10c, com. 2c per 100 lb., misc. charges $5.

(*d*) 80 A. land @ $50, com. 4%, transfer and survey $35.

1. Compute agent's commission in each case.

2. Determine amounts paid and received by each party.

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EXTRACT FROM QUOTATION LIST.

Chicago markets yesterday:

Wheat	Opening.	Highest.	Lowest.	Closing.
July.....	.89⅛	.90⅛	.89	.90⅛
Sept.....	.91	.92⅝	.90⅝	.92⅜
Dec.....	.94⅞	.96⅜–½	.94⅝	.96¼
May.....	.99⅜	1.01	.99¼	1.00⅞
Corn				
July.....	.51⅜	.52½	.51⅜	.52½
Sept.....	.51⅜	.52¾	.51⅜	.52⅝
Dec.....	.48½	.49⅝	.48½	.49½
May.....	.50¼	.50⅞–.51	.50	.50⅞
Oats.				
July.....	.42	.43	.41¼	.43
Sept.....	.37⅞	.38¾	.37⅝	.38⅝
Dec.....	.38½	.39¼	.38⅜	.39⅛
May.....	.40⅝	.41	.40	.40⅜–.41
Pork..				
Sept.....	16.55	16.55	16.52	16.52

EXERCISE.

(See Quotation list where necessary.)

1. Invest $5000 in July wheat, opening quotation, brok. 1/8c—no fraction of 100 bu. to be bought. Number of bushels?

2. What is the purchasing power of $2000 in September corn, at highest quotation, brok. 1/4c.

3. Determine gross cost of 500 bbl. pork, lowest quotation, com. 2½c a bbl.

4. An agent, charging 4½% com., draws on me for what sum to cover the full cost of 200 cr. tomatoes—$2.25 expressage and drayage 40c each?

Using *account sales* form as a model, prepare an account purchase for this transaction.

5. My draft for $700 will pay for how many crates of eggs, 30 doz. each, @ 18c a doz.; com. 4%; drayage 25c?

6. A receives from his principal a draft for $12,000 with orders to secure three months' options on as many acres of certain coal lands as possible. A allows $50 for travelling expenses and 4% for commission. He secures options on 80 A. at 15% of the selling price, which is $420 per A.

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The balance he spends in the purchase of 12% options on land priced at $210 per A. On how many acres of this latter tract can he secure options?

7. I bought through an agent 60 cr. choice tomatoes @$2.75; com. 4%; expenses $20. I sold 25 crates direct @ $3.50 and the balance through an agent at $3.80 on 5% com. Find net returns.

8. In accordance with orders, an agent disposes of 200 A. of a tract of 280 A. @ $68, and the balance @ $43, investing the net proceeds in options on timber lands @ $10 per A. His commission being 4% each way and his expenses $50, he can secure options on how many acres?

9. An advertising solicitor agrees to secure advertising for a booklet, on 25% commission. He secures 35 full page advertisements @ $11.50 per inch. The pages are two column width and 10 inches depth of column. His total commission is $——.

10. Extend this statement of a real estate agent to his principal, charging 5% on collections.

WASHINGTON, D. C., Feb. 2, 191–.

Mr. JAMES PARKER,
1382 13th Street, N.W.

Dear Sir

Enclosed find my check per statement below.

Respectfully ROBERT CRANE, *Agent.*

Tenant	Premises	Explanation	Receipts	Charges
C. P. Lewis	1491 N. St. N.W.	Rent to Feb. 1, 19—	60 —	
		Repairs to range		3 60
James Farmer	1280 L St.	Rent to Feb. 1, 19—	85 50	
		Plumbing repairs		7 35
		Commission 5%		$ —
		Balance due		$ —

PROFIT AND LOSS.

EXERCISE.

(Solve mentally when possible.)

Determine the rate of gain or loss on:

1. 500 lb. Rio coffee bought @ 8c, on 5% com., and sold @ 9c direct.

2. Quinces bought @ $2 and sold @ $3 on 5% commission.

3. Determine the profit or loss on 1000 bu. corn bought @ 70c and sold @ 71½c; brokerage 1/4c each way.

4. Spring bran bought @ $19.60 per ton, com. 5% and drayage $1.00 per ton, must be retailed at what rate to yield 15% profit?

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5. What per cent. of profit is obtained by selling @ \$3.20 apples bought through a commission merchant @ \$2.50, on 5% commission? Expenses of sale equal 5% of the selling price.

6. What profit is realized in retailing @ \$1.50, through an agent charging 20% commission, a subscription book that costs 92 cents to publish?

7. An article, costing \$1.80 to manufacture, is to be sold at a price to yield the manufacturer 20% profit, after allowing selling agents 25% commission. Find list price.

8. A sewing machine costs \$11.90 to manufacture. Find marked price to allow for 25% profit and 20% commission to agents.

9. How much must the sales of an agent increase to make it profitable to lower his rate of commission from 5½% to 4½%?

10. A book costing 90 cents to publish is sold @ \$1.50 through agents who are paid 30% commission. How is the publisher's rate of profit affected by lowering the commission to 20%?

REVIEW DRILL EXERCISE.

1. Buy 40 bbl. @ \$5, com. 5%.

Find: gross cost; prime cost; commission; sum agent pays seller; sum agent charges principal; sum agent earns; largest sum involved. Repeat, if commission rate is (1) 8c. bbl.; (2) 1/4%.

2. Sell 2000 bu. @ 60c, com. 1/4c, storage 1/2c.

Find: sum buyer pays; sum agent earns; sum handled by agent; net proceeds; the cost of the sale to the principal.

3. State the values that may be computed from these facts. Formulate and solve simple problems involved in them. State whether purchase or sale.

(*a*) Commission, \$4; gross proceeds, \$40.
(*b*) Commission, \$12; net proceeds, \$188.
(*c*) Commission, \$60; prime cost, \$2000.
(*d*) Commission, \$90; quantity 400 bbl.
(*e*) 5% commission on a sale is \$22.
(*f*) An agent sends his principal \$388 after reserving \$12 for himself.

4. Give illustrations showing what values can be determined from:

(*a*) The commission and a "cent" rate.
(*b*) The commission and a rate per cent.
(*c*) Number of articles and price.
(*d*) The net and gross proceeds.

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(*e*) The sum for which an agent sells goods and the sum he remits his principal.

5. Give illustrations showing from what known values the following may be computed:

(*a*) The commission on a sale; on a purchase.
(*b*) The sum an agent receives in making a purchase.
(*c*) The sum the principal receives from a sale.
(*d*) The gross cost.
(*e*) The sum paid by an agent in making a purchase; a sale.

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CHAPTER XXXIII.

INSURANCE.

To PUPILS. Collect and bring to class, for reference, old fire, health, accident and life policies, insurance folders, and magazine advertisements that give insurance information.

INTRODUCTORY EXERCISE.

1. What is your own definition of "insurance"?

2. Into what two classes can you divide all the classes of insurance?

3. Name several kinds of personal and of property insurance.

4. Give five reasons for taking out life insurance.

5. Give other uses for life insurance than simply to produce a fixed sum at death.

6. Read over a life, accident or health policy. Are any conditions imposed on the person insured, as to occupation, place of residence, etc.? Why should such conditions be made?

7. Give several reasons for insuring one's property.

8. Distinguish between general and special property insurance.

9. Study a fire insurance policy and name at least eight conditions imposed on the insured. State good reasons for these conditions.

10. State several ways in which one might "forfeit" his policy.

11. Does property insurance protect the insured property against damage? What does it do? Has an insurance comp ny a right to make repairs rather than *pay* damages?

12. How do strict policy conditions tend to prevent losses?

13. Why do insurance rates vary on different kinds of property?

14. Why should a person always read a policy carefully before taking out insurance under it?

A *contract* is an agreement between two or more parties to do or not to do a particular thing. It is evident that the parties must agree as to the thing desired. It is generally considered, also, that there must be proper *consideration*—that

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is, a fair return in service or value, for what is received. Naturally, contracts may be of most varied kinds. Among the commonest are those for the sale or rental of property, for the payment of money, for employment and for *insurance*. Contracts in the form of notes, bonds, etc., have been touched on elsewhere.

Contracts for insurance are agreements whereby one party agrees to pay another party a specified sum of money in the event of certain happenings, such as the death or injury of specified persons, or the destruction or loss of property through fire, water, theft or accident.

The written or printed document containing the terms is the *insurance policy*. The *face* of the policy is the sum, or limiting sum, which the insurer agrees to pay. The *premium* is the payment made by the insured to the insurer, for protection. Payments of premiums after the first period are termed *renewals*.

Life insurance policies run for a term of years, or for the life of the insured; health and accident insurance policies generally for one year; transit insurance (on goods being transported) generally for the period of travel; fire insurance for terms of one, two, three or five years.

The rates of premium on personal insurance are usually expressed in dollars per thousand; on property insurance in dollars per $100 or $1000, or as a per cent. of the face of the policy.

The computations involved in the general offices of insurance companies are exceedingly involved and require a high degree of mathematical knowledge. From the standpoint of the insurance clerk and the insured, the computations are very simple and require no new principles.

It is evident that insurance is naturally divided into two great groups—*personal insurance* and *property insurance*. These will be noticed briefly.

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PERSONAL INSURANCE.

Besides general life insurance, there are many classes of personal insurance, of which the following are well known:

Accident Insurance policies provide for payment to the insured, or to specified beneficiaries, of sums varying with the character and severity of the injury to the insured. There are provisions for payments of surgeons' fees. Payments may be made in a lump sum, or weekly for a specified number of weeks.

Health Insurance may be written in connection with accident insurance, or as a separate contract. One form provides for weekly or monthly indemnity for illness arising from a limited number of diseases; others are much more strict in limitations. The contract usually provides for payments for a period not greater that twenty-six weeks and for a specified general payment in case of permanent disability.

Liability Insurance is written for employers to cover settlements with employees for injuries for which the employer is responsible. A policy covering all or certain classes of employees in a manufacturing plant, for example, is paid for by a premium based on the total pay roll. The insurance company defends for the insured any damage suits brought by employees on account of injuries, and settles all legal or just claims.

Automobile liability insurance is also written for any owner of an automobile to indemnify him for liability on account of injuries he inflicts on others while running his machine.

Fidelity and Surety Insurance. Fidelity insurance is written to protect the insurer against loss through employees who do not faithfully perform their duties. It covers the bonding of employees who handle money, and is often paid for and taken out by the ones bonded. Surety insurance involves also the issuing of general court bonds, and bonds for the faithful performance of any service or contract. Here the insurance

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company may demand from the person it bonds not only a premium but also certain approved securities on which it can realize funds with which to complete the contract in case of the failure of the bonded person or company.

Credit Insurance is in a sense, also, a class of property insurance. This is issued to merchants to protect them from excessive loss from bad debts. Merchants who sell on credit and desire such insurance submit their books to the credit insurance company which averages the losses from bad debts for a series of preceding years. A premium rate is then fixed, in return for which the insuring company agrees to make good any loss from bad debts during the year, in excess of the average loss. The credit insurance company places restrictions on the merchant, however, to prevent the careless granting of credit to unreliable customers.

EXERCISE.

1. Compute the cost of a combined accident and health policy for $7500, if the premium is $5.60 per unit of $500.

2. What should the insurance company pay the holder of the above policy for a serious illness of 9 weeks 3 days, at the rate of $37.50 per week?

3. Compute the cost of a $75,000 bond for a cashier, secured from a bonding company at a rate of $2.45 per $1000?

4. The pay roll of a manufacturing company amounts to $175,000 per annum. A liability insurance company agrees to assume the employer's liability for this force at an annual premium of 3½%. What is the premium?

5. What does the insurance company gain in the above case, if it pays damages of $4580 and incurs legal expense in defense and examinations amounting to $547.50?

6. A credit insurance company examines the books of a mercantile house and finds the normal loss from bad debts to be $4500. It issues a policy for an excess loss of $8000, at a premium of 4%. What is the premium? For the year covered by the policy the loss is $7896. What sum does the insurance company pay?

7. Compute the cost of a $7500 excess policy at 3½%. Compute the normal loss in this case if the respective losses for the six preceding years was $2512, $2681, $2512, $2800, $3126, and $2107.

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Life and Endowment Insurance. Insurance on the life of a person is issued under policies of most varied form. The most common classes might be termed *straight life, limited payment life, endowment* and *term.* Life policies guarantee the payment of a specified sum at death of the insured, to the beneficiary named in the policy, in consideration of premiums paid during the life of the insured, or during a given period. Ten-payment and twenty-payment life policies are common. Term policies limit the guarantee payment in case of death within a fixed number of years. Endowment policies, in consideration of annual premiums paid for a fixed number of years, guarantee a fixed payment to the beneficiary at death of the insured, or *to the insured at a certain age.*

Premiums are usually expressed on a basis of $1000 of insurance, and vary with the age of the insured and the character of the insurance agreement. They are most carefully compiled and are based on the chances of life and the earning power of the premiums or investments of the company. The processes of computation are exceedingly complicated.

The *reserve* of an insurance company is the portion of income from premiums that it is required by law to set aside as a fund for the payment of policies when they fall due. The *surplus* of a company is the excess of assets over liabilities. In the case of mutual companies, or those issuing *participating* policies, a certain portion of the surplus is returned to policyholders whose contracts entitle them to share in it.

Many causes may lead or force a policyholder to give up his policy, or to cease paying premiums on it. (Name some.) However, as a rule, if he has paid two or more annual premiums, he receives some return for premiums already paid. Thus, he may get cash, on surrendering his policy, equal to practically its reserve value. He may secure a smaller policy in exchange, all "paid up," for whatever amount the reserve

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will purchase; or he may secure extended insurance for a certain number of years. Many companies will also lend money to the insured, on his policy, up to its cash value. Some companies allow a fixed number of days of grace, say thirty, for the payment of premium after due. Some reinstate, after lapse of a payment, on proof of insurability and payment of back premiums with interest.

A good, modern twenty-year endowment policy, for example, at age 35 years, for $2000, in a certain company, if the insured live, will cost $100.72 per year for 20 years, at the end of which time it will produce to the insured $2000 and its share of surplus or accumulation.

STATEMENT OF GUARANTEED VALUES.

No. of Annual Prem. Paid.	Cash or Loan. Value.	Paid up Insurance. (Participating.)	Upon Surrender. Participating Term. Insurance for	
2	$112	$180	6 yr.	21 days.
3	186	292	10	259
4	262	400	14	258
5	342	512	15	0
6	424	62)	14	0
7	508	724	13	0
8	596	828	12	0
9	688	932	11	0
10	784	1032	10	0
11	884	1136	9	0
12	988	1236	8	0
13	1096	1336	7	0
14	1210	1432	6	0
15	1328	1528	5	0
16	1448	1620	4	0
17	1572	1722	3	0
18	1762	1804	2	0
19	1838	1892	1	0
20	2000 and accumulations, shares in dividends, in addition.			

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EXERCISE.

Assume that the above policy was taken out on July 17, 1910.

1. What sum could be borrowed on the policy at the end of 7 years? On August 18, 1917?

2. If the insured surrenders the policy on May 14, 1916, what amount of paid up insurance should he receive?

3. What sum can he borrow on the policy after 12 premiums have been paid?

4. If the insured surrenders the policy on May 19, 1918, he can secure for it insurance for what term? How much is paid then in case of death 6 months later, no allowance being made for accumulations?

5. In 1912 his dividend was $19.66. He allowed it to go to reduce his premium. What was his net premium in this year?

6. What is the corresponding premium per annum for a policy of $17,000?

7. What cash surrender value has this new policy after 9 annual premiums have been paid?

8. What is the premium on a $12,000 policy at same rate? If the insured dies after the payment of five annual premiums, what sum do the heirs receive? What sum in excess of premiums paid in? What sum in case accumulations during the period of insurance have amounted to $1,345.61?

ANNUAL PREMIUM RATES FOR DIFFERENT TYPES OF POLICIES.
Per $1000 of Insurance.

Age.	Ordinary. Life.	10 Pay. Life.	20 Pay. Life.	10 Year Endowment.	20 Year Endowment.
21	$18.50	$46.40	$28.30	$101.68	$47.70
25	20.26	49.34	30.17	102.00	48.10
28	21.73	51.82	31.76	102.29	48.47
30	22.90	53.62	32.92	102.52	48.78
35	26.48	58.68	36.27	103.25	49.80
40	31.12	64.69	40.43	104.33	51.43

EXERCISE.

1. Find the cost of a policy of each kind for $4500 issued at age 28.

2. A man at age 30 takes out a 20-payment life policy. When he has completed payments on it, how much less than the face of the policy has he paid in?

3. Why does the age of the insured affect the amount of the premium?

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4. Give general reasons for the difference in amount of premium for policies of different kinds, at any fixed age.

5. Suppose a man, at age 21 years, had taken out a $5000 policy of each of the above kinds. Suppose that he died at the age of 50 years. What benefit would he have had from each policy during his life? What benefit would his heirs receive in each case?

6. Why is an insurance company able to pay large sums at death of the insured when it receives only small premiums?

7. Name several advantages of insuring at as early an age as possible.

8. Why are intending policyholders required to undergo a strict medical examination?

Income Insurance. In return for the payment of annual premiums varying as usual with the age of the insured, the insurance company agrees to pay the beneficiary, on the death of the insured, a fixed income per month, for a fixed number of years, or for life, in place of paying a lump sum. Under some policies both a lump sum and an income is provided. If the income is payable during the lifetime of the beneficiary, then the premium rate depends in part on the age of this person. Income insurance can be arranged in *income multiples* of $10.

Old Age and Savings Insurance. This insurance is increasing in this country and is common abroad. For the payment of small monthly premiums, ordinary straight life and endowment policies are issued. There are also combined insurance and annuity policies, involving payment in full at death before 65 years of age, and in full, less annuities paid, for death between, say, 65 and 69. Annuities may be paid after 65 years, the annual payment being a certain fraction of the face of the policy. Straight old age pension policies are also issued, wherein a certain annuity is given after the age of 60 or 65, in return for annual premiums paid up to that age. In case of death before the annuity age begins, the amount of the premiums paid in is turned over to the heirs. The maximum annuity is often a small sum, say $200, and the maximum

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insurance limit is very small. No medical examination is required for old age pensions.

EXERCISE.

1. Compute the annual cost of an old age pension policy ensuring an annuity of $200, purchased at an age of 36 yr., at the rate of $3.14 per month. How much will have been paid in in premiums by the time the insured reaches the age of 60?

2. Anderson takes out an old age policy guaranteeing $400 annuity after the age of 60 years, and the face of the premiums in case of death at an earlier age. He is 34 years old and pays a monthly premium of $1.12 per $100 annuity. Compute his monthly and yearly premiums. In case of the death of the insured at the age of 57 years, what sum should be paid his beneficiaries?

3. Roberts, aged 26, takes out a $400 savings endowment policy payable in twenty years or at death. What is his monthly premium, if the rate is $2.19 per $500 insurance? What is his annual premium?

4. Newton, aged 40 years, takes out an income policy, guaranteeing a monthly income of $60 at his death, to his beneficiary for 20 years, at a premium rate of $5.76 per $10 income unit. What is his annual premium?

5. Osgood, aged 30 years, takes out an income policy, calling for the payment to his beneficiary after his death, of a monthly income of $75 for 20 years, at a premium rate of $33.47 per $10 income unit. What is his annual and his quarterly premium? In case of death, at age 47, what will he have paid in premiums? What sum will his beneficiaries receive during the following 20 years? Why is the insurance company's loss less than the difference between these two amounts?

PROPERTY INSURANCE.

EXERCISE.

1. Is death *sure*, sooner or later, to every insured person?

2. Is loss by fire certain to occur, sooner or later, to every building insured against it? Is lightning sure to strike every building?

3. Name differences between life and property insurance, as regards probable losses payable by insurance companies?

4. Why are property insurance rates lower than personal insurance rates?

5. Why should fire insurance rates vary with the (*a*) use of a building; (*b*) with its surroundings; (*c*) with the character of material and construction of it?

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6. Why is a brick paper mill a greater "risk" than a brick residence?

7. Why do property insurance companies make restrictions as to how an insured building shall be built or used?

8. Why do fire insurance companies favor and work for strong fire departments?

9. Why are country dwellings insured at a higher rate than city dwellings of the same class?

10. Read the conditions of a policy and determine some of the duties of the person whose property is insured, in case of damage to it which is covered by insurance.

As in the case of life insurance companies, property insurance companies may be either *stock companies*, owned and managed for the stockholders' profit, under certain legal restrictions; or mutual companies, in which the policyholders share in gains and losses. Some companies are partially mutual.

Policies vary greatly in form and conditions. They may be *valued* policies, stating a fixed amount to be paid in case of loss; or *open* policies, in which a maximum payment is named but the actual payment is based on evidences of loss.

ILLUSTRATIONS. Insurance on ships is frequently covered under valued policies Insurance on cargoes, and ordinary fire insurance is generally covered by open policies. In case of total or partial loss under open policies, the insurance company pays that part of the maximum stated in the policy that will indemnify the insured. Thus, if $2500 damage is caused by fire to a building insured for $6000, the entire damage is paid; but if the damage is $6800, only the maximum of the policy, $6000, is paid.

Some policies contain a co-insurance clause requiring that the property shall be kept insured up to a certain per cent. of its value—otherwise that per cent. only of the loss is paid that the insurance named is of the per cent. required.

ILLUSTRATION. If 80% is the per cent. required to be insured then, on a $15,000 building, for example, $12,000 insurance must be carried to receive full benefit. In case of $10,000 insurance, five-sixths of $12,000, only five-sixths of any loss, up to the face of the policy will be paid.

Some policies contain an *average clause*, specifying the

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payment of that part of the loss that the insurance is of the value of the property.

ILLUSTRATION. If the insurance on a piece of property is four-ninths of its value, four-ninths of any loss will be paid.

Insurance companies reserve the right of replacing or repairing damaged property covered by their policies, in place of paying cash for losses. In case the property is insured in several companies, the loss is usually apportioned among them according to the faces of the policies they issued on it. Sometimes a single company issues a policy on a valuable property, for a heavy amount, and re-insures a part of its "risk" in another company. Losses are appraised by special inspectors and often agreements are reached with owners, as to the loss payable.

The rate of premium varies with the character of the risk and the period of the policy. It may be for a specified number of days, for a year or for a period of years, or in the case of merchandise in transit, for the length of a certain voyage. "Short rates" are often charged for terms under one year, and are relatively higher than the yearly rates. Rates are expressed in cents per $100, or as a rate per cent.

Many of the computations of premiums and settlements of losses are based on the simple principles of ratio and of percentage. No new process is involved in any problems in this book.

SOME CLASSES OF PROPERTY INSURANCE.

Fire Insurance is insurance of property against loss by fire. The liability covers damage by water in fighting the fire, or destruction of property in the same cause. Sometimes, it covers damage caused by lightning, or by wind storms.

Marine Insurance is insurance of vessels and cargo against the perils of navigation. This includes damage by fire, or storm, shipwreck, etc. Marine policies frequently contain the average clause.

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Transportation Insurance is insurance of goods in transit by land, or by land and water, against loss resulting from the ordinary dangers of such traffic.

Live Stock Insurance is issued to cover loss by casualties to horses, cattle, etc.

Miscellaneous Special Types of Insurance. It is said that it is now possible to insure any property against any damage. The following titles illustrate common types, and are practically self-explanatory: Automobile and equipment insurance, tourists' (personal effects) insurance, motor boat and equipment insurance, steam boiler insurance, explosion insurance, theft insurance, plate glass insurance, derailment insurance, fly wheel insurance, etc.

ORAL EXERCISE.

Find the cost of insurance:

	Face of Policy.	Premium.		Face of Policy.	Premium.
1.	\$ 2,000	1¼%	5.	\$ 4,000	\$1.25 per \$100
2.	\$ 3,600	¾%	6.	\$ 3,000	.80 per \$100
3.	\$50,000	2½%	7.	\$12,000	1.75 per \$100
4.	\$ 8,000	1¼%	8.	\$ 7,500	2.50 per \$1000

Find the rate of premium and express in two ways:

	Face.	Premium.		Face.	Premium.
9.	\$4,000	\$120	11.	\$12,000	\$150
10.	\$6,000	45	12.	\$ 5,000	75

13. \$400 will purchase \$? insurance at 2%.

14. \$600 will purchase \$? insurance at 1¼%.

15. \$90 is the premium on a policy of \$? at \$1.50 per \$1000.

EXERCISE.

1. Johnson takes out \$2500 insurance on his touring automobile at 2% at a cost of \$——.

2. What will it cost Robinson to place \$1500 insurance on his stable for one year at 25c per \$100, and \$800 on his horses and carriages at 40c per \$100?

3. Newton takes out an 80% policy on his brick house, valued at \$7500, at a rate of \$2.75 per \$1000. It costs him \$——.

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4. A factory owner takes out \$40,000 insurance on his buildings at $3\frac{1}{2}\%$; and \$3000 special insurance on his fly-wheel, at \$12.50 per \$1000. His total payment is \$——.

5. James Sanborn insures his property through a broker, as follows: Brick and stone house, valued at \$24,000, at 75% of its value at \$3.75 per \$1000; a broad plate glass window, for \$400 at $1\frac{1}{4}\%$; furniture, valued at \$12,500, at \$5.75 per \$1000 for five years; a brick stable and garage, for \$2000, at \$2.50 per \$1000 one year policy; horses and vehicles, \$2000 at \$4.25 per \$1000; automobile and equipment, \$2000 at $3\frac{1}{2}\%$; frame boat house \$600, at 60c per \$100; motor boat and equipment, \$1000 at $2\frac{3}{4}\%$. Compute his total annual cost of insurance.

6. A shipment of goods to Manila, valued at \$85,000, is insured for 90% value at $1\frac{1}{4}\%$. What does the insurance cost?

EXERCISE.

1. On a house insured for \$7500, damage is caused to the extent of \$3748. What should the insurance company pay?

2. Randall insures his brick house, valued at \$9000, at 4/5 value at 2%. The property is damaged to the extent of \$8500. What should the insurance company pay? What is his net loss and that of the company, allowing for premiums?

3. Property worth \$12,000 is insured for \$7500, under a policy containing an 80% clause. In case of proved damage to the extent of \$2750, what should the insurance company pay?

4. On a stock of goods amounting to \$30,000, insurance is carried as follows: Company A, \$4000; Company B, \$6000; Company C, \$15,000. The property is damaged by fire to the extent of \$16,500. What should each company pay?

5. A stock barn, insured for \$2500 against damage by tornados at 80c per \$100, is damaged in such a storm to the extent of \$3750. What is the real loss of the owner?

6. Prepare an illustrative example showing how the losses paid to a policyholder are met by other policyholders in the same company.

7. Robinson bought merchandise abroad to the value of £3000. (£1 = \$4.86). He paid a commission of 2%, and insurance of 3/4 value, at $1\frac{1}{4}\%$ (average clause). The entire shipment cost him \$——. Fire broke out in transit and the goods were damaged to the extent of \$5860. What should the insurance company pay? What was Robinson's net loss?

8. Fire broke out on the estate of James Sanborn (ex. 5 of previous exercise), causing loss of \$19,200 on the house; destroying all but \$800

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worth of furniture; destroying the plate glass window; damaging the stable to the extent of $1500; destroying the boat house, and damaging the motor boat to the extent of $350. The insurance company repaired the motor boat, and sent a check for the other losses for which it was responsible to Mr. Sanborn. The face of check was $——.

9. The "Warfield," freighter, valued at $180,000, is insured under average clause policies, for $40,000 in Company A and for $65,000 in Company B— the former at 2¼% and the latter at 2½%. (Premiums?) On one trip, the vessel carried a shipment of 28,000 bushels of corn, valued at 61c per bu. and insured at 1¼% to cover goods and charges. (Face of policy and premium?) The vessel also had aboard some general merchandise valued at $85,000 and insured for 3/4 value at 1¾%—average clause. (Premium?) Fire broke out on this trip. The general merchandise was destroyed or thrown overboard. The insurance company sold the grain that was not destroyed, amounting to 7520 bu., at 31c per bu. The damage to the vessel was appraised at $57,800. Compute the payments by each company, assuming that Company B carried the insurance on both grain and merchandise. Compute the net losses of the companies and of the owners, taking premiums into account.

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CHAPTER XXXIV.

TAXATION AND PUBLIC REVENUE.

GENERAL INTRODUCTORY EXERCISE.

1. What protection does a city government give its inhabitants?
2. What besides protection does it supply?
3. What does it do for the young people?
4. How does the local government aid travel and intercourse?
5. How are your public officers chosen?
6. Who can vote? Does it cost anything to vote?
7. How is the money for local government expenses secured?
8. What kind of property is taxed?
9. Why must many business men pay for a license to do business?
10. What does your state government do for its citizens?
11. How does it raise money for these objects?
12. Name twelve public objects for which the United States Government pays money. How are the people benefited in each case?
13. How does the National Government aid commerce and industry?
14. How does the National Government raise its funds?
15. Why does the cost of government increase steadily?
16. In what ways are governments doing more than they used to do for their people?

1. LOCAL AND STATE TAXATION.

The cost of local and, to some extent, of state governments is met by a tax or charge levied on *real estate*, in proportion to its value; and on general *personal* or movable *property*—such as furniture, machinery not built in as a part of a building, horses and wagons, and sometimes, stocks and bonds. A further income is derived from *fees* for *licenses*, or permits to conduct special businesses; from *assessments*, or charges

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against private real estate, for improvements in the form of sidewalks, water mains, etc.; and from *fines* and *permits*. *Polls* or capitation taxes are levied on all qualified voters.

EXERCISE.

Secure a printed statement of your own, or a nearby, city showing total expenditures for a recent year for governmental purposes. Show what are the largest items of expense. Find the average expenditure per person.

The greatest source of public income is *realty*. The process of taxing realty in one Eastern city is as follows: A complete assessment (valuation) of real property "at not less than two-thirds of the real value thereof," is made every three years by tax assessors. The chief assessor and his assistants then sit as a Court of Equalization and Review to hear owners and witnesses, readjusting any seemingly unfair assessment. Their decisions are then confirmed by the governing officials. Special assessments are made annually of property that has changed in value $500 or over, owing to its improvements, or to destruction of buildings. The tax, $1\frac{1}{2}\%$ of assessed value, is payable to the collector of taxes in May for the year ending June 30th.

Tax rates on real or personal property, expressed as mills or cents per $1.00 or $100, or as a rate per cent., are based on the assessed value. Thus, $1\frac{1}{2}\%$, 15 mills ($1\frac{1}{2}\%$ of $1.00) or $1.50 (on $100) are equivalent rates. Several taxes, such as local, county and state, may be levied on the same property, and collected at one time.

ORAL EXERCISE.

Express each rate in two other forms:

1.	1.8%.	**3.**	$1\frac{1}{3}\%$.	**5.**	12 mills ($1.00).	**7.**	$22\frac{1}{2}$ mills.
2.	3/4%.	**4.**	$1\frac{1}{2}\%$.	**6.**	16 mills.	**8.**	$1.75 ($100).

Find total rates equivalent to these series:

11. 2/5%, 8 mills, $1.50.

12. $1\frac{1}{3}\%$, 3 mills, $1.25.

13. $1\frac{1}{4}\%$, 75c, $7\frac{1}{2}$ mills.

14. 75c, $1.25, $2\frac{1}{4}\%$.

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The computation of the tax involves simple percentage or multiplication.

ILLUSTRATION. Compute state and local tax at $1\frac{1}{2}\%$ and 8 mills, on property assessed at $4200.

Solution. State tax $= 1\frac{1}{2}\%$ of $4200 = $63.00

Local tax = 8 mills per $100 = $4200 \times \$.008 = \$33.60$.

NOTE. The total tax may be found directly. 8 mills $= 4/5\%$. $1\frac{1}{2}\% + 4/5\% = 2\frac{3}{10}\%$. $2\frac{3}{10}\%$ of $4200 = $96.60.

ORAL EXERCISE.

Find total tax.

	Rates.	Assessed Value.		Rates.	Assessed Value.
1.	2/5%	$4500	5.	8 mi.	$24,500
2.	$1\frac{1}{2}\%$	7280	6.	1/4% and $1.50	6000
3.	$1.25	14,500			
4.	.50c	8420	7.	15m., $2\frac{1}{2}$m.	8400

EXERCISE.

Find missing values.

State each example in form of a question, and show how it differs from preceding problems of a similar nature.

1. John Sampson's property is assessed for $12,600, at $1\frac{1}{4}\%$. His tax is $——.

2. C. P. Crane has assessed at $1.25 per sq. ft., a lot measuring 240 ft. $\times$ 80 ft. His tax, at $1\frac{1}{4}\%$, is $——.

3. Robert Walter owns three lots, assessed at $8420, $9200 and $14,620. On the latter is a building assessed at $16,500. He has personal property assessed at $4500, but $1000 is exempt by law. His total tax at $12\frac{1}{2}$ mills is $——.

4. A Permanent Improvement Fund tax of $2\frac{1}{2}$ mills, levied by a western city on property assessed at $26,450, costs its owner $——.

5. A Current Expense Fund is created, in a Minnesota city, by a tax of 6/10%. On a realty assessment of $99,547,484 and a personal property assessment of $29,049,250, this would yield what sum if every person paid? In this city, the delinquent tax averages $4\frac{3}{4}\%$. What net tax should the property yield?

6. A western city recently levied this general tax per $1000. What does it cost C. A. Mansfield whose property is assessed at $8455?

State revenue	$1.50
State School and University Fund	2.23

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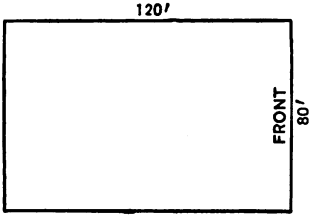
County revenue	2.75
School revenue	6.00
City funds	14.37
	?

7. James Field, collector for the town of Mayfield, receives a salary of 3% of his collections. If he collects a $1.25 tax on $364,000 of property, he earns $——.

NOTE. Penalties are charged for delinquent (unpaid) taxes. In one city, penalty interest of 1% per month is charged. In default of payment, the property is sold, but the purchaser gets simply a tax deed bearing 1% per mo. interest, and the owner may redeem his property within two years.



8. The property here shown is assessed at $30 per *front foot*, and taxed 8 mills. The tax due on Jan. 1, was not paid until Oct. 1, incurring a penalty, at 3/4% per month, of $——.

9. A tax certificate of $241.60, bearing 1% interest per month, dated Dec. 10 last, is worth to-day $——.

10. J. C. Parker leaves his wife by will, $42,000 in property; his son, $12,600, and a niece $5200. The state has an inheritance tax of 1½% on bequests to lineal heirs, and 6% to others. The tax on these bequests yields the state $——.

11. The Randolph Manufacturing Co. is taxed 1% of its legal capital, on issuance of its state charter. If the authorized capital is $240,000, the tax is $——.

12. An interurban trolley road is taxed 90c per $100 on its equipment and 1/2% on its gross earnings. Last year its equipment was assessed at $8,240,000 and its gross earnings were $6,547,325.60. Its state tax amounted to $——.

Many problems arise in tax offices when making up advanced estimates. A few are stated below.

EXERCISE.

1. What rate in mills and per cent. is sufficient to raise $12,460 on an assessed value of $800,000?

2. In a city having a fixed legal tax rate of 15 mills, the total assessed value must be fixed at what sum, in order that an income of $172,000 may be secured?

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3. Last year, in a Nebraska town, the legal rate of 3½ mills yielded $142,500. This year the income required being only $122,000, the assessed value may be reduced to approximately ——%.

4. A Pennsylvania town must meet the following appropriations by a tax of ? mills, on an assessed valuation of $1,246,300: salaries $2140; schools, $8720; roads, $2465; general expenses $5840. Assume 5% delinquent tax.

5. 520 polls @ $1.50; license fees, $1260; and a $1.25 tax on realty and personalty, assessed at $1,652,000, will yield an annual income of $ ——.

For convenience in calculation, tax computation tables are often prepared for the total tax rate of a town or city. The following table gives the tax, at the given rate, on any sum from $10 to $99. The tax on other sums is found by "pointing off" and by addition. The original table must be computed with great care, and written, preferably, in six decimal places.

TAX TABLE

7½ mills — Per $100

	Unit lines										
	0	1	2	3	4	5	6	7	8	9	
1	.075	.0825	.0900	.0975	.0150	.1125	.1200	.1275	.1350	.1425	1
2	.150	.1575	.1650	.1725	.1800	.1875	.1950	.2025	.2100	.2175	2
3	.225	.2325	.2400	.2475	.2550	.2625	.2700	.2775	.2850	.2925	3
4	.300	.3075	.3150	.3225	.3300	.3375	.3450	.3525	.3600	.3675	4
5	.375	.3825	.3900	.3975	.4050	.4125	.4200	.4275	.4350	.4425	5
6	.450	.4575	.4650	.4725	.4800	.4875	.4950	.5025	.5100	.5175	6
7	.525	.5325	.5400	.5475	.5550	.5625	.5700	.5775	.5850	.5925	7
8	.600	.6075	.6150	.6225	.6300	.6375	.6450	.6525	.6600	.6675	8
9	.675	.6825	.6900	.6975	.7050	7125	.7200	.7275	.7350	.7425	9
	0	1	2	3	4	5	6	7	8	9	

ILLUSTRATIONS. The tax on $82 is found on the horizontal line "8" at its intersection with the vertical unit line "2." It is $.615 or 62c.

The tax on $8200 (100×82) = $61.50. The tax on $972 = tax on $900 + tax on $72, etc.

EXERCISE.

1. Show how the above table may be constructed by addition. Show how short methods may be used for construction or checking.

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2. Read from the table, the tax on assessed values of:

$840	$24,200	$1,220	$ 815
1200	36,450	19,600	2,290
960	8,000	452	42,600

3. Use the table to read the tax on these same values at 15 mills.

4. Construct tax tables for these rates: (*a*) 1½%; (*b*) 60c; (*c*) $1.45; (*d*) $27.60 (per $1000).

Licenses. Many municipalities secure an income from a charge for licenses to conduct certain businesses. An eastern city received from this source, in one year, $572,473.43. These are a few of its rates:

Business.	Rate.	Remarks.
Apothecaries	$ 6.00	per annum.
Auctioneers	100.00	per annum.
Boarding houses, hotels, public	1.00	per guest room.
Brokers, real estate	.50	per annum.
Carriages, for hire	6.00	one horse.
Carriages	9.00	more than one horse.
Circuses	200.00	per day.
Confectioners	12.00	per annum.
Livery stables	25.00	per annum 10 stalls.
Livery stables	2.00	each additional stall.
Passenger transfer lines	6.00	each vehicle seating ten or less.
Passenger transfer lines	12.00	each vehicle seating over ten.

EXERCISE.

Using these license rates, tabulate the income derived by a city from 61 apothecaries, 18 auctioneers, 529 street cars, 63 confectioneries, 17 livery stables (under 10 stalls), 8 stables—total of 286 stalls, 282 carriages for hire; 41 brokers; 21 hotels, guest rooms respectively: 149, 217, 300, 84, 29, 80, 120, 360, 400, 210, 88, 146, 97, 63, 48, 88, 95, 85, 66, 22, 38.

Assessments. In some cities a portion of the cost of public improvements and of repairs, such as curbing, sidewalks, water mains, sewers, alleys, etc., is levied by assessment against property that is especially benefited. If an owner *asks* for improvements, he must deposit his estimated share of the cost in advance. If he does not ask for the improvements, his payments may usually extend over two years. The

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proportion paid by private owner and the methods of assessment vary greatly in different cities. Several different methods are illustrated in the following problems.

EXERCISE.

1. John Evans is assessed $? for a water main, at a rate of $1.10 per front foot, on two lots of 85 ft. and 40 ft. frontage respectively.

2. 320 ft. of curbing are laid at a cost of 90c per foot. The frontage benefited is 300 ft., of which Adams owns 65 ft. He is assessed one-half the pro rata cost, or $ ——.

NOTE. The cost of grading and paving streets is sometimes paid in full by the city or town; sometimes assessed one-half on abutting property, according to frontage, sometimes one-half according to area, sometimes part by frontage and part by area. Owing to the fact that corner lots may be assessed for improvements on two streets, their rates are sometimes reduced. Areas are measured from established building lines.

3. Rockford Avenue (see page 276) is paved with asphalt from First to Second street at a cost of $2.25 per square yard. The area paved is 60 ft. by 360 ft. If one-half of the cost is assessed on property owners, what sum must be met by the owners of frontage on *one* side of the street? If this cost is shared according to frontage, what *lots* must be assessed?

Andrews, who owns lots No. 266 and No. 265, pays $?; Chapman, for No. 270, and No. 271, pays $?; Eastman, the owner of No. 267, 268, 269, 270, and No. 273 pays $?.

4. Find assessments (ex. 3) if corner lots are assessed on basis of 1/2 frontage.

5. Determine assessments if 25% is levied by frontage and balance by area.

6. A curbing is laid from the alley around these lots, 25 ft. from building line, at a cost of 35c per ft. Assess one-half the cost against the lots according to frontage.

7. The property fronting on Rockford Avenue is assessed 65c per front foot for water mains. What does each owner pay?

8. The alley is paved at a cost of $1.20 per sq. yd. from 1st to 2nd street. One-fourth of the cost is assessed by area against lots Nos. 264–273. Determine the "lot" assessments.

II. NATIONAL GOVERNMENT INCOME AND EXPENSE.

The main sources of National Government income and expense are shown in the tables that follow. There are many minor sources also, such as profits on coinage, fees, sales of

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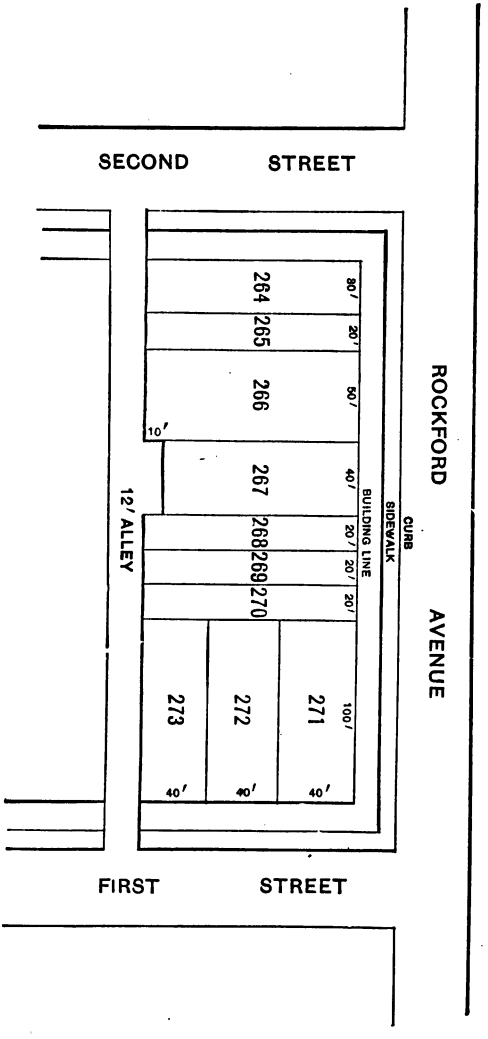

public land, etc. The internal tax on liquors, tobacco, etc., is a stamp tax, and is not of such general character as to require notice. The other taxes are paid in money.

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RECEIPTS OF THE UNITED STATES GOVERNMENT IN FIVE RECENT YEARS.

Source.	I	II	III	IV	V	Last Yr. Per Capita.
Customs revenue	$300,251,878	$332,233,363	$286,113,130	$300,711,934	$333,683,445	?
Internal revenue	249,150,213	269,666,773	251,711,127	246,212,644	289,933,519	?
Postal service	167,932,783	183,585,006	191,478,663	203,562,383	224,128,658	?
Miscellaneous	45,315,851	61,225,524	63,236,466	56,664,912	51,894,751	?
Total						

Per capita basis: Population of 92,175,000.

MAIN EXPENDITURES OF THE UNITED STATES GOVERNMENT IN THE SAME YEARS.

Source.	I	II	III	IV	V	Last Yr. Per Capita.
Legislative	$ 11,239,772	$ 11,695,867	$ 13,723,103	$ 13,678,999	$ 13,616,496	?
Executive	698,131,220	714,907,782	790,559,651	844,042,405	861,621,418	?
Judicial	7,967,217	8,686,485	8,298,300	8,165,422	8,596,135	?
Total						

EXERCISE.

1. Complete the tables.
2. Define the terms in the first column.
3. Where does authority for expenditures rest? What is an appropriation?

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Customs.

The National Government levies duties or customs on many classes of articles, for revenue and for protection of home industry. Congress has adopted a *tariff* or schedule showing the rates of duties on imports; and a *free list*, or schedule of articles imported exempt from duty.

Duties are either *ad valorem* (levied as a per cent. of their net value) or *specific* (levied by measure). Both classes of duties are imposed on some imports. In general, classes of goods varying markedly in quality, and therefore in value, are taxed "ad valorem." These frequently are manufactures, such as carpets, crockery, knives, etc. Raw materials, and many articles of fairly uniform grade, are taxed specifically, for example, rivets, gold leaf, eggs, wheat.

Ad valorem duties are not computed on fractions of a dollar, less than 50 cents being discarded, and 50 cents or more being considered an additional dollar. The same rule applies, in specific duties, to fractions of the unit measure. The Government employs the long ton (2240 lb.) in weighing, but makes allowances, in many cases, for tear and breakage.

Rates from a United States Tariff.

Camphor, refined, six cents per pound.

Castor oil, thirty-five cents per gallon.

Paris green and London purple, fifteen per centum ad valorem.

Castile soap, one and one-fourth cents per pound; medicinal or medicated soaps, twenty cents per pound; fancy or perfumed toilet soaps, fifty per centum ad valorem.

Lime, five cents per weight of one hundred pounds, including weight of barrel or package.

Cast polished plate glass, finished and unfinished, or unsilvered, not exceeding three hundred and eighty-four square inches, ten cents per square foot, above that, and not exceeding seven hundred and twenty square inches twelve and one-half cents per square foot; all above that twenty-two and one-half cents per square foot.

Pen knives, pocket knives, clasp knives, valued at not more than forty cents a dozen, forty per centum ad valorem; valued at more than forty

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cents per dozen, and not exceeding fifty cents per dozen, one cent per piece, and forty per centum ad valorem.

Gold leaf, thirty-five cents per one hundred leaves.

Shingles, fifty cents per thousand.

Horses and mules, valued at one hundred and fifty dollars or less per head, thirty dollars per head; if valued at over one hundred and fifty dollars, twenty-five per centum ad valorem.

Buckwheat, fifteen cents per bushel of forty-eight pounds.

Carpets, woven whole for rooms, and Oriental, Berlin, Aubusson, Axminster and similar rugs, ten cents per square foot, forty per centum ad valorem.

Firecrackers of all kinds, eight cents per pound, the weight to include all coverings, wrappers and packing material.

EXERCISE.

1. Define: Income, duties, revenue, tariff, free list, imports, ad valorem, and specific.

2. Which is simpler to levy, a specific or an ad valorem tax?

3. State the kind of duty specified in each of the given sections of the tariff. Suggest a reason for the kind of duty.

The country is divided into collection districts, each containing a *port of entry* and one or more *ports of delivery.* All imports must be brought first to the port of entry at which is situated a *custom house* for the collection of duties and for the official entry or clearance of vessels. After duties are paid and permission given to unload, goods may be *discharged* at any port of delivery. Thus the collection district of the City of New York has two ports of entry, New York and Jersey City, which, as usual, are also delivery ports; and has additional ports of delivery at New Windham, Albany, Cold Spring, Port Jefferson and Patchogue.

In the larger ports, a *Collector of the Port* is the chief officer, having under him *appraisers* to determine values, and *surveyors, weighers, gaugers,* etc., to determine quantities.

In brief, the process of importing goods is as follows: The seller or shipper at a foreign port makes out an *invoice* of the marks, number, quantities, value, etc., of the goods shipped,

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weights and values being stated in the standards of the foreign country. The invoices, if over $100, are certified by the local U. S. consul who retains a copy and sends a second copy to the collector of the port at which the goods are to be entered. On loading the goods aboard ship, the shipper receives duplicate *bills of lading*, or receipts involving a contract to deliver at a specified port. The shipper forwards copies of invoice and of bill of lading to the purchaser in the United States. The latter fills out certain forms and presents them, with the invoice and bill of lading, at the custom house, pays the estimated duties, and receives a permit to land the goods subject to examination and verification.

The permit goes to the *Surveyor* who is in direct control of the vessel and its cargo. The goods are then landed and examined and appraised, to determine and check quantities and dutiable value. One package in each invoice, one package out of every ten of the same kind and same price of each class of material, is examined carefully. The master of the vessel also submits to the Collector of the Port a manifest, or statement of the name of the vessel, of the names of consignors and consignees, and of the items of his cargo.

If the importer does not care to use and pay duty on his goods, at the time of receipt, he may *give bond* to pay the duties, and may store the goods in *bonded warehouses*, maintained by the Government. All or part of the invoice, within certain minimum limits, may be withdrawn as desired on payment of duties and reasonable charges for storage and incidental labor. Such goods may be withdrawn for export without being subject to duty. If the goods are retained in storage over one year, however, the duty is increased 10%. After three years, the goods are subject to forfeit. If duties have been paid and imports afterwards exported, a *drawback* or refunding of the duties is granted.

For the collection of the excise, or internal revenue tax, the

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country is divided into districts each in charge of a collector. In many cases, the duty is paid by affixing to packages of manufactured goods the requisite amount of tax in the form of adhesive stamps which may be purchased in quantities and used as needed. The stamps must be destroyed when the packages are opened. Drawbacks are allowed on goods made in this country that are exported after the excise tax has been paid.

As the invoice value of imports is expressed in the coinage of the exporting country, it is necessary to reduce such values to United States standards. Coinage reduction tables are issued periodically by the Director of the United States Mint.

VALUES OF FOREIGN COIN IN UNITED STATES GOLD.

(Foreign *gold* coins unless otherwise stated.)

Country.	Coin.	Abbreviation.	U. S. Value in Gold
Argentine Republic	Peso	p.	$.965
Austria	Krone	kr.	.203
Belgium, France, Switzerland	Franc	fr.	.193
Brazil	Milreis	ml.	.546
Chili	Peso	p.	.365
German Empire	Mark	M.	.238
Great Britain	Pound Sterling	£	4.8665
Italy	Lira	l.	.193
Japan	Yen	y.	.498
Mexico	Peso, Dollar (sil.)		.498
Netherlands	Florin	fl.	.402
Philippines	Peso	p.	.500
Russia	Ruble	r.	.515

Computations under this table involve simple multiplication.

ILLUSTRATION. Determine the U. S. value of a shipment of hides from Argentine, invoiced at 4200 pesos.

Solution. From the table, 1 peso = $.965. 4200 × $.965 = $4053, the value of 4200 pesos.

ORAL EXERCISE.

Determine value of these invoices in U. S. gold.

1. From Argentine, 2000 pesos.
2. From England, £ 600.
3. From France, 1200 fr.
4. From Germany, 5000 M.

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5. From Chili, 250 p.
6. From Mexico, 1000 pesos.
7. From Netherlands, 300 fl.
8. From Russia, 400 r.
9. From Philippines, 720 p.
10. From Japan, 10,000 yen.

EXERCISE.

Determine the value in U. S. gold of these invoices:

1. From Russia, 7246.5 r.
2. From Japan, 864.36 yen.
3. From Switzerland, 15,329 fr.
4. From Germany, 2468 M.
5. From London, £365.82.
6. From Amsterdam, 4368.20 fl.
7. From Havre, 32,129.6 fr.
8. From Naples, 673.80 l.
9. Of bone and tallow from Argentine, 6754 pesos.
10. Of nitrate of soda from Chili, 73,854 pesos.
11. Of coffee from Brazil, 785.25 ml.
12. Of knives from Sheffield, £542.10.
13. Of glass from Bohemia, 529.65 krone.

Specific Duties.

When specific duties are charged, the reduction of foreign measures to U. S. standards is necessary. Great Britain uses measures similar to our own, and most other exporting countries use the metric system.

Once the invoice quantity is expressed in U. S. standards, the computation of the specific duty is a matter of direct multiplication, except where the wording of the U. S. tariff necessitates a further computation.

Illustrations. (1) What is the duty on 50 M. shingles at 50c per M.?

Solution. At 50c per M. the duty on 50 M. $= 50 \times 50c = \$25.00$.

(2) What is the duty on 7200 lb. buckwheat at 15c per bu. of 48 lb.?

Solution. The number of bu. $= 7200 \div 48 = 150$. $150 \times 15c = \$22.50$ duty.

ORAL EXERCISE.

See Tariff, pp. 278, 279 for missing rates.

Compute the duty on:

1. 2400 bu. wheat @ 25c.
2. 60 gr. pens @ 12c.
3. 2750 lb. rivets @ 1½c.
4. 16 T. grindstone @ $1.75 per T.
5. 4600 lb. Castile Soap.
6. 625 lb. camphor.
7. 3000 lb. lime.
8. 28 pkg. gold leaf.
9. 10 pc. plate glass, 24 in. × 30 in.

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EXERCISE.

Compute the duty on:

1. 246 pc. plate glass, 16″ × 24″.
2. 7500 ft. bar iron, weighing 1.2 lb. per foot, @ 6/10c per ft.
3. Three cases of fire crackers, weighing respectively 215, 129, 142 lb., including all packing material, @ 8c per lb.
4. 1520 short tons of barley, @ 30c per bu. of 48 lb.
5. 320 bx. Castile soap, averaging 110 lb. per bx., allowance for tare 5%.
6. 22 long tons of lime.
7. 60 gr. bottles of mineral waters @ 20c per doz.
8. Find the entire duty on this invoice.

SHEFFIELD, Nov. 17, 191—.

INVOICE OF HARDWARE PURCHASED.

By *James Evanston of Baltimore, Md.*
From *The Peter Mfg. Co. of Sheffield.*
To be shipped per Cambria.

Marks.	Quantity.	Description.	Price. £	s	d	Amount. £	s	d	Consular Corrections.
◆68	6	gr. Files,		3	0	xx	x	x	
◆69	160	lb. Bolts,		1	6	x	x	x	
◆70	42	lb. Wood screws,		1	9	x	x	x	
		Total.							
		U. S. Money.							
		Duty on files, 25c doz.							
		Duty on bolts, 1½c lb.							
		Duty on screws, 8c lb.							

AD VALOREM DUTIES.

The computation of ad valorem duties often involves the reduction of money values to U. S. gold standard, and always involves percentage.

ILLUSTRATION. Compute the duty (40%) on forty watch cases invoiced @ 36 fr.

Solution. The value of forty watch cases = 40 × 36 fr. = 1440 fr. 1 fr. = $.193. 1440 fr. = 1440 × .193 = $277.92. The dutiable value is $278. The duty = 40% of $278 = $111.20.

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EXERCISE.

Compute the duty on:

1. An invoice of London purple, valued at £46 8s.
2. 8 doz. pen knives @ 0/1/6 per doz.
3. 40 head of horses from Canada, @ $135.
4. An invoice of crockery, 675 fr. Duty 60%.
5. A collection of orchids from Brazil, @ 526 ml. Duty 25%.
6. An invoice of swords, valued at $4264.14. Duty 50%.
7. A set of "standard authors" valued at £64. Duty 25%.
8. Compute ad valorem duty at 60% on this invoice:

MME. S. RENANT.

Robes Manteaux Modes. *52. Place du Trocadéro*

Madame C. Fanning, Dr.

	Paris, le 18 Novembre, 191—	Fr.
2	Robes, garni motif	775
1	Robe, en foulard	310
1	Robe, en draps bronze, Princesse	400
1	Robe, drap gris, brader	526
1	Blouse, en crepe de chene blanc	92
1	Blouse, taffeta bleu	65

NOTE. Both ad valorem and specific duties are levied on many imports.

9. A rug, 6′ × 8′, valued at 146 fr. (see tariff).

10. 86 lb. celluloid manufactures, valued at 756 fr. Duty 65c per lb. and 30%.

11. 1200 m. oil cloth, 2 yd. wide, @ 2.4 M. per m. Duty 6c sq. yd. and 15%.

12. Extend this form:

Manifest No. 868. Invoiced at Amsterdam, Netherlands, Nov. 29, 19—.

INWARD FOREIGN ENTRY OF MERCHANDISE.

Imported by *J. J. Jervis* In the steamer *Kriedam*

A. B. Roe Master From Amsterdam. Arrived Dec. 15, 19—

Mark	No.	Packages and contents	Quantity	Free List	44 ¢ sq. yd. and 40% ad valorem	40 ¢ sq. yd. and 40% ad valorem	Duty	Total
[2]	920–1	2 cas. Brussels carpet 3/4 yd. wide.	400m.		600 fl. $?			
[5]	922–3	2 cas. Tap. 3/4 yd. wide.	420m.			840 fl. $?		
		40% of $——					$?	
		? yd. @ 44c					?	
		? yd. @ 40c					?	$——

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CHAPTER XXXV.

INTEREST.

Interest ranks with the fundamental processes, and with simple percentage, as one of the most useful branches of arithmetic. It has more general and varied applications than simple percentage, and its use is being constantly extended. It is the great interpretative branch of arithmetic and has to do, especially, with the study of values.

INTRODUCTORY EXERCISE.

1. Give business reasons for borrowing money.
2. Need a prosperous business ever borrow money?
3. Why is money paid for the *use* of others' money?
4. I owe Smithson $500, due to-day. If I delay payment for a month, does Smithson gain or lose? Why?
5. Does the time I keep another's money affect the charge for its use?

Interest has double meaning. Strictly, it is *the use of money.* Commonly, it is *the sum paid for that use.* The sum on which interest is charged is the *principal.* Interest is computed as a per cent. of this principal per year, with relatively less amounts for portions of a year. Each state has a *legal rate* of interest, used when no rate is stated in interest-bearing obligations. By agreement, a lower rate may be charged; or in some states, a higher rate up to a maximum fixed by law may be charged. A requirement to pay higher than the legal maximum is termed *usury.*

Commercially, the interest year is reckoned as 360 days, or 12 months of 30 days each. If, however, the time is a year or more, calendar years are taken for the full years. The United States Government, and some banks, reckon interest

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on a basis of 365 or 366 days. Interest tables commonly are used to save computation. For long periods, interest time is computed by compound subtraction; for short periods the exact number of days is used, as a rule, this depending, however, on the custom of the business or the character of the obligation.

ORAL EXERCISE.

1. Determine the interest on \$6000 for one year at 6%. At 8%.

2. Find the interest on \$1200 for 1 year at 5%; for 2 years; for 6 months; for 2 months.

3. What part of the interest for a year is the interest for 3 mo.? 2 mo., 4 mo., 60 da., 90 da., 80 da., 120 da., 30 da.?

4. Determine the interest on \$800, at 4%, for 1 yr., 3 mo., 4 yr., 2 mo., 90 da., 6 mo., 30 da. For a fixed rate and principal, how is interest affected by a change of time?

5. How does interest at 4% compare with interest at 5%, on the same principal?

6. If the interest for a fixed time, on a given principal is \$32, at 4%, find the interest at 5%, at 6%, 2%, 8%, 1/2%, 4½%. What is the effect of increasing or decreasing the rate, if principal and time remain constant?

7. The interest on a given principal, at a fixed rate, for 20 days, is \$1.60. Find the interest for 5 da., 10 da., 30 da., 40 da., 2 mo., 80 da., 11 da., 3 mo. What is the effect of change of time?

8. If rate and time remain unchanged, how will the increase or decrease of principal affect the interest? If the interest on \$1200, for a certain time, is \$36, determine the interest on \$900, \$800, \$700, \$6000, \$720, \$400.

Most practical methods of computing interest depend on the principles just emphasized, are closely related, and are based on a year of 360 days. It is best to understand several methods and to select for general use the one that seems most natural and simple.

I. CANCELLATION METHOD.

(Based on 360 or 365 days to the year.)

ILLUSTRATION. Example. Compute the interest on \$720 for 84 days at 6%.

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Analysis and Solution. The interest for 1 year is 6/100 of $720. For 84 days, it is 84/360 of this amount.

$$\frac{\overset{2}{\cancel{\$720}} \times .06 \times 84}{\underset{1}{\cancel{360}}} = \$2 \times .06 \times 84 = \$10.08.$$

EXERCISE.

Compute the simple interest on:

1. $720 for 36 da. at 8%.
2. $925 for 2 mo. 6 da. at 6%.
3. $456.50 for 8 mo. at 7½%.
4. $862.85 for 7 mo. 3 da. at 6%.
5. $1260 for 0 mo. 15 da. at 6%.
6. $829.60 for 39 da. at 4%.
7. $3600 for 20 da. at 5%.
8. $240 for 18 da. at 9%.
9. $1200 from Jan. 5 to Nov. 21 at 6%. (Approximate time.)
10. $12,900 from Aug. 21 to Dec. 3, at 4%. (Approximate time.)
11. $270 from Mar. 13 to May 24 at 7%. (Exact time.)
12. $1200 for 75 da. at 5%
13. $320 for 3 mo. 5 da. at 8%.

II. SIXTY-DAY METHOD.

The sixty-day, or bankers' method, is one of the most effective in use. By it, interest is first computed at 6% and then reduced to the required rate.

INTRODUCTORY EXERCISE.

1. If the interest rate is 6%, what per cent. of the principal is the interest for 6 mo.? 2 mo.? 8 mo.? 1 mo.?
2. What per cent. for 60 da., 30 da., 90 da., 20 da.?
3. For what period, in months, is the interest 1% of the principal? For what period in days?
4. What is the simplest method of finding 1% of a number?
5. Find the interest at 6%, for 60 days, on $1200, $800, $300.

ILLUSTRATIVE EXAMPLE. Compute the interest on $7200 for 96 days at 6%. At 3%.

Solution. The interest for 60 days at 6% equals 1% of the principal.
$7200.00 = the principal.
72.00 = interest for 60 da. (1% of principal)
36.00 = " " 30 " (30 da. = 1/2 of 60 da. ∴ interest equals 1/2 of interest for 60 da.)
7.20 = " " 6 " (6 da. = 1/10 of 60 da.)
$115.20 = " " 96 " Ans. (6%)
The interest at 3% = ½ the interest at 6% or $57.60.

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EXAMPLE 2 Compute the interest on $840 for 4 mo. 24 da. at 6%.

Solution.

$840	= the principal.		
8 | 40	= interest for	2 mo.	(Use a line to mark decimal point.)
8 | 40	= "	"	"
2 | 80	= "	"	20 da. (20 da. = 1/3 of 2 mo.)
| 56	= "	"	4 " (4 da. = 1/5 of 20 da.)
$20 | 16	= "	"	4 mo. 24 da.

Check. Examples may be checked by changing time components.

ILLUSTRATIONS. (Checks on example 2)

4 mo. 24 da. = 144 da.

(1)

$ 8 | 40	= interest for	60 da.	
16 | 80	= "	"	120 da. (Twice interest for 60 da.)
2 | 10	= "	"	15 " (1/4 interest for 60 da.)
| 84	= "	"	6 " (1/10 " " 60 ")
| 42	= "	"	3 " (1/2 " " 6 da.)
$20 | 16	= "	"	144 "

(2)

$ 8 | 40	= interest for	60 da.	
21 | 00	= "	"	150 " (2 1/2 times interest for 60 da.)
| 84	= "	"	6 "
$20 | 16	= "	"	144 " (By subtraction.)

NOTE. This method substitutes simple addition and division for more complicated operations, and offers positive check solutions. If times are large, 600 days may be used as a unit time in place of 60 days; if the time is small, three places may be pointed off in order to find interest for a unit time of 6 days.

It is generally advisable to select such sub-times that the interest at each step may be found from the previous step, or from the unit. See ex. 1, above.

ORAL EXERCISE.

Subdivide these times into parts, and also suggest a "check" series:

1. 84 da.	**5.** 5 mo. 7 da.	**9.** 3 mo. 21 da.
2. 90 da.	**6.** 87 da.	**10.** 48 da.
3. 11 da.	**7.** 4 mo. 15 da.	**11.** 37 da.
4. 21 da.	**8.** 7 mo. 3 da.	**12.** 16 da.

ORAL EXERCISE.

Compute the interest at 6% on:

1. $1200 for 60 da.	**3.** $ 960 for 6 da.
2. $ 840 for 30 da.	**4.** $ 120 for 20 da.

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5. $ 600 for 3 da.
6. $640 for 45 da.
7. $ 900 for 2 mo. 6 da.
8. $ 126 for 600 da.
9. $ 884 for 30 da.
10. $ 390 for 2 da.
11. $ 900, 2 mo. 20 da.
12. $1600 for 15 da.

Compute the interest on:

13. $300, 4 mo., 5%.
14. $240, 30 da., 7%.
15. $800, 45 da., 4½%.

EXERCISE.

Compute the interest on the following, checking by change of time:

	Principal.	Time.	Rate.		Principal.	Time.	Rate.
1.	$ 542.50	17 da.	6%.	5.	$2164	23 da.	6%.
2.	$3690	97 da.	5%.	6.	$ 980	3 mo. 8 da.	6%.
3.	$ 452.80	48 da.	4%.	7.	$ 452.75	5 mo. 6 da.	6%.
4.	$ 725	39 da.	6%.	8.	$3726.	6 mo. 13 da.	4%.

Compute the *amount*, principal plus interest, on the following:

9.	$ 426	11 da.	6%.	13.	$1268	132 da.	5%.
10.	$ 300	25 da.	8%.	14.	$ 282.90	3 mo. 17 da.	6%.
11.	$1285	34 da.	6%.	15.	$ 562.80	1 mo. 5 da.	6%.
12.	$ 950	68 da.	6%.	16.	$1262.50	7 mo. 15 da.	7½%.

Using approximate time, compute the interest on:

17. $9650 from Jan. 27 to Oct. 19, at 6%.
18. $248.20 from Nov. 12 to Jan. 13, at 8%.
19. $3242.90 from Mar. 28 to Dec. 19, at 6%.

Using exact time, compute the amount of:

21. $1220 from Mar. 22 to Apr. 14, at 6%.
22. $356 from Feb. 12, 1909, to June 14, 1909, at 7%.

In order to simplify computation, the numerical value of the principal and time (in days) may be interchanged without affecting the resulting interest. In finding the *amount*, however, it is necessary to remember to add the original principal.

ILLUSTRATION. The interest on $120 for 17 days is the same as that of $17 for 120 days. Thus in the first case, the factors are

$$\frac{120 \times 17 \times .06}{360},$$

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and in the second case, they are simply interchanged in order,

$$\frac{17 \times 120 \times .06.}{360}$$

EXAMPLE. Compute the interest on \$72 for 185 da.

Solution. Read the example: Compute the interest on \$185 for 72 da.

\$1.85 = interest for 60 da.
.37 = " " 12 "
\$2 22 = " " 72 " on \$185, or for 185 da. on \$72.

ORAL EXERCISE.

By interchanging terms, compute interest at 6%, on:

1. \$60 for 29 da.
2. \$20 for 96 da.
3. \$30 for 47 da.
4. \$120 for 38 da.
5. \$15 for 92 da.
6. \$12 for 132 da.
7. \$600 for 71 da.
8. \$150 for 42 da.

EXERCISE.

Compute interest and amount on:

1. \$126 for 197 da. at 6%.
2. \$540 for 83 da. at 8%.
3. \$3000 for 179 da. at 6%.
4. \$ 450 for 168 da. at 6%.

III. SIX PER CENT. METHOD.

In case the interest term is greater than one year, the 6% method is frequently used. Some prefer it, also, for short terms.

INTRODUCTORY EXERCISE.

1. The interest on \$1.00 for one year at 6% = ?

2. The interest on \$1.00 for 2 mo. = ?. Find the interest for 30 da., 6 mo., 4 mo. What is the interest on \$1.00 for any multiple of 2 mo. or 60 da.?

3. The interest on \$1.00 for 6 da. = ?. How may it be found for 2 da., 6 da., for any multiple of 6 da.?

4. The interest on \$1.00 for 4 mo. = ?. From it, determine the interest on \$400 for the same time.

Theoretically, the 6% method consists in applying the 60-day method to building up the interest on \$1.00, multiplying the result by the number of dollars. For practical use, the following values should be kept in mind:

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$.06 = interest on $1.00 for 1 yr. at 6%.
.01 = " " " " 2 mo. " "
or .005 = " " " " 1 " " "
.001 = " " " " 6 da. " "
or .000$\frac{1}{6}$ = " " " " 1 " " "

ILLUSTRATION. *Example.* Compute the interest at 6% on $700, for 1 yr. 7 mo. 18 da.

Solution. $.06 = interest on $1.00 for 1 yr.
.035 = " " " " 7 mo. ($3\frac{1}{2}\times$int. for 2 mo.).
.003 = " " " " 18 da. ($3\times$int. for 6 da.).
$.098 = " " " " 1 yr. 7 mo. 18 da.
$700\times\$.098$ = interest on $700 for given time = $68.60.

NOTE. Interest at other rates may be obtained by ratio, applied either to the interest on $1.00, or to the total interest. Thus 2/3 of either $.098, or of the product, $68.60, equals the interest at 4%.

ORAL EXERCISE.

Find the interest on $1.00, at 6%, for:

1. 88 da.
2. 92 da.
3. 17 da.
4. 14 da.
5. 29 da.
6. 62 da.
7. 54 da.
8. 2 da.
9. 1 mo. 9 da.
10. 2 mo. 27 da.
11. 3 mo. 14 da.
12. 1 yr. 4 mo. 11 da.

Compute the interest on:

13. $200 for 24 da.
14. $1000 for 3 mo. 6 da.
15. $500 for 2 yr. 6 mo. 12 da.
16. $400 for 96 da.

EXERCISE.

Compute the interest on:

	Principal.	Time.	Rate.
1.	$60432	1 yr. 2 mo. 8 da.	6%.
2.	$ 98.25	2 yr. 5 mo. 3 da.	6%.
3.	$ 7294	3 yr. 1 mo. 15 da.	6%.
4.	$ 520	8 mo. 18 da.	3%.
5.	$ 2460	1 yr. 16 da.	7%.
6.	$ 590.84	1 yr. 4 mo. 12 da.	5%.
7.	$ 1580	3 mo. 16 da.	5%.
8.	$ 2050.40	5 mo. 13 da.	6%.

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	Principal.	Time.	Rate.
9.	$ 840	7 mo. 24 da.	5%.
10.	$ 1292	2 yr. 5 mo. 20 da.	6%.

11. $36,432, from Jan. 29, 1908, to Oct. 17, 1909, at 6%.

12–21. Solve by 6% method, examples 1 to 8 and 21–22, page (289).

Find the interest on the following by the 6% method and "check" by the 60-day method:

22. $324 for 1 yr. 2 mo. 8 da. at 6%.

23. $98.40 for 2 yr. 6 mo. 15 da. at 8%.

24. $7250 for 3 mo. 21 da. at 4 %.

Find the *amount* of the following, using 60-day method and checking by the 6% method:

25. $76.80 from Jan. 12 to Aug. 17, at 6%.

26. $358.25 from Feb. 14 to Dec. 29 at 6%.

IV. TABLE METHOD.

Interest tables of many forms are in common use among bankers and brokers. They usually require decimal division of the principal and ordinary subdivision of time. The following outline illustrates one common form.

SIX PER CENT. INTEREST TABLE.

Time.	$10	$20	$30	$40	$50	$60	$70	$80	$90
1 da.	.00167	.00333	.00500	.00667	.08333	.01000	.01167	.01333	.01500
2 da.									
3 da.									
1 mo.									
2 mo.									

NOTE. The practical tables give interest for each day from 1 to 30. Such tables should be constructed with the utmost care and should read to five or more decimal points, to avoid errors when applied to large amounts.

ILLUSTRATION OF ERROR. Suppose an interest table is constructed to show interest at 6% for twenty days, on amounts of $1.00 to $99.00. In a three-place table the interest for $10.00 would be $.033; in a seven-place table, $.0333333. Applying the table to determine interest on $100,000, it is necessary to move the decimal four places to the right. By the small table this would give $330; by the seven place, $333.33—a difference of $3.33.

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EXERCISE.

1. Complete the table (p. 292) by computation, and from known values.

2. Show how, from the given times, to obtain the interest for any time. Find the interest on \$10 for 17 da. (Suggestion, 17 da. = 10 da. plus 7 da.), 23 da., 4 mo. 6 da.

3. Show how to use the table in finding interest on any principal, (*a*) if a decimal of a given principal; (*b*) if an irregular amount. Find the interest on \$40,000 for 1 da.; on \$43 for 1 da. (Suggestion. \$43 = \$40 + \$3.)

4. Using the table just constructed, determine the interest in ex. 1–11 of the preceding exercise.

Exact Interest.

Exact interest, based on 365 days, may be computed by the cancellation method, or it may be obtained from interest computed on a 360-day basis. Thus 1 day's interest, on a business basis, is 1/360 of a year's interest; on an exact basis, it is 1/365. The difference is 1/360 − 1/365, or 1/73. That is, exact interest is 1/73 less than common interest.

Illustration. *Example.* Compute exact interest on \$450 for 84 da., at 6%.

Solution.
\$4.50 = interest for 60 da.
1.50 = " " 20 "
.30 = " " 4 "
\$6.30 = " " 84 "
\$6.30 ÷ 73 = \$.09.
\$6.30 − \$.09 = \$6.21, the exact interest.

EXERCISE.

Find the exact interest on:

1. \$582.90 for 39 da., at 6%.
2. \$1254 for 87 da., at 5%.
3. \$632.96 for 215 da. at 6%.
4. \$98.25 for 49 da., at 4%.
5. \$2960 from Aug. 17 to Dec. 29, at 6%.
6. \$382.50 from Jan. 5 to June 26 at 6%.

Interest Problems.

Interest is a truer measure of profit and loss than simple percentage, and is practically the common instrument for

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the computations incident to financial operations. The problems that follow are selected at random to show common uses of interest.

EXERCISE.

1. I lend James Brown $2400 on Jan. 6, 1909, which he repays with interest on Aug. 15, 1910. What does he pay in settlement? What does he pay for the use of the money?

2. I borrow $1365, on my written promise to pay in 120 days with 5% interest. At the end of the period, what is due in settlement?

3. What is the quarterly income from a $20,000 loan placed at 4½% interest?

4. $824, if placed in a savings bank paying 3½% interest, will amount to what sum in 6 months?

5. If money is worth 8% to a buyer, is it better for him to pay $880 cash, or to wait 3 months and pay $900?

6. Determine the per cent. of gain on a four months' investment of $1280, which yields 5% interest.

7. If goods are bought for $420, is it better to sell them in three months at 5% profit, or at 5% interest on cost?

8. I am thinking of buying a piece of property for $8750 which, it is claimed, will rent for enough to yield 9% net interest on investment. What sum must be cleared? If repairs, taxes, etc., are estimated at $240 per year, what monthly rental should be charged?

9. A bill of crockery, $752.80, was bought Jan. 27, on 2 months credit. It was not paid until June 17. What was then due? Interest, 6%.

10. I lent through a broker, for a term of 6 months: $500 at 5% interest; $800 at 4¼%; $1200 at 5½%. The broker charged 1/4% interest as his commission. Determine my net income.

11. If this account draws 6% interest, what is due on Aug. 10 of the year following?

S. R. Ramey

191—	Jan. 17	300.—	
	Mar 15	286.—	
	July 12	125.—	
	Aug. 10	85.60	

12. The Montrose County School Fund is invested in:
$10,000 City of Minneapolis 3½% bonds.
$40,000 Morristown Water Works bonds, bearing 4½%.
$120,000 Lewiston Gas Co. bonds, bearing 5%.
What is the annual income from the fund?

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13. I owe a London exporter £266 5 s. 10 d., with interest at 6% from Mar. 12 to Jan. 8. What is due?

CREDIT PRICES.

ORAL EXERCISE.

1. Is it better for a merchant to sell an article for $400 cash, or for $400 credit?

2. If he gives credit what does he lose?

3. Should a credit price be greater, or less, than a cash price?

4. If the worth of money to a merchant be known, how may a credit price, equivalent to a cash price, be determined?

NOTE. While many merchants add arbitrary amounts to the cash price when selling on credit, others use a carefully computed rate of interest in determining credit prices.

Determine credit prices for the following:

Cash Price.	Credit Term.	Money Worth.
5. $ 8.40	2 mo.	6%.
6. 80.00	1 mo.	7½%.
7. 6.50	30 da.	12%.
8. 5.00	3 mo.	8%.
9. 240.00	75 da.	9%.

EXERCISE.

(Use 6% as a rate unless otherwise stated.)

1. Does it pay to buy flour on 5 months credit, at $6.50, if it can be bought for $6.00 cash, and money is worth 8%?

2. What profit results from buying a $16.50 article on 5 months credit, and immediately selling it at the same price on 1 month credit?

3. What is a fair settlement for a bill of merchandise amounting to $1240 paid 3 months before its 4 months credit term is up?

NOTE. Subtract interest for period before due. This is the interest-discount method.

TO DETERMINE RATE OF INTEREST.

INTRODUCTORY EXERCISE.

1. If the interest on $800 for 1 year is $36, what is the rate of interest?

2. If the interest on $600 for 3 months is $15, what is the interest for a year. The year's interest is what per cent. of the principal?

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3. What is the interest on \$1200 for 9 days at 6%? At 1%? \$7.20 is the interest *at what rate* for the same time?

It is evident that if the interest for a year, on any principal, is known, or can easily be obtained, the rate of interest is determined by dividing the known interest by 1% of the principal. If the time is irregular, it is usually easier to measure the given interest by the interest on the given principal, at 1% for the given time.

ILLUSTRATION. At what rate does \$720 earn \$4 in 40 days?

Solution (1).

\$ 4.00 = interest for 40 days.
36.00 = " " 360 " (1 yr.)
\$36.00 ÷ \$720 = .05.
The rate is 5%.

Solution (2). The principal is \$720.

\$7.20 = interest at 6% for 60 da.
4.80 = " " 6% " 40 "
.80 = " " 1% " 40 "
\$4 (given interest) ÷ \$.80 (interest at 1%) = 5.
The rate is 5%.

ORAL EXERCISE.

Principal and time being constant.

1. If the interest at 6% is \$48, \$60 is the interest at ? %.
2. \$3.60 is the interest at ? %, if \$9 is interest at 2½%.
3. 84c is the interest at ? % if 16 c is the interest at 2%.

Find the rate of interest if

4. \$1200 earns \$72 in one year.
5. \$800 earns \$20 in 6 months.
6. \$400 earns \$3 in 2 months.
7. \$600 earns 36c in 12 days.

EXERCISE.

1. If the Russian Government sells its last 6% bond issue at 10% below face, what rate of interest does it really pay on the funds that it secures?
2. Discuss the investment value of a 6% investment running one year with a 5% one running ten years.
3. The offer of \$80 for the use of \$1500 for 45 days is equivalent to an offer of ? % interest.

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4. $800 is loaned from January 15 to September 12 at 6%, and from October 1 to January 1, at 5%. What annual rate is earned?

PROFIT AND LOSS MEASURED BY INTEREST.

INTRODUCTORY EXERCISE.

1. Is it better to allow 3 months or 6 months credit on merchandise sold at a fixed price?

2. Is it better to earn 5% profit, or 5% interest, on an article that is held 2 years before sale? On one held 6 months?

3. When does a rate of interest yield a higher profit than the same rate of gain?

4. Which more clearly shows the real return from an investment—the rate of interest, or the rate of gain? Why?

The quickness of sale, after purchase, greatly affects the profits of a business. This is the period of "turn-over of capital." The above exercise emphasizes, as in percentage, the value of a rapid turn-over, and shows the mathematical basis for the old business motto "Quick sales and small profits."

EXERCISE.

1. A merchant invests in articles costing $2.40, which sell, on an average, in 2 mo. 6 da., at 3% profit. What is his annual rate of income on investment?

2. Property bought January 29, for $5000, and sold June 26 for $5400, yields what rates of gain and interest?

3. A house, bought for $9600, is rented for $50 per month, except for 2 months of the year. Taxes are 1½% on an assessed value of $8000, and repairs cost $118.60. What is the per cent. of income on investment?

4. Is it better to make 5% interest or 3% profit on a 7 months investment. Illustrate.

5. A real estate dealer advertises as a 10% investment for $5000, a house that rents for $50 per month. Taxes are 1½% on an assessed value of $4200; insurance is 1¼% on $3000. Is allowance made for repairs?

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TO FIND THE PRINCIPAL.

INTRODUCTORY EXERCISE.

1. What is the interest on \$1 at 6% for 2 months? \$4 is the interest on what principal for the same rate and time?

2. What is the amount of \$1 for 6 months at 10%. \$525 is the amount of how many dollars, same time and rate?

3. What is the measure and quantity measured in examples 1 and 2?

4. Explain: $\text{Principal} = \frac{\text{Total interest}}{\text{Interest on \$1}}$ (for same time and rate).

ILLUSTRATION. (1) What principal earns \$18.30 in 12 days at 6%?

Solution.

Int. on \$1 for 12 da. is \$.002.
\$18.30 ÷ \$.002 = \$9150.
The principal is \$9150.

If \$1 under given conditions, earns \$.002, \$18.30 represents the earning of as many dollars as \$.002 is contained times in that amount.

ILLUSTRATION (2). \$976.32 is the amount of what principal for 3 mo. 12 da. at 6%?

Solution.

The amount of \$1 for 3 mo. 12 da. at 6% is \$1.017.
\$976.32 ÷ \$1.017 = \$960. The principal is \$960.

ORAL EXERCISE.

Find the principal.

	Int.	Time.	Rate.		Int.	Time.	Rate.
1.	\$30.	18 da.	6%.	4.	\$3.20	24 da.	6%.
2.	\$48.	6 mo.	8%.	5.	\$9.	1 yr. 6 mo.	3%.
3.	\$ 9.60	36 da.	6%.	6.	\$2.40	3 mo.	8%.

7. What principal amounts to \$550 in 2 years at 5%?

8. What principal amounts to \$2080 in 6 mo. at 8%?

EXERCISE.

1. On January 1, 1911, a corporation must meet a debt of \$25,000. What sum set aside on October 15, 1909, at 4% will amount to the debt when due?

2. What sum must be loaned at 5¼% to earn a quarterly income of \$180?

3. Capitalize at 6%, semi-annual earnings of \$84,200.

NOTE To capitalize earnings or expense is to determine the capital, or principal that, at the given rate, will produce the stated earnings.

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4. Capitalize at 5% annual charges for rented buildings for which a city government pays $18,600.

5. By borrowing at 4%, a city government might have how much money with which to secure its own buildings, for the $16,500 it now pays annually in rentals?

The *present worth*, or true value, of a future payment or debt, is the principal that, at the "worth of money," will amount to the payment when due. Present worth is sometimes used in place of interest-discount, in reckoning advance payments, in comparing bids and investments.

ILLUSTRATION. Compare the present value of a $960 cash offer and a $964.25 credit price for 6 months, money being worth 6%.

Solution.

The amount of $1 for 3 months at 6% is $1.015.

$964.25 ÷ $1.015 = 950.

The present value of the credit price is evidently $950. The credit price is better for the buyer than the cash price.

Question. Why is the *present worth* method more accurate than the *interest-discount* method?

EXERCISE.

1. Money being worth 8% to me, what do I gain by buying property for $960, on 6 months credit, and immediately disposing of it for cash, at the same price?

2. By present worth, at 6%, compare these bids for the construction of a building:

C. Smith & Son, $12,000: 1/4, cash, 1/4, 6 mo.; balance, 1 yr.
James Andrews; $11,750: 1/3, 4 mo.; balance, 1 yr.
A. B. Norton, $11,500: 1/2, 4 mo.; balance 1 yr.

3. If money is worth 6%, what sum fairly settles, on June 15, a payment of $1200 not due until August 20?

4. What sum settles in cash for a purchase of $4284, on terms of 1/4 cash, balance 90 days. Money is worth 5%.

TO FIND THE TIME.

INTRODUCTORY EXERCISE.

1. If 6c is the interest for 1 day, on a certain principal, 54c is the interest for how many days?

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2. Find the interest of \$720 for 1 day. 84c is the interest on the same principal for how many days?

3. Explain. Time, in days, $= \frac{\text{Total interest}}{\text{Int. for 1 da.}}$ (same prin. and rate).

4. Show how to use one month, or one year, as a measure, in place of one day.

ILLUSTRATION. In what time does \$840 earn \$28 at 6%?

Solution.

The interest on \$840 for 1 da. = 14c.

\$28 ÷ \$.14 = 200.

The interest term is 200 da., or 6 mo. 20 da.

ORAL EXERCISE.

Find the time, using 1 day.				Find the time using 1 month.		
	Prin.	Int.	Rate.	Prin.	Int.	Rate.
1.	\$ 600	\$1.50	6%.	5. \$ 420	\$ 6.30	6%.
2.	\$1200	\$6.00	9%.	6. \$ 800	\$90.	6%.
3.	\$ .720	\$1.32	6%.	7. \$1200	\$80.	5%.
4.	\$3000	\$7.50	6%.	8. \$2000	\$75.	9%.

9. In what time does a principal double itself (simple interest) at 4%, 5%, 6%, 8%, 7%, 10%, 9%, 3%?

EXERCISE.

1. On January 25, \$7200 is invested at 6% simple interest to remain until it amounts to \$7500. Find the interest period.

2. On what date will \$3200, invested July 8, 1909, at 6%, amount to \$5000?

PERIODIC INTEREST.

Periodic interest is the interest on a principal, payable at stated intervals, plus simple interest on overdue interest payments. It can be legally enforced in but few states, but it is collected indirectly by means of non-interest notes, given for the regular payments, which draw simple interest when overdue.

ILLUSTRATION. What is due after one year and six months on a \$1200 loan, at 6% interest, payable quarterly, nothing having been paid?

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Solution.

Interest on \$1200 for 3 mo. = \$18, the quarterly payment.

Interest on \$1200 for 1 yr. 6 mo. = \$108, the total simple payments.

At the end of 1 yr. 6 mo.,

The first	payment	is overdue	1 yr.	3 mo.
" second	"	"	1 "	0 "
" third	"	"		9 "
" fourth	"	"		6 "
" fifth	"	"		3 "
The sum of overdue payments				3 yr. 9 mo.

The total overdue interest on the several equal payments equals the interest on one payment for the total time, or for 3 yr. 9 mo.

Interest on \$18 for 3 yr. 9 mo. =	\$ 4.05
Regular interest payments,	108.—
Principal,	1200.
Amount due,	\$1312.05

EXERCISE.

1. On June 1, 1908, \$3000 was lent at 5%, payable semi-annually. Nothing having been paid, what sum is due in full settlement, on December 1, 1909?

2. Find the amount due in full settlement for a loan of \$7000, at 6%, payable quarterly, on which nothing has been paid for a term of four years?

3. Repeat example 2, on a basis of semi-annual payments.

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CHAPTER XXXVI.

COMPOUND INTEREST.

Compound interest is interest computed on the sum of a principal and its due and unpaid interest. Interest must be "compounded," or added to the principal, at stated intervals, as quarterly, semi-annually, or annually. Other periods may be fixed by agreement. While not recoverable by law, in most states, the accepting of compound interest does not constitute usury. If a new obligation is given at the end of any of the fixed periods for the amount then due, the obligation is binding.

Compound interest is used by savings banks, in reckoning interest on deposits, by insurance companies and by business men generally, in computing the cumulative value of investments at a fixed rate, where earnings are not withdrawn.

ILLUSTRATION. $12,000 invested at 6%, and compounded semi-annually for two years, will amount to what sum?

Solution.

$12,000	= original principal.
360	= interest for 6 mo. (3%).
12,360	= amount, end of 6 mo.
370.80	= interest for 6 mo. on last amount (3%).
12,730.80	= amount, end 1 yr.
381.92	= interest for 6 mo. on last amount (3%).
13,112.72	= amount, end 1 yr. 6 mo.
393.38	= interest for 6 mo., on last amount (3%).
$13,506.10	= final amount.

ORAL EXERCISE.

1. The interest for 4 years at 8%, compounded semi-annually, equals the interest for ? semi-annual periods, at ? %.

2. In compounding quarterly for 8 years at 6%, there are ? interest periods at ? % per period.

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Compound Interest Table.

Showing the amount of $1.00 for any number of periods up to 25, at the given rates.

No. Period.	Rate Per Period. 1 %.	1½ %.	2 %.	2½ %.	3 %.	3½ %.	4 %.	4½ %.	5 %.	6 %.
1	0.010000	1.005000	1.020000	1.025000	1.030000	1.035000	1.040000	1.045000	1.050000	1.060000
2	1.020100	1.030225	1.040400	1.050625	1.060900	1.071225	1.081600	1.092025	1.102500	1.123600
3	1.030301	1.045678	1.061208	1.076891	1.092727	1.108718	1.124864	1.141166	1.157625	1.191016
4	1.040604	1.061364	1.082432	1.103813	1.125509	1.147523	1.169859	1.192519	1.215506	1.262477
5	1.051010	1.077284	1.104081	1.131408	1.159274	1.187686	1.216653	1.246182	1.276282	1.338226
6	1.061520	1.093443	1.126162	1.159693	1.194052	1.229255	1.265319	1.302260	1.340096	1.418519
7	1.072135	1.109845	1.148686	1.188686	1.229874	1.272279	1.315932	1.360862	1.407100	1.503630
8	1.082857	1.126493	1.171659	1.218403	1.266770	1.316809	0.368569	1.422101	1.477455	1.593848
9	1.093685	1.143390	1.195093	1.248863	1.304773	1.362897	1.423312	1.486095	1.551328	1.689479
10	1.104622	1.160541	1.218994	1.280085	1 343916	1.410599	1.480244	1.552969	1.628895	1.790848
11	1.115668	1.177949	1.243374	1.312087	1.384234	1.459970	1.539454	1.622853	1.710339	1.898299
12	1.126825	1.195618	1.268242	1 344889	1.425761	1.511069	1.601032	1.695881	1.795856	2.012197
13	1.138093	1.213552	1.293607	1.378511	1.468534	1.563956	1 665074	1.772196	1.885649	2.132928
14	1.149474	1.231756	1.319479	1.412974	1.512590	1.618695	1.731676	1.851945	1.979932	2.260904
15	1.160969	1.250232	1.345868	1.448298	1.557967	1.675349	1.800944	1.935282	2.078928	2.396558
16	1.172579	1.268986	1.372786	1.484506	1.604706	1.733986	1.872981	2.022370	2.182875	2.540352
17	1.184304	1.288020	1.400241	1.521618	1.652848	1.794676	1.947901	2.113377	2.292018	2.692773
18	1.196148	1.307341	1.428246	1.559659	1.702433	1.857489	2.025817	2.208479	2.406619	2.854339
19	1.208109	1.326951	1.456811	1.598650	1.753506	1.922501	2.106849	2.307860	2.526950	3.025600
20	1.220190	1.346855	1.485947	1.638616	1.806111	1.989789	2.191123	2.411714	2.653298	3.207136
21	1.232392	1.367058	1.515666	1.679582	1.860295	2.059431	2.278768	2.520241	2.785963	3.399564
22	1.244716	1.387564	1.545980	1.721571	1.916103	2.131512	2.369919	2.633652	2.925261	3.603537
23	1.257163	1.408377	1.576899	1.764611	1.973587	2.206114	2.464716	2.752166	3.071524	3.819750
24	1.269735	1.429503	1.608437	1.808726	1.032794	2.283328	2.563304	2.876014	3.225100	4.048935
25	1.282432	1.450945	1.640606	1.853944	1.093778	2.363245	2.665836	3.005434	3.386355	4.291871

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3. Compute the amount of a $4000 savings deposit, compounded quarterly for one year, at 4% per annum.

EXERCISE.

1. A fund of $16,000 at 6%, compounded semi-annually, amounts in 5 years to $—.

2. $5000 invested Jan. 1, 1906, at 4%, compounded quarterly, will amount to $—— on March 21, 1907.

NOTE. Compute interest for fractional term on last full amount.

Compound interest is computed, as a rule, from tables similar to the one on page 303.

ILLUSTRATION. Find the amount of $2500 for 6 years at 6%, compounded quarterly.

Solution.

In 6 years there are 4×6, or 24, interest periods.

The rate of 6% per year equals $1\frac{1}{2}$% per quarterly period.

The problem is to find the amount of $2500 for 24 periods at $1\frac{1}{2}$%.

$1.429503 = amount of $1.00 for 24 periods at $1\frac{1}{2}$%.

2500

$3573.76 = amount of $2500 for 24 periods or 6 years.

It is evident that time and rate should be reduced to periods and period rates, and that the amount of $1.00 for this number of periods should be multiplied by the number of dollars in the principal. If the number of periods is greater than twenty-five, find the amount for twenty-five periods, and then find the amount of this amount for the remaining periods.

ORAL EXERCISE.

Read from the table the amount of $1000 if compounded:

1. Quarterly at 6% per annum for 6 years.
2. Semi-annually, at 8% for 5 years.
3. Bi-monthly, at 6% for 3 years.
4. Every 4 months, at 9% for 6 years.

EXERCISE.

Compute the amount of

1. $5950, for 3 yr. 6 mo. at 5%, compounded semi-annually.

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2. $12,600 for 6 yr. 6 mo., at 8%, compounded quarterly.

3. $7200, from July 1, 1904, to October 27, 1909, at 6%, compounded semi-annually.

Compute the compound interest on

4. $1200 for 8 yr. 4 mo. at 6%, compounded quarterly.

5. $2500 for 2 yr. 6 mo., at 9%, compounded every 4 months.

6. $3250 from October 1, 1909, to November 10, 1913, at 5% compounded semi-annually.

What sum must be invested at

7. 6%, compounded quarterly, to amount to $7500 in 6 years?

8. 8%, compounded semi-annually, to amount to $12,500 in 4 yr. 6 mo.?

9. 9%, compounded semi-annually, to amount to $8000 in 3 yr. 6 mo.?

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CHAPTER XXXVII.

LOANS AND PAYMENTS.

INTRODUCTORY EXERCISE.

1. What may lead one, in his private capacity, to borrow money?

2. Name objections to such borrowing.

3. What may cause a business man, or organization, to borrow?

4. Does a prosperous business ever need a loan?

5. When might a business borrow in order to pay cash for goods bought on credit?

6. How may a loan spread the cost of an improvement, for which it pays, over a long period?

7. What is the danger of lending money on a verbal promise to repay?

8. What business paper commonly is used as evidence of a loan, or of a payment due in the future?

9. How else may loans be "secured"?

ORAL EXERCISE.

1. I borrow, to-day, \$1200 for 3 mo. at 6%. I receive \$?, but must pay back \$?, paying \$? for the use of the money.

Determine interest and amount due on these loans:

	Loan.	Period.	Rate.		Loan.	Period.	Rate.
2.	\$ 800	9 mo.	6%.	6.	\$2500	24 da.	6%.
3.	1200	36 da.	5%.	7.	540	30 da.	6%.
4.	400	5 mo.	9%.	8.	250	2 yr.	4%.
5.	1500	2 mo.	4%.				

EXERCISE.

1. What interest has accrued to date, at 4½%, on a loan contracted Feb. 11 of last year, for \$820?

2. On Mar. 18, I must meet a loan of \$840, contracted Oct. 21 last, and drawing 6% interest. If I have \$520, what additional sum must I raise?

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3. Settlement of a 3% loan of $90,000, dated Jan. 20, is demanded Jan. 25. What is due?

4. A 3% loan of $6000, payable on demand, is contracted on February 16, and renewed on February 25 at 4%. Settlement is made, March 2, by paying $—.

5. My brokers lend for me, on a commission of 1/2% interest:
Jan. 19, to C. Warren, $900 at 6%.
Feb. 3, to M. C. Poston, $1250 at 5%.
Feb. 16, to R. Fallon, $1920 at 6%.
Determine my net interest on the following January 1.

PROMISSORY NOTES.

A promissory note is a written promise to pay a specified sum of money, on demand, at a specified time, with or without interest. It is a common evidence of debt.

ILLUSTRATION.

$560xx *Cincinnati, O., Mar. 21, 1913.*

Sixty days after date *I* promise to pay to the order of *James Fielding* at *756 Fifth Ave.* *Five hundred sixty xx*/100 Dollars.

Value received with Interest at *6* % per annum.

Robert C. Evans

No. *97*.... Due. *May 20*....

NOTE. Robert C. Evans, who signs this note, is the *maker;* James Fielding is the *payee.* If the note had read "On demand, I promise," etc., it would have been payable whenever presented to Evans.

Notes are *negotiable,* if transferable by the payee to other parties by endorsement of his signature on the back. To be transferable, notes must read "Pay to ———, or order," "Pay to the order of ———," or "Pay to ———, or bearer." If the words "or order," or "or bearer" are omitted the note is not transferable.

The following are the common forms of endorsement:

(1) *Endorsement in Blank.* This transfers right of ownership to the bearer.

(1) James Fielding.

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(2) *Full Endorsement.* Here James Fielding transfers the note to Robert Anderson, but since the expression "pay to the order of" is used, Anderson may transfer the note to others.

(2) Pay to the order of Robert Anderson,
James Fielding.

(3) *Qualified Endorsement.* The note is transferred by Fielding to Rankin, but the former by use of the phrase "without recourse" refuses to agree to pay the note when due, in case the maker fails to pay.

(3) Pay to the order of C. P. Rankin, without recourse to me
James Fielding.

The last legal holder of the note secures payment from the maker at maturity. If the maker fails to pay, each endorser in turn, unless his endorsement is qualified, may be held for payment. In case of non-payment, the note may be given to a *notary public*, who formally demands payment. If payment is refused, he legally *protests* the note, issuing notice to each indorser and thus binding them to payment. This protest is a basis for legal action to compel payment. The amounts collectible are the face, interest if any, and legal charges. Non-interest-bearing notes draw interest after due.

Notes signed by two or more parties and reading "we jointly promise," are termed joint notes, each party being held for his proportional part of the note (see p. 314). If the note reads "We jointly and severally promise," each signer is held for full settlement. Some states hold joint notes as similar to joint and several notes.

Collateral notes contain a permit to enable the holder, in case of failure to pay at maturity, to dispose of certain property (collateral) of the maker, and placed in possession of the payee,

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as security, at the time of giving the note. One form of collateral note is shown on page 346.

When a loan is made on real estate, notes usually are given, secured by a mortgage or deed of trust, under which the property may be disposed of in order to meet payments. Receipts for goods in warehouse or grain elevator likewise, may be endorsed to the lender as security for loans.

The *bonds* of corporations are simply corporate notes, and frequently are secured by mortgages.

A *commercial draft* is an order written by one person requesting a second person to pay a stated sum of money to the order of the former, or to a third person. The parties to a draft are the *drawer*, or maker, the *drawee*, or person drawn on, and the payee. When *accepted* by the drawee, commercial drafts usually are negotiable. Accepted drafts are termed *acceptances*.

ILLUSTRATION (1). Andrews sells Parker $646 worth of merchandise, on terms of "60 day acceptance." He sends Parker an invoice, accompanied by the draft shown on page 310. Parker, finding the goods satisfactory, writes "Accepted" across the face of the draft and signs his name, thus binding himself to pay the paper when due. He then returns the draft to Andrews. By writing "Accepted, Payable at the First Nat. Bank, Chas. Parker," Parker could have turned the settlement over to his bank. Practically, the draft is now a note.

ILLUSTRATION (2). Andrews might wish to transfer payment to Henry Connors, a creditor of his. In that case he might make out the above draft and transfer it by endorsement, or he might make out the draft in the form shown on page 311. In the latter case he would probably send the draft to Connors and the latter would present it to Parker for acceptance.

When drafts read "after sight," in place of "after date," the acceptance must be dated, and maturity counts from that date. Drafts reading simply "at sight," are payable at once without acceptance. They are often drawn by merchants against customers, and are sent through a bank for collection. If the terms of a sale read, "subject to draft after 30 days" it is understood that a *sight draft* will be drawn at the *end* of the period.

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$646 $\frac{xx}{100}$ Philadelphia, Pa., Mar. 16, 1912

Sixty days after date Pay to the

Order of myself

Six hundred forty six $\frac{xx}{100}$ Dollars

Accepted Chas. Parker

Value received and charge the same to account of

To Chas. Parker
No. 87 Balt. Md.

H. M. Andrews

Google

Philadelphia, Pa., Mar. 16 1912 No. 87

At sixty days' sight Pay to the order of

Henry Connors $ 646 XX/100

Six hundred forty-six XX/100 Dollars

IN COLLEGE CURRENCY

Value received and charge the same to account of

To Chas. Parker
Baltimore, Md.

H. M. Andrews

Accepted Mar. 20 1912 Payable at Fidelity Bank Baltimore Md.

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Factors Affecting the Maturity of Negotiable Paper.

The wording of negotiable paper affects the date of maturity. On papers reading "— months after date" and "— days after date" maturity is reckoned from the date of the *note*, the former by approximate and the latter by exact time. A few states still allow "three days of grace," after date of maturity, before payment must be made.

If drafts read "after sight," or "at — days sight," the maturity is reckoned from the date of acceptance. Thus the draft on page 311 is payable 60 days from March 20.

After the calendar date of maturity has been reckoned, the actual payment date must be determined with due regard to the varying state laws concerning Sundays, holidays, etc. Thus, in Pennsylvania, a note coming due Saturday is not payable until the following Monday, while if Monday should happen to be a holiday, it would become payable on Tuesday.

Summary of Laws Relating to the Maturity of Negotiable Instruments.

Note. These laws are in process of change, in many states. Ask a banker for the latest regulation.

For states allowing days of grace, or for local laws, consult a local banker or business man.

Requirement.	State or Country.
Due Sunday or holiday, pay next business day.	Ala., Ariz., Cal., Col., D.C., Fla., Hawaii, Ill., Ind., Ia., Kas., Ky., La., Me., Minn., Mo., Mont., N.H., N.M., Okla., S.C., S.Dak., Tex., W.Va., Wis., Canada.
Due Sunday or holiday, pay preceding business day.	Alaska, Ark., Del. (Kent and Sussex Co.), Ga., Miss., Nev., Philippines, P.R., Cuba, Mexico.
Due Saturday, Sunday or holiday, pay next business day.	Conn., Idaho, Md., Mass., Neb., N.J., N.Y., N.C., N.Dak., O., Oreg., Pa., R.I., Mich., Tenn., Ut., Vt., Va., Wash., Wyom.

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HOLIDAYS.

Day.	Country or State.
January 1.	All states (except Mass. and Kansas), Canada, Cuba, and U. S. possessions.
February 22.	All states and U. S. possessions.
May 30.	All states and possessions, except Ala., Ark., Fla., Ga., La., Miss., N. C., Okla., S. C. and Tex.
July 4.	All states and U. S. possessions.
Labor Day (first Monday in September).	All states (except Md., Wis., Wyom.) and U. S. possessions, except Philippines.
Thanksgiving Day (by proclamation, usually last Thursday in November).	All states and possessions.
December 25 (Christmas).	All states and U. S. possessions, Canada and Cuba.

Many states or counties have local holidays other than those named above.

EXERCISE.

Find the dates of maturity of this commercial paper, consulting the above summary. Use a calendar and allow for holidays, etc.

MATURITY IN YOUR OWN STATE

1. 90 day note, dated January 27.
2. 60 day note, dated Dec. 24.
3. "Three months after date" paper, dated Mar. 4.
4. "Sixty days after date" draft, dated June 14, accepted June 17.
5. "At 90 days' sight" draft, dated Aug. 31, accepted Sept. 3.

MATURITY IN DIFFERENT STATES.

6. "60 days' sight" draft, dated May 4, accepted May 5, payable in Minnesota.
7. A "90 days after date" draft dated September 3, accepted September 4, payable in Ohio.
8. A 4 months note date Mar. 4, 1909, payable in Texas.
9. A 120 day note, dated April 24, 1909, payable in Canada.

QUESTIONS FOR DISCUSSION.

1. As a rule, which allow a longer time for payment, notes given for months, or for the same number of thirty day periods?

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2. Is thirty day paper ever better for the payer than one month paper?

3. Give reasons for allowing days of grace. Why are they being abolished?

4. Ordinarily, do the regulations as to Sundays and holidays favor payer or payee?

5. Why should the date of payment be fixed beyond question

EXERCISE.

1. What yearly saving in interest results from refunding at 3½%, $52,500 worth of 5% bonds?

NOTE. Refunding means re-issuing instead of paying at maturity.

2. What sum should be paid on June 20, in full payment for a 90 day non-interest note for $1200, dated February 11? Interest, 6%.

3. What sum will settle this note to-day:

520\frac{XX}{100}$ WASHINGTON, D. C., August 12, 1909.

On demand, after date, we jointly promise to pay to the order of C. M. Fairley.....

Five hundred, twenty $\frac{XX}{100}$——————Dollars.

Value received, with interest at 6% per annum.

A. B. MORTON,
HENRY C. MASON.

4. I lend C. P. Rogers $2260, at 5%, taking a first mortgage on his farm. What quarterly payment should he make?

At the end of two years, nothing having been paid, the mortgage is foreclosed, and the property sold for $3675, the charges for the sale being $61.90. What is due Rogers?

INCOME AND OUTGO.

5. An investment of $19,700 in 5% mortgages yields a semi-annual income of $—.

6. A broker lends for a customer $18,000 at 5%, charging a commission of 5% of the first year's income. What is the client's net income the first year?

7. On February 19, I borrowed $12,000, at 3½%, giving my demand note. On March 3, I lent the money at 5%. On December 15, my note being presented for payment, I called in my loan. Net gain?

PARTIAL PAYMENTS.

Partial payments may be made, by agreement, before or after the maturity of a note or bond. Such payments should

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be endorsed on the back of the paper in one of the forms shown.

Face.

$1800.$\frac{XX}{100}$ Washington, D. C., June 10, 1909.

One year after date I promise to pay to the order of C. P. Crandall.....
Eighteen hundred _____Dollars.
With interest at 6% per annum.
Robert Davis.

Back.

Paid, July 12, 1909, $200.
Paid, Oct. 12, 1909, $300.
Paid, Mar. 10, 1910, $400.

Other forms of Indorsement.

(1)

July 12, 1909, paid $200.

(2)

Rec'd, 7/12/09, $200.

(3)

Rec'd on the within note, July 12, 1909, three hundred dollars.

QUESTION. Why is no signature to an endorsement of a partial payment necessary?

The partial payment introduces several factors into the computation of the amount due, the question arising as to whether payments shall be applied to principal or to interest, or to both. State laws govern the matter, but many states have now legalized the United States rule, established by the Supreme Court of the United States. The so-called Merchants' rule is also frequently used. The United States rule is very commonly used for long term paper.

INTRODUCTORY EXERCISE.

1. What is due at the end of 6 months on a 6% note for $1000?

2. If $230 is then paid, what sum will pay the interest due? What is left to reduce the face? How much of the debt is still outstanding? On what sum should interest now be paid?

3. What interest is due 6 months later? What amount?

4. If, now, $324 is paid, the new principal becomes $——.

5. The debt is finally settled a year later, by paying $——.

ILLUSTRATION. UNITED STATES RULE.

EXAMPLE. Find the balance due at maturity, on the note shown above.

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Solution.

Face of note	$1800.—
Interest, June 10 to July 12, 2 mo. 2 da	18.60
Amount due, July 12	1818.60
Payment, July 12	200.—
New Principal	1618.60
Interest on new principal, July 12 to Oct. 12, 3 mo.	24.28
Amount due, Oct. 12	1642.88
Payment, October 12	300.—
New principal, balance due	1342.88
Interest on new principal, Oct. 12 to Mar 10, 1910, 4 mo. 28 da.	33.12
Amount due, Mar. 10	1376.—
Payment, Mar. 10	400.—
New principal, balance due	976.—
Interest from Mar. 10 to maturity, June 10, 3 mo.	14.64
Amount due at maturity	$ 990.64

NOTE. Under this rule, a payment is first applied to pay off interest, and then to reduce principal. In case any payment is less than the interest accrued, the principal remains unaltered until such a date that the sum of payments exceeds the interest accrued. For example, if the first payment had been $10, it would have been added to the following $300 payment, and interest found on the original face from June 10 to October 12.

QUESTION. What would be the result of using a payment less than the accrued interest?

EXERCISE.

Use approximate time for ex. 1–3; exact time for ex. 4–5.

1. Date of loan, June 26, 1908; amount, $3000; interest 6%. Payments: Aug. 15, 1908, $450; June 21, 1909, $800; Aug. 21, $300; July 16, 1909, $425. Find the balance due Oct. 21, 1909.

2. A two year note for $3760, dated Oct. 15, 1911, and bearing 6% interest, has these endorsed payments. What is due at maturity?

Paid, 11/8/11, $400
" 12/9/11, $200
" 7/3/12, $600
" 11/6/12, $ 20
" 1/7/13, $520

3. On Oct. 27, Robert Evans paid $1200 on a demand note for $3000, dated Jan. 17, and bearing 6% interest. On Nov. 5, he paid $200, and gave a new 3 mo. note for the balance due. Determine the face of the new note.

4. Determine the amount due on this note at maturity.

$3500.— BALTIMORE, MD., July 15, 1911.

Fifteen months after date, I promise to pay to the order of Elton Howells, thirty-five hundred_____Dollars.
Value received, with interest at 5%. JAMES ELWELL.

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Payments endorsed as follows:

Paid, Nov. 5, '11, $ 800.
" Mar. 17, '12, $ 250.
" Nov. 12, '12, $1500.

5. From this Loan Book record, find the balance due, Oct. 1, 1912, at 3%:

Loan to C. P. Davidson,

No.	Date.	Amount.	Date Payment.	Amount Payment.
136	Oct. 1, '10	$40,000	Jan. 15, '11	$5,000.
			Mar. 20, '11	10,000.
			Aug. 21, '11	10,500.
			Jan. 1, '12	3,000.

INTRODUCTORY EXERCISE.

1. What is due at maturity on a 6 months note for $1200, dated June 1, and bearing 6% interest?

2. On Oct. 1, a payment of $600 was made. If this payment was allowed to draw interest, to what would it amount when the note was due?

3. What is the difference, at maturity, between the value of the note and the value of the payment?

The Merchant's rule is commonly used in banks and commercial houses, especially for short term notes.

ILLUSTRATION. MERCHANT'S RULE.

EXAMPLE. Determine the balance due on the note, page 315.

Solution.

Face of note		$1800.—
Interest to maturity—1 year		108.—
Amount due at maturity		$1908.—
First payment	$200.—	
Interest on payment to maturity, July 12, 1909, to June 10, 1910, 10 mo. 28 da.	10.93	
Second payment	300.—	
Interest, October 12 to June 10, 1910, 7 mo. 28 da.	11.90	
Third payment	400.—	
Interest, Mar. 10 to June 10, 3 mo.	6.—	
Total value of payments at maturity	$928.83	928.83
Difference between value of note and of payments		$ 979.17

NOTE. Under this method, the face draws interest until the date of settlement, and each payment is allowed interest until the date of settlement or maturity. The balance due is the difference at time of settlement, between the value of the face and the value of the payments.

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EXERCISE.

1. Compute by both rules, the balance due March 29, 1912, on this note.

$2400.— Cincinnati, Ohio, Sept. 7, 1911.

On demand, after date, I promise to pay to the order of S. P. Campbell..... Twenty-four hundred.....Dollars with interest at 6% per annum until paid.

A. B. NORTON.

Rec'd, Nov. 5/11, $300.
Rec'd, Jan. 16/12, $580.
Rec'd, Mar. 3/12, $627.50

Solve the following, using exact time.

Loan No.	Date	Name	Amount	Rate	Date Pay.	Am't	Settled
2	1911 Oct. 8	James Baker	$36,000	3%	Nov. 5/11 Dec. 21/11 Mar. 20/12	$4000 6000 8000	June 21/12
3	Oct. 11	Robert Jones	$4,500	6%	Jan. 15/12 Mar. 12/12 Apr. 21/12	$ 400 1150 1200	July 17, '12
4	Oct. 15	Henry Sanders	$1850	6%	Dec. 20/11 Jan. 15/12 Feb. 20/12 Apr. 14/12	$200 185 300 400	May 21/12

5–9. Solve all the examples given under the U. S. rule by the Merchants' rule.

QUESTIONS FOR DISCUSSION.

1. Compare the illustrative examples for the two rules and describe the different treatment of payments.

2. How are interest periods obtained under the U. S. rule? Under the Merchants' rule?

3. Why does it make no difference, under the Merchants' rule, whether the payments are greater or less than the interest?

4. Explain the reason for the difference in the final amount due under the two rules.

BANK DISCOUNT.

A dealer buys "on credit" in order that he may dispose of his goods, in part at least, before the bill for them becomes due. He thus makes them "pay for themselves" and lessens the amount of his necessary capital. The manufacturer, however, pays *cash* for his heavy labor expense, and for some supplies.

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Were he obliged to wait for payments from customers, while paying out cash, he might be forced to carry extra heavy capital, on which he could not earn a fair return on investment. To avoid this, he takes notes from his customers, or drafts on them, and sells these to a commercial bank, at a slight discount. This discount is computed as interest for the time the note has to run. The bank collects the full amount at maturity, from the maker, the discount representing its profit, while the manufacturer, although selling on credit, gets the immediate use of most of his money.

Commercial banks are chartered by law to receive and lend money and to make collections. A large part of the income of many banks is obtained by loans through discounts. The bank's directors, or some designated officials, examine all paper offered for discount, as to its makers, endorsers, or security. Some banks discount only for their regular customers. In such cases, the loan, or proceeds, is placed to the credit of the discounter, who "checks out" the money as needed. The bank gains slightly from the extra use of the deposited money.

QUESTIONS FOR DISCUSSION.

1. Of what advantage is it to a dealer to buy on credit, rather than for cash?

2. What are the objections to selling "on credit"?

3. What is the disadvantage of buying goods for cash and selling on time?

4. What is meant by "tying up" one's capital?

Discount Terms and Rules.

The *term of discount* is the time between date of maturity and date of discount. A note or draft is discounted for its "full term" when it is discounted for the time stated in it.

The *amount or value at maturity* is the face, plus interest, if any.

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The *discount* is the sum deducted by the bank.

The *proceeds* is the sum credited, or paid, to the person discounting.

Commercial banks vary greatly in their rules for discounting. Some use exact time, others approximate time; some have a minimum discount of 25c, etc. The general rules given below may be modified to conform to local custom:

1. Discount is reckoned on the amount due at maturity.

2. Notes discounted for full terms are discounted for the time stated in them.

3. Notes discounted for partial terms are discounted for the exact number of days between the date of discount and date of maturity.

EXCEPTION. When interest-bearing notes are discounted, at the same rate as the interest rate, for a full term, the proceeds equal the face.

ILLUSTRATIONS. (1) *Non-interest note.* Discounted for full term at 6%.

$840 $\frac{XX}{100}$ *Philadelphia, Pa., Jan. 27,* 190—

Three months——.....after date..*I*..promise to pay to the order of.....*James C. Leonard*.....

at.....*The Cross National Bank*——.....

Eight hundred forty——.....Dollars

Value received with Interest at —% per annum.

Chas. R. Carleton.

No. *89*....Due *Apr. 27*

Solution.

Discount term, 3 mo.		$840.00 = face.
Amount due at maturity,	$840.—	$ 8.40 = discount, 2 mo.
Discount, for 3 mo.	12.60	4.20 = discount, 1 mo.
Proceeds,	$827.40	$ 12.60 = discount, 3 mo.

(2) The same note, discounted February 9.

Date of maturity, April 27.

Discount term, February 9 to April 27, 77 days.

Amount due,	$840.—	$840.— = base, for discount.	
Discount,	10.78	$ 8.40 = discount for 60 da.	
Proceeds,	$829.22	$ 1.40 = "	" 10 "
		.84 = "	" 6 "
		.14 = "	" 1 "
		$ 10.78 = "	" 77 "

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(3) *Interest note.* Discounted for full term at 6%.

$784\frac{xx}{100}$ *Cleveland, O., October 14,* 190—

.....*Sixty days*_____ after date....*I*....promise to pay to the order of ....*Chas. Morgan & Co.*_____

at....1685 *Payne Ave.*_____

*Seven hundred eighty-four*_____Dollars

Value received with Interest at 6% per annum.

Henry T. Carmody

No. *9* Due *Dec. 13* 1685 *Payne Ave.*

Solution. See rule 3, exception. The proceeds equal $784.—

(4) The same note, discounted for full term at 8%.

Solution.

Face of note	$784.—	Discount term, 60 da.	
Interest, 60 da., 6%	7.84	$791.84	= base for discount.
Amount at maturity,	$791.84	$ 7.9184	= discount, 60 da. 6%.
Discount,	10.56	2.6394	" " 2%.
Proceeds,	$781.28	$ 10.56	= " " 8%.

(5) The same note. Discounted, November 10, at 6%.

Solution.

Date of maturity, December 13.

Discount term, Nov. 10 to Dec. 13 = 33da.

Amount due at maturity, as above,	$791.84	$791.84	= base for discount.
		$ 7.9184	= discount, 60 da., 6%.
Discount,	4.36	3.9592	= " 30 "
Proceeds,	$787.48	.3959	= " 3 "
		$4.36	= " 33 "

ORAL EXERCISE.

Find date of maturity.

	Date of Note.	Term.
1.	Jan. 27.	2 mo.
2.	Jan. 31.	1 mo.
3.	Jan. 30.	60 da.
4.	Feb. 28.	4 mo.
5.	Jan. 30.	6 mo.

Find time of discount.

	Date.	Term.	Date Discount.
6.	Apr. 9.	60 da.	
7.	Feb. 17	1 mo.	Mar. 6
8.	Mar. 12.	30 da.	Mar. 26.
9.	Apr. 19.	60 da.	May 7.
10.	July 14.	6 mo.	July 29.

Find the discount and proceeds of these non-interest notes for full term

	Face.	Term of Note.	Rate Discount.
11.	$960	60 da.	6%.
12.	$600	2 mo.	5%.
13.	$420	30 da.	6%.
14.	$840	60 da.	7%.

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	Face.		Term of Note.	Rate Dis.
15.	$285		4 mo.	6%.
16.	$1200, discounted for		30 da.	6%.
17.	$ 840.		12 da.	6%.
18.	$ 920.		20 da.	6%.
	Face.	Date.	Term of Note.	Date Dis.
19.	$ 600	Mar. 8.	90 da. (6%.)	Mar. 17.
20.	$1200	June 19.	90 da. (6%.)	July 6.

EXERCISE.

1. Discount 5% for full term, the draft on page 310.

2. Discount, at 7%, for full term, the draft on page 311.

3. Discount at 5%, for full term, the note on page 307.

4. Discount, April 14, at 5%, the acceptance on page 311.

5. Discount February 19, 1912, at 6%, the note on page 316.

6. A merchant discounts on May 16, at 6%, the following notes and acceptances. What sum should be placed to the credit of his account?

(*a*) A 3 mo. note for $725.60, dated Apr. 12.

(*b*) A 60 day sight draft, for $852, dated Apr. 20 and accepted Apr. 23.

(*c*) A 4 mo. note for $192.68, dated May 16.

(*d*) A 30 day acceptance, for $365.42, dated May 15.

ORAL EXERCISE.

Find the amount due at maturity on the following notes:

	Face.	Term.	Rate Int.
1.	$ 246.	2 mo.	3%.
2.	$ 88.72	60 da.	6%.
3.	$ 900.	90 da.	4%.
4.	$1260.	30 da.	6%.
5.	$ 425.	4 mo.	6%.
6.	$ 960.	20 da.	6%.
7.	$ 528.	90 da.	6%.
8.	$ 720.	60 da.	5%.

Find the discount and proceeds of the following notes, bearing interest at 6% and discounted at the same rate:

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	Face.	Term.	Term. Dis.
9.	$ 600.	4 mo.	60 da.
10.	$1200.	60 da.	20 da.
11.	$ 900.	30 da.	30 da.

	Face.	Date.	Term.	Date Dis.
12.	$ 840.	Jan. 6.	60 da.	Feb. 5.
13.	$1200.	June 5.	30 da.	June 25.
14.	$ 600.	Aug. 10.	60 da.	Aug. 30.

EXERCISE.

1. Find the discount and proceeds on the note on page 321, if discounted November 4, at 6%. If discounted October 27, at 6%.

2. Determine the proceeds of the same note, if discounted at 5% on Nov. 3.

3. Find the proceeds of the following firm note, if discounted March 17, at 5%:

$582.$\frac{00}{100}$ *Kansas City, Mo., Mar.* 5, 1913

.....*Ninety days*_____after date____we....promise to pay to the order of_____*John Sheldon*_____

at.....*our office*.....

*Five hundred, eighty-two*_____$\frac{00}{100}$Dollars.

Value received with Interest at 6% per annum.

J. M. Brownson & Bro.

No. 189 Due.....

4. James Devlin is obliged to pay his 90 day, 5% interest-bearing note, for $960, on Nov. 19. He has only $564.30 to his credit in the bank where his note becomes payable. On Nov. 18, therefore, he discounts the note shown below at 6%. He also deposits checks for $82.67 and $91.80. After his own note has been paid, what balance remains to his credit?

$468.$\frac{40}{100}$ *Baltimore, Md., Nov.* 18, 190—

.....*Eighty days*_____after date.....*I*.....promise to pay to the order of.....*James Devlin*_____

at....*Third Nat. Bank*.....

....*Four hundred sixty-eight*_____$\frac{40}{100}$Dollars

Value received with Interest at 5% per annum.

Chas. D. Warrenton.

No. 82 Due....

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5. Extend this form:

DISCOUNT TICKET.

THIRD NATIONAL BANK.

March 16, 190...

NOTES AND DRAFTS LEFT FOR DISCOUNT.

By *James Fielding* ....

Note or Draft.	Maker or Drawer.	Face.	Int. to Maturity.	Amount.
30 d. n.	S. P. Norton	840 —	4 20	?
60 d. dft.	Johnson Bros.	1765 80		?
4 mo. n.	Roberts and Bro.	882 17	? (6%)	?
3 mo. n.	Champlain Mfg. Co.	1206 50	? (5%)	?
	Total,			?
	Discount, @ 6% (full term)			? —
	Proceeds,			?

7. Complete the extension of the bank discount register on page 325.

ORAL EXERCISE.

1. If a non-interest note is discounted for its full term of 4 mo., at 6%, the proceeds equal what per cent. of the face? If the proceeds are $980, what is the face?

2. A 60 day, 6%, interest-bearing note is discounted for 30 days at 6%. The proceeds equal what per cent. of the face?

3. Knowing the proceeds of a note, how may its face be determined?

EXERCISE.

NOTE. The per cent. method, suggested above, may be used, or the proceeds of a dollar note, under the given conditions, may be determined. The proceeds divided by the proceeds of a dollar note will determine the face.

1. What must be the face of a non-interest-bearing note for 3 mo., that will yield $750 proceeds when discounted for its full term at 6%?

2. On January 17, I sold A. B. Norton a bill of merchandise amounting to $762.50, less 20%. Terms, 10 days. At the end of the period, Norton, being temporarily short of cash, offers his non-interest, 30 day note for such an amount that when discounted for its full term at 6%, the proceeds equal the face. This offer being accepted, the note should be made out for what sum?

INDIVIDUAL ORIGINAL WORK.

Report on local business methods of computing discounts on notes, drafts and business paper. Give local process of discounting.

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Discount Register.

DATE DIS.	MAKER	DISCOUNTED FOR	OUR NO.	TIME	MATURITY DATE	FACE	RATE OF INT.	INTEREST	AMOUNT	TIME OF DISC. DAYS	DISCOUNT	PROCEEDS	DATE NOTE
Feb.17	C.Barker	R.Corning	682	3 m.		1264 80	6						Feb.14
17	James Dean	"	83	90 d.		900 –	6						12
17	F.C.Cross	"	84	30 d.		247 85	5						17
17	H.M.Mason	"	85	6 m.		1306 90	–						17
17	S.P.Roch	A.B.Hoskins	86	45 d.		207 56	4						9
17	Jas. Elvin	Kenton Bros.	87	1 yr.		482 –	–						1
17	Chas. Evans	R.Corning	88	60 d.		1288 75	–						Jan.16
Feb.18	R. Harper	Jas. Chase	89	4 m.		960 –	5						29
18	F.D.Dent	"	690	1 m.		1540 62	6						Feb.16
18	James Rowe	"	91	90 d.		2060 50	4½						18
18	S.P.Carlin	"	92	60 d.		3000 –	8						18
18	E.K.Young	Henry Field	93	45 d.		3880 –	6						3
18	James Bros.	J.T.Roome	94	20 d.		4500 –	–						Jan.29
18	The Crane Co.	E.P.Collins	95	80 d.		6750 –	–						Feb.6
18	James Best	Benton Bros.	96	5 m.		205 40	5						18
18	Louis Cates	Jas. Camp	97	6 m.		1654 –	–						16

Note: Discount rate, 6%

CHAPTER XXXVIII.

SAVINGS ACCOUNTS.

"The best way to accumulate money is to resolutely save and bank a fixed portion of your income, no matter how small the amount."—*The World's Work.*

INTRODUCTORY EXERCISE.

1. Name three good reasons for saving money from one's income.

2. If a journeyman carpenter saw a chance to become a contractor on a small scale, he would have what reason for saving money?

3. Direct savings increase rapidly. A salesman, setting aside ten cents per day, saves $—— per month. An iron-worker, saving $3.50 per week, has $—— at the end of one year. It takes an office clerk, who is saving 20% of his weekly salary of $12, —— years to save the $600 necessary to secure an interest in a small independent business.

4. What are the dangers of simply "hoarding" one's money?

5. What difficulties lie in the way of investing small sums as they are saved?

6. What advantages result from placing those small savings in a bank?

7. What rates of interest do your local banks and trust companies pay?

8. What is the smallest sum accepted by them to open an account? What is the smallest sum accepted thereafter as a deposit?

9. Give reasons for one's withdrawing his deposit — (*a*) in prosperous times; (*b*) in times of panic; (*c*) in times of business depression.

Practically all investment by wage-earners starts in the *savings bank,* an institution chartered by the State for receiving savings deposits, on which it pays interest. Savings banks receive enormous deposits of individually small amounts. A New York bank, in a poor district, has over $100,000,000 on deposit.

The savings banks usually have the primary quality of safety, and are more carefully regulated and guarded by law than are other banking institutions. Most savings banks are

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partly or entirely mutual and are run, in such cases, for the benefit of the depositors. A few are stock companies. Unless run for the benefit of stock holders, the banks place the major portion of their funds in safe mortgages, or in government, or other "gilt edge" bonds. Often the state laws specify very definitely the class of investments that may be made. It is well to examine the laws, or to get reliable business advice before depositing in any bank. If good banks are not within easy reach, deposits may be sent to them by mail.

Many trust companies also accept savings deposits. They generally hold deposits "subject to check," but are not always as closely safeguarded as savings banks.

EXERCISE.

1. Eleven savings banks of Pennsylvania recently had 398,885 depositors, an average of ? per bank.

2. Complete this tabulation concerning the savings banks of the country:

Year.	No. Banks.	Deposits.	Depositors.	Av. per Bank.	Av. Depos.
1905	1237	$3,093,077,357	7,693,229	$——	$——
1910	1759	4,070,486,247	9,142,908	$——	$——
5 yr. increase		?	?	?	?
% of increase		?	?	?	?

3. Account, in some way, for these increases.

Most deposits are for long periods. Withdrawals are at infrequent intervals, and are further restricted by the fact that a bank may require, if it desires, 10 days to 60 days notice before making a payment. This rule is seldom enforced except in times of money stringency or sudden panic. On notice of expected withdrawal, the bank begins to save incoming deposits to meet this sum. It thus leaves practically its entire savings receipts free for investments. Money that it cannot immediately invest, it may lend "on call" (see p. 345), or lend to commercial banks at 1% to 2% interest.

Whether mutual or stock organizations, savings banks

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usually pay from 3% to 5%, the average for the country being $3\frac{1}{3}\%$. Interest is added to deposits at periodic intervals. Since the period of compounding has some effect on income, it is well to note this, as well as the method of computing interest, before selecting a bank. The method is generally stated in the rules of the bank.

EXERCISE.

(Use compound interest table, page 303.)

Find the amount for one year:

1. Of \$400 compounded semi-annually at 4%.
2. Of the same sum compounded annually.
3. Of \$364 compounded quarterly at 4%.
4. Of \$942 compounded semi-annually at $3\frac{1}{2}\%$.
5. Find the amount of \$368 for three years if compounded annually at 3%.

Since, in most cases, the amount on deposit is constantly changing, a special method is necessary for computing the simple interest for each term. Many savings banks allow interest on the smallest balance during any interest period. Sums deposited early in a period draw no interest until the next period, unless deposited the first day of the period. Fractions of a dollar do not draw interest.

ILLUSTRATION I.

NORTHERN SAVINGS BANK.

In account with *James P. Norman.*

Date.		Withdrawn.	Deposited.	Interest.	Balance.
190—					
Jan.	1				320
	29	10			310
Feb.	17		462		772
Mar.	2	90 50			681 50
Apr.	1			3 10	684 60
	4		40		724 60
June	28	160			564 60
July	1			5 64	570 24

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EXPLANATION. The deposits and withdrawals are entered in order of dates. The rate in this bank is 4%, compounded quarterly, or 1% per quarter. The first interest period extends from Jan. 1 to March 31. The smallest balance on hand during the period is $310, from Jan. 29 to Feb. 17. The interest is 1% of $310, or $3.10, which is added to the balance, as a new deposit, on Apr. 1. The next interest period extends from Apr. 1 to June 30. The smallest balance is $564.60, on June 28, and the interest is 1% of that balance, or $5.64. This interest is added July 1.

If compounded semi-annually, the smallest balance for the entire period is $310, and the interest is 2%, or $6.20, a loss to the depositor, by change of compounding, of $2.54.

Some savings banks reckon interest on monthly or quarterly balances, but compound it, or add it to the deposit, only at semi-annual, or annual, intervals.

ILLUSTRATION II.

WASHINGTON SAVINGS BANK.

In account with *Henry C. Brewster.*

Date.		Withdrawn.		Deposits.		Interest.		Balances.	
191—									
July	1							560	
Aug.	5			246	10			806	10
Sept.	8	26						780	10
	19			129	80			909	90
Oct.	31	40						869	90
Dec.	14			37	43			907	33
Jan.	1					14	29	921	62

EXPLANATION. Here assume interest at 4%, on quarterly balances, compounded semi-annually. The lowest balance in the quarter ending Sept. 30 is $560, and the interest is 1%, or $5.60. The lowest balance in the fourth quarter, ending Dec. 31, is $869.90 and the interest is $8.69. The total interest, compounded on Jan. 1, is $5.60+$8.69, or $14.29.

EXERCISE.

1. What is the effect of a heavy withdrawal just before the end of an interest period?

2. Would Norman (illus. 1) have gained by delaying his withdrawal of June 28, until July 1?

3. When would an additional withdrawal of $400 have caused the greatest "interest" loss in Brewster's account?

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4. Determine the interest and final balance of Brewster's account, if the interest is reckoned on monthly balances.

5. Balance this account. Rate 3%, reckoned and compounded quarterly:

THORNTON SAVINGS BANK.

In account with *John Kensington.*

Date.		Withdrawals.		Deposits.		Interest.		Balances.	
190—									
Jan.	1							362	40
	8	40							
	31			72	50				
Mar.	3			80	45				
May	19	162	65						
June	12			321					
	30	40							

6. Balance this account. Rate, 4%, reckoned on quarterly balances, and compounded semi-annually.

ILLINOIS CENTER SAVINGS BANK.

In account with *C. R. Martin.*

Date.		Withdrawn.		Deposited.		Interest.		Balances.	
July	5			328	45				
	29	102	10						
Aug.	15			107	38				
Sept.	29			54	13				
Nov.	10	127	69						
	28			24	63				
Dec.	12			132	31				
	18			46	50				

7. Extend this account. Rate, 4%, reckoned on monthly balances and compounded semi-annually.

EXCHANGE SAVINGS BANK.

In account with *Robert S. Fenwick.*

Date.		Deposits.		Interest.		Payments.		Balance.	
Jan.	1							582	61
Mar.	3	45							
May	16	312	62						
June	21					46	58		
Sept.	17					92	46		
Oct.	20	545	20						
Nov.	10					416	92		
Dec.	21	38	80						

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NOTE. Notice change in column headings.

8. Balance this "subject to check" trust company account. Interest, 2%, compounded semi-annually. $240 is on hand January 1, 1912.

Name *Robert B. Thornton*

No. *72426* Address *1429 Hinckley Ave.*

Occupation *Contractor*

DATE		WITHDRAWN	DEPOSITED	BALANCE
1912				
Jan.	*17*		*2 3 2 1 4*	
	28	*6 5 3 0*		
Feb.	*17*		*3 1 1 5 0*	
Mar.	*8*	*1 4 2 5 1*		
	26		*1 2 0 7 3 8*	
Apr.	*4*	*2 9 5 0*		
	14	*1 3 2 5 0*		
May	*11*		*1 2 6 4 3*	
Jun.	*8*		*4 6 5 9*	
	16	*4 8 5 0*		
	29		*3 6 6 0*	
Oct.	*17*		*2 5 9 1 0*	
	23	*1 1 1 6 3*		
Nov.	*13*	*4 1 6 9*		
	21		*2 0 0 0 0*	
Dec.	*11*		*3 1 0 5 0*	
	16		*4 2 5 0*	
	29	*8 1 1 6*	*3 0 0 0 0*	
	30		*6 8 8 2*	
Jan.	*1*	*Interest*		

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CHAPTER XXXIX.

STOCKS AND BONDS.

The proprietor of a business who finds a need for increased capital may secure it by taking into partnership the one who supplies it. But partnerships are often unsatisfactory, because each partner is usually responsible for the acts of the others and in most cases is liable for all debts. Each change of partners, moreover, causes a new adjustment of interests and often interrupts business. These objections increase with the enlargement of a business and often seriously affect ease and continuity of management.

The *corporation* meets many of the objections to a partnership. It is *an association of individuals recognized under the law as a single artificial person.* The capital, or *capital stock*, of such an organization is divided into small equal amounts called *shares.* A single person may hold one or more shares. Many individuals may be interested in large companies. A single, well-known railway corporation has 63,000 *shareholders* or *stockholders.*

The steps in the organization of a corporation are:

(1) A specified number of people (at least three, in New York) come together and draw up a certificate, stating (*a*) the name of the proposed corporation, usually including the word "Company"; (*b*) the place of business; (*c*) the nature of business; (*d*) the amount and face (par) value of shares; (*e*) duration of corporation; (*f*) other information required by law.

(2) This certificate is filed with the Secretary of State, who issues a license to these people to open books for *subscriptions* to the capital stock.

(3) When the legal amount of subscriptions have been made, and in some instances paid in, a meeting of subscribers is called, officers are chosen and by-laws adopted. Usually the stockholders elect *directors*, each stockholder having as many votes as he holds shares, and the directors elect officers.

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(4) A record of these proceedings is filed with the Secretary of State who then issues a certificate to the effect that the corporation is legally organized and ready for business.

Certificates of stock are issued to subscribers when subscriptions are paid, and these certificates may be transferred at will thereafter, unless the stockholder is indebted to the corporation. In such cases, transfer may be permitted by special vote.

Advantages. The principal advantages of corporations to their partners (stockholders) are: (1) Financial responsibility for only the par value of stock (twice the value in bank stock) until paid; (2) small shares, permitting the purchase of interests in many prosperous businesses where a partnership interest could not have been bought; (3) a simple method of transfer of ownership; (4) possibility of holding an interest in a business for which one has neither time nor qualifications for direct control; (5) the value of stock as an investment for surplus funds. The advantages to the business are (1) possibility of indefinite expansion as to capital and organization; (2) unity of direction and management, enabling the employment of experts.

Shares. The *par*, or *face value* of shares in the larger corporations is usually $100, but may be any simple multiple or aliquot part of $100, as $1, $5, $10, $500, etc. A majority of shares outstanding control the company. Thus, a few large shareholders may control a company in which a host of small holders are interested.

EXERCISE.

Find the missing values:

	Capital.	Par. Val. per Sh.	No. Shares.
1.	$ 200,000	$100	——
2.	$4,000,000	$ 50	——
3.	$ 80,000	$ 10	——
4.	$ 250,000	——	500

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5.	$ 600,000	$200	——
6.	————	$400	1500

7. Andrews holds 1200 shares ($100) in a $250,000 company. He must buy —— shares, in order to control it.

8. Name the number of shares necessary to control each of the companies in ex. 1–7.

9. In the reorganization of a $50,000 company, the par value of shares is changed from $20 to $100. Snell, who owns 400 $20 shares, should receive ——$100 shares.

Preferred and Common Stock. Usually the original issue of stock is called *common* stock and has full voting power. Sometimes *preferred* stock is issued at time of organization and may or may not have voting power. Preferred stockholders must be paid a certain stated proportion of profits before common stockholders receive anything. Thus, "4% preferred stock" entitles the holders to 4% of the face value in profits, annually, *but no more.* The preferred stock may be cumulative, carrying over to a succeeding year its right to its share for a previous year, even if in those years there was no profit. Preferred stock is sometimes non-cumulative.

Preferred stock is issued, sometimes, to meet special needs for additional capital, and is in the nature of a perpetual loan, the holders having no right to vote. If the common and preferred stockholders agree, a second issue of preferred stock may be made that will have rights over previous issues.

Raising the Capital. Some states require that the original capital stock shall be paid in full; in other states it may be sold for what it will bring. The market price is termed a *quotation.* If the stock is sold at face value, quotation is "at par"; if below, it is "below par," or at a discount; if in advance of par it is "above par," or at a premium.

At times, private bankers will take over a large block of stock, not originally subscribed for, at a fixed price and sell it to the public. Often a commission is paid, in money or stock.

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EXERCISE.

Find missing values.

	Capital Stock.	Shares.	Orig. Selling Quotation.	Actual Cash Cap.
1.	$200,000	$100	$100	$——
2.	$500,000	$100	$98	$——
3.	$2,000,000	$50	$47	$——
4.	$500,000	$100	$103	$——

5. C. P. Jacobs has subscribed for 500 $100 shares in a new company, payable in full. He turns over a small factory, which is accepted at a valuation of $36,492. What must he pay in cash?

6. The partnership firm of Adams and White is reorganized as a corporation of $60,000 capital, in $100 shares. Adams' present worth in the old firm was $18,500 and White's $32,000. What number of shares should each receive for his present worth, and what number of shares should go into the treasury for future sales?

7. Suppose the full number of shares is divided between Adams and White in proportion to their present worth, Adams paying White in cash for the fraction of a share, what number of shares should each receive? The capital stock is *watered*, or in excess of the real capital by $——.

8. Martin agrees to take 80 shares ($100), Brown 450 and Baker the balance, in an $80,000 company. They decide to pay for their stock at 15% premium, in order to create a surplus or extra working capital. Determine the capital, surplus and payment of each subscriber.

Distribution of Profits and Losses. In case of losses, if stock is assessable, stockholders, by vote of directors, may be required to pay a certain assessment, levied as an amount per share, or as a certain per cent. of par value. The profits, on the other hand, are held as *undivided profits* or *surplus*, until *dividends* are declared by the directors. Dividends are more commonly expressed in dollars per share, or as a per cent. of par value of stock. Dividends must be paid, first, on preferred stock, and then on common stock. The directors are not forced to dispose of all profits as dividends.

ORAL EXERCISE.

1. The Crown Manufacturing Co., capitalized at $1,000,000 ($100 shares) declares a dividend of 5% ($5 per share). The total dividend

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in money, is $——; and the dividend of Jamison who owns 60 shares is 60×$5, or $——.

Find the missing values.

	Capital.	No. Sh.	Par Val.	Div. %	Div. $	Div. of A who owns
2.	$200,000	1000	?	4%	?	40 shares, $——
3.	$500,000	?	$100	5%	?	180 " $——
4.	$80,000	?	$50	?	$1.50	90 " $——
5.	$2,400,000	?	$100	?	$4.50	2000 " $——
6.	$60,000	?	?	4¼%	?	200 " $——

EXERCISE.

NOTE. Assume par value as $100 per share, unless otherwise stated.

1. What sum of money is required to pay a 3½% dividend on a company capitalized at $4,000,000?

2. A 7% dividend on 670 shares of P. R. R. stock will yield the owner $——.

3. A quarterly dividend rate of 1¾% will yield the owner of 250 shares $—— per year.

4. What rate of dividend, in dollars per share, must be declared in order to distribute $42,000 to stockholders in a $500,000 company?

5. An assessment to meet a loss of $36,000 must be levied against a capital stock of $400,000 ($50 shares). Determine the assessment rate A must pay $—— on his holdings of 120 shares.

6. An industrial corporation, capitalized at $2,000,000, has net earnings of $172, 541.50, for the year. Of this sum, 20% is transferred to surplus. What is the largest whole per cent. of dividend that can be declared from the balance? If this is declared and paid, what is the balance of undivided profits?

7. The Concklin Wheel Co. has a capital of $200,000, one-fourth consisting of 5% preferred stock. Its gross income for one year is $729,342, and its gross expense $512,860. The directors set aside 10% of profits for surplus, and after paying preferred dividends, declare the largest possible dividend (in per cents and fourths). The rate declared is ——%.

NOTE. At times, when profits are very high or when, for any reason, a cash payment is not advisable, *stock dividends* are sometimes declared. Additional stock, thus issued, participates in all future dividends. When business is profitable, but there is a temporary money stringency, dividends are sometimes paid in *scrip*, or certificates cashable after a certain date.

8. The Lamberton Electric Co , capitalized at $500,000, has decided to distribute $80,000 in dividends. The highest rate it pays in cash is

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8%, the balance being paid in new stock. Harper, who owns 400 shares, receives $—— cash and —— new shares. The new capital stock of the company is $——.

Bond Issues. When a government or corporation borrows large sums of money it usually issues a formal promissory note called a *bond.* The bonds usually are issued in series of like tenor and for equal amounts, with interest payable annually, semi-annually or quarterly. The face value is usually $1000, but $500 bonds are common, and smaller ones are issued.

The payment of industrial bonds is secured by a mortgage on the property of the corporation. Minor municipal bonds are sometimes paid from a *sinking fund* raised by taxation. Government bonds and securities, and corporation debenture bonds have no security. *First mortgage* bonds usually have a first right to lien on property; succeeding issues, second, third, etc., have rights only after the first issue is paid.

Bonds are issued commonly in registered or coupon form. If *registered,* the names of the owners are recorded in the books of the company, as in the case of stock; the bonds are made payable to the specified buyer, and are transferable by endorsement and office record. Interest is sent by check to the recorded owner. *Coupon* bonds have attached coupons or vouchers, one for each interest period during the life of the bond. These are detached as they become due, and are presented for payment, or collected through a bank.

Many bonds derive their name from their rate of interest, and from their term and time of maturity. "D. C. 4s of 1925" are District of Columbia 4% bonds due in 1925.

While bonds are usually payable at a fixed time, provision is sometimes made to pay off a certain number of an issue each year. Sometimes the company reserves the option of paying off between certain dates, as "after ten years but not later than twenty years."

Bonds are similar to preferred stock in that they pay a fixed

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rate of dividend or interest. They have a limited life and they confer no voting power, but they have a right to the payment of interest ahead of the preferred stock.

EXERCISE.

1. The Selford Electric Co. sells a $40,000 6% bond issue at $1050 per $1000 bond, receiving $—. The interest on the issue will cost the company $ — annually.

2. The Crane Hardware Co. issues 5% bonds running five years for a face value of $120,000. The company creates a 4% sinking fund to meet the issue at maturity. If it sets aside equal amounts at the end of each year and also pays interest, what is the annual expense?

3. The Western Banking Corporation underwrites the bonds of the West River Lumber Co., par value $50,000, at 90% of face value. The banking corporation disposes of the issue at 93% of face value, thus making a profit of $ —, and paying the lumber company $ — .

Buying and Selling. The market quotations are affected by many varied factors. Primarily, the price should depend on earning power, present and prospective. Thus, price is influenced indirectly by the capital of the company, for overcapitalization means less income per share. The price may be seriously, though temporarily, influenced by speculation, or by an attempt of parties to gain control of a company. The general prosperity or depression of industry affects quotations seriously. A financial depression may cause *liquidation* or the selling of stock to meet financial debts — sales often made at a heavy loss. A sudden loss of confidence in one stock may affect others. The fear of injurious legislation, the threat of war, the falling off of exports, the closing down of a great factory are other illustrations of the many influences affecting market price.

Quotations. General purchases and sales are governed largely, as to price, by the quotations of the stock exchanges. The quotation lists, as printed, often vary in form, but usually show the price in dollars per share, at several times of the

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business day. For bonds, the price is quoted per $100 of par value.

STOCK QUOTATION LIST.

(From newspaper lists. Consult newspapers for current quotations.)

	Open.	High.	Low.	Close.
Mo., Kan. & T., com.	26	26	25½	25½
M. S. P. & S. S. M.	141⅞	142	141	142
Missouri Pacific	44	44¼	43⅞	43⅞
N. Y. Air Brake	82⅜	82⅝	82⅜	82⅝
New York Central	109½	110	107	107⅛
N. Y., Ont. & West	33	33¼	32¾	32¾
Norfolk & Western	113	113	113	113
Northern Pacific	126¾	127⅜	125⅝	126
Pennsylvania R. R.	124	124⅛	123⅛	123¼
Pressed Steel Car	35	35	35	35
Rail's Steel Springs	36	36	35	35
Reading	164½	165¼	162⅞	163⅛
Rep. S. & I., pfd.	89	89	89	89
Rock Island, com.	24¼	24¼	24	24
Rock Island, pfd.	47⅞	47⅞	47⅝	47¾

BONDS.

C. R. I. & P. 4s	69	69	68½	68½
Atcn. 4s. con.	104½	104½	104½	104½
Nor. & West 4s	98½	98	98½	98¾
U. S. Steel 5s	101⅞	102	101¾	102

GOVERNMENT BONDS.

United States 3s, coupon	104¾	104¾	104¾	104¾	5
United States 4s, coupon	114½	114½	114½	114½	15
Imp. Japanese Gov. 6s ctfs. full paid	90¾	92	91⅛	92	678
Republic of Cuba 5s, full in. paid	103	103	102½	102⅝	232

In the above stock list, N. Y. C. stock, for example, is quoted at the opening of the day at $109.50. During the day, the highest price was $110 and the lowest, $107. At the close of the day, the price was $107.125. Such stock is usually bought or sold through brokers. The common charge is 1/8% of par value, or $1/8 on a $100 share. Small brokers, handling small lots, sometimes charge 1/4%.

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ILLUSTRATIONS. (1) Determine the cost of 300 shares of P. R. R. stock, at closing quotation, brokerage 1/8.

Solution. By reference to list, the closing quotation is \$123.25. Brokerage is \$1/8 or \$.125.

The entire cost of 1 share is \$123.25 + \$.125, or \$123,375. 300 shares cost 300 × \$123.375, or \$37,012.50.

(2) What should I receive from the sale of 400 shares Reading at opening quotation, brokerage 1/8?

Solution. Opening quotation, $164\frac{1}{2}$.

Net proceeds of 1 share, $\$164\frac{1}{2} - \$\frac{1}{8} = \$164\frac{3}{8}$.

Net proceeds of 400 shares = $400 \times \$164\frac{3}{8} = \$65{,}750$.

ORAL EXERCISE.

1. Which of the stocks in the quotation list are *preferred?*

2. Was the general tendency of prices, during the day, upward or downward?

3. If I buy Reading at highest quotation, brokerage 1/8, I pay \$ — per share. If I had sold at the same price I would have received \$ — net proceeds.

4. Brokerage being 1/8, determine the cost of

	No. shares.	Stock or Bond.	Quotation.
(*a*)	100	N. Y. Air Brake	Closing.
(*b*)	1000	Mo. Pac.	Open.
(*c*)	10	Penn. R. R.	High.
(*d*)	2	No. Pac.	Low.
(*e*)	\$10,000, par. val.	Nor. & Western	High.
(*f*)	\$10,000, par val.	U. S. 4s,	Open.

5. Determine the net proceeds, brok. 1/8, from the sale of

	Shares or Val.	Stock or Bond.	Quotation
(*a*)	200 sh.	U. S. Steel,	$68\frac{1}{8}$.
(*b*)	\$ 1,000.	Southern Pacific,	$106\frac{3}{4}$.
(*c*)	\$10,000.	S. L. & S. F., com.,	16.
(*d*)	\$10,000.	U. S. 3s.,	Closing.
(*e*)	100 sh.	Reading,	Closing.

6. Determine the brokerage at 1/4 on lots of: 80 sh., 25 sh., 1000 sh., 250 sh.

7. If brokerage is 1/8, one way only, find the net profit or loss per share on:

	Buying Quot.	Selling Quot.		Buying Quot.	Selling Quot.
(*a*)	$79\frac{1}{4}$	$81\frac{1}{4}$	(*c*)	8 prem.	6 prem.
(*b*)	$111\frac{3}{4}$	112	(*d*)	$5\frac{1}{4}$ dis.	1 prem.

8. Express these discounts in other forms: 105 (Ans. 5 prem. or 5 above par), 96, $110\frac{1}{4}$, 8 dis., $7\frac{1}{4}$ prem.; $5\frac{1}{8}$ below par, 100.

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EXERCISE.

1. Determine the cost at opening quotation, brokerage 1/8, of 200 sh. M. K. & T. com.; 350 sh. Mo. Pac.; 3000 sh. Reading; $10,000 par of C., R. I. & P. 4s; $20,000 par U. S. Steel 5s.

2. Determine the net proceeds received from the sale, at high quotation, brokerage 1/8, of: 3000 sh. N. Y. C.; 2000 sh. Nor. & West.; 800 sh. Rock Is. com.; 3000 sh. P. R. R.; 18,000 sh. Pressed Steel Car; $500 Japanese Gov't 6s.

NOTE. In determining the gain or loss from a purchase or sale, it is advisable to find the net gain or loss per share, and to multiply by the number of shares.

ILLUSTRATION. Determine the profit or loss from 800 sh. bought at $98\frac{1}{8}$ and sold at 101, brok. $\frac{1}{8}$ each way.

Solution. The cost $= 98\frac{1}{8} + \frac{1}{8} = 98\frac{1}{4}$.
The net proceeds $= 101 - \frac{1}{8} = 100\frac{7}{8}$.
The net profit per sh. $= 100\frac{7}{8} - 98\frac{1}{4} = 2\frac{5}{8}$.
The total profit $= 800 \times \$2\frac{5}{8} = \$2100$.

3. Find the profit or loss in the following, brokerage $\frac{1}{8}$ one way:

	No. Sh. or Val.	Stock.	Buy. Quot.	Sell. Quot.
(*a*)	200 sh.	Penn. R. R.	Open	$113\frac{7}{8}$
(*b*)	$8000	Japanese 6s.	High	Close.
(*c*)	500 sh.	No. & West.	Open	Close.
(*d*)	1200 sh.	No. Pac.	Open	Close.
(*e*)	3000 sh.	Reading.	$104\frac{7}{8}$	Close.

(*f*) On $5000 of each of the bonds in the quotation list, buying at opening and selling at closing price.

4. I directed my broker to sell $20,000 U. S. Steel 5s and invest proceeds in Reading. Buying quotation, $104\frac{7}{8}$; selling quotation, $88\frac{1}{8}$; brokerage 1/8 each way. Number of shares bought? Surplus?

NOTE. No fraction of a share is bought. In purchases through exchanges, lots of *hundreds* of shares are usually dealt in.

5. I ordered my broker to sell 2600 shares N. Y. Ont. & W. ($32\frac{1}{4}$) and to buy 2000 shares N. P. ($125\frac{1}{8}$). He should draw on me for what sum to settle the balance?

Find missing values:

6. Carroll County issues ten-year coupon school bonds, bearing 6% interest, payable semi-annually, which are disposed of at $102\frac{1}{4}$, brok. $\frac{1}{8}$. The bonds produce a fund of $——, but the County must meet interest charges of $—— every six months, or $—— in ten years. At the close of the final year it must pay out $——. The bonds have —— coupons attached, each worth $——, if the face value is $500 each. Abbott, who

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buys $12,000 worth, pays $——, and gets an annual income of $——, or —% on his original investment.

Stock Exchanges. Stock exchanges are organizations, formed for the purpose of supplying a regulated market for the purchase and sale of stocks and bonds. All cities of any size have such organizations, but the New York Stock Exchange is by far the best known, and is national in scope. This has 1000 members. Membership is called a *seat*, and may be sold. The buyer of a seat must be elected by the Committee on Admissions.

The strictest regulations are in force in regard to methods of doing business, and the organization maintains the highest standard of commercial integrity among its members. Transactions involving hundreds of thousands of dollars are contracted verbally and are carried out to the letter. Severe penalties are inflicted for any breach of rules.

An Exchange handles listed and unlisted securities. It does not guarantee the value of securities that it admits or lists, but before listing, it makes examinations of the companies whose stock is offered for listing. Railway companies, for example, must file statements giving location and detailed description of property, and a financial statement of liabilities, assets, earnings, mortgages, etc. An industrial corporation must prove that it is legally organized, and must also make a statement of its financial and business affairs. An Exchange makes further requirements in regard to regular reports, notices to stockholders, etc., thus forcing the company to make public its actual condition, before listing is granted.

For corporations of standing that are unable, or unwilling, for some reason, to conform to the strict requirements of "listing," an "unlisted department" is made. Less strict statements are required, but the stocks are dealt in in practically the same way. Unlisted securities are not considered of so high a rank as listed securities, and are not so highly esteemed by bankers as collateral for loans.

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On one Exchange business is done by members for outsiders at 1/8% commission; by members for named members, at 1/50% and by members for unnamed members at 1/32%. Lots of 100 shares, or $10,000 par of bonds, are usually dealt in. Trading must be done between 10:00 A.M. and 3:00 P.M. Brokers often meet "on the curb," or in the street in the neighborhood of the exchange, and buy or sell small lots, unlisted securities, etc. This is the "Curb Market."

All transactions are by word of mouth. There are posts in places in the Hall of the Exchange where specified stocks are traded in. Brokers desiring to buy or sell the given security congregate at these posts and shout their orders or sales. A nod, a sign with the finger, or the word "sold," may be the sign of a transaction involving thousands of dollars. Memoranda are roughly made on paper, at the first possible moment, and later compared. Prices rise or fall by 1/8%. If a broker bids 98 for a stock and no one offers to sell, his next higher bid is $98\frac{1}{8}$. During the times a corporation's books are closed, its stock sells "*ex-dividend*"—that is, the last holder gets the dividend.

Stock is sold "cash" for delivery the same day; "regular," to be delivered the day following; "at three days"; or at "buyer's option" or "seller's option"—usually 4 to 60 days. Most sales are "regular."

Practically all speculative exchange business is on "margin." Thus, on January 21, C. Brown may order his broker to buy 100 shares D. & M., whenever securable at 186. He deposits 10% of the money, or $1860 (called the margin). The broker supplies the rest of the money needed. The stock is bought the same day. On January 29, when the stock is quoted at 189, he orders the broker to sell. The broker charges 1/8% brokerage each way, and 6% interest on the money lent to complete the transaction, rendering a statement as follows:

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PLEASE EXAMINE.

NEW YORK, January 29, 190—.

Mr. C. Brown
In account current with
John Doe, Broker.

Date.		Charges.	Amount.		Da	Inter't.		Date.		Credits.	Amount.		Da	Int.	
Jan.	21	To 100 sh. D. & M., 186	18600	—	8	24	80	Jan.	21	Margin	1860	—	8	2	48
		Brokerage	12	50					29	100 D. & M. 189	18900	—			
	29	Brokerage sale	12	50					29	Bal. int.				22	32
	29	Net interest	22	32											
	29	To check herewith	2112	68											
			20760	—							20760	—			

NOTE. The broker gives credit for interest on *margin*. The difference between debit and credit interest is entered in debit amount column.

The broker usually requires 10% margin on active stocks and 20% on inactive. In case of a serious fall in quotation, he can demand additional *margin*, and if it is not forthcoming, can sell the stock for what it will bring, the customer meeting the loss.

The speculator who buys for a rise is a "bull." One who contracts to sell what he has not, is a "bear." The bull works to raise prices in order to sell at a profit; the bear works to lower prices so that he can buy at a lower price the stock needed to fill his earlier selling orders. When a bull sells at a higher price he is said to "realize" his profits; when at a lower price he "liquidates." A bear buying either at a gain or loss, "covers" his previous sale.

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EXERCISE.

1. On March 17, I purchased through a broker 600 sh. Reading, at 163⅝—buyer's option at twenty days. I called for the stock March 26, and directed its sale at 164¼. Brokerage 1/8. Determine my net profit.

2. On May 9, my broker bought for me 800 sh. P. R. R. at 123⅜ on 10% margin, selling it May 14 at 125⅛. Interest, 6%; profit, $?.

3. On June 9 a broker sold short, on 10% margin, 500 sh. N. P. at 125⅝. The stock declined so that on June 16, his principal ordered his sale covered (121⅛). The principal's net gain was $?.

4. Prepare a broker's statement for his principal, A. C. Benson, from the following memoranda: Margin 10%; interest, 6%. Bought: Jan. 11, 200 sh. N. Y. C., regular, 106¾; Jan. 18, 500 sh. N. P., regular, 127⅛; Sold: Jan. 22, 500 sh. N. P., 131¾; Jan. 23, 200 sh. N. Y. C., 105¼.

Collateral Loans. Many business men invest surplus funds in dividend-paying stocks and bonds. In times of financial stringency, they borrow of their banks on time or demand, depositing this stock as security or *collateral.* These stocks and bonds are accepted, if reliable, on a margin of 10% to 20% (*i. e.*, at 10% or 20% below market quotation). No margin is required on government bonds. In case of failure to pay the loan, the securities are sold by the bank, the debt is cancelled and any balance is returned to the borrower. If the market value of the securities falls while they are being held as collateral, additional collateral must be deposited or a portion of the loan repaid.

Stock brokers use the certificates of stocks and the bonds they buy for customers as temporary collateral for loans of this order—termed *call loans.* The banks lend at the prevailing money rate and have the privilege of changing the rate from day to day as the financial market varies. They may also "call" for payment at any time. Rates are "easy" at 1% to 3%; "firm" at 6% to 8%, and "stringent" at higher rates. During a marked panic some money was lent at the rate of 100%.

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FACE OF A COLLATERAL NOTE.

$—— WASHINGTON, D. C.....190...

.....promise to pay to THE —— NATIONAL BANK of WASHINGTON CITY, or order at said Bank .....Dollars for value received with interest at.....per centum per annum, having deposited with said Bank, as collateral security for the payment of the full sum of principal, with interest and cost due on this note, and also as collateral security for all other present or future demands, of any and all kinds, of the said Bank, against the undersigned, due or not due, the following to-wit:.....

.....

.....

.....

with full power and authority to said bank to sell the whole or any part of said security, or any substitutes therefor, or any additions thereto, at public or private sale, at any time, and at the option of said Bank or its assigns, on the non-performance of this promise or any part thereof, or the non-payment of any other present or future demands of said bank as aforesaid, and without advertisement or notice to the undersigned; and upon such sale the holder thereof may purchase all or any part of said securities, discharged from any right of redemption. After deducting all proper costs and expenses, the residue of the proceeds of sale shall be applied to the payment of the note, and of any other present or future demands of said Bank as aforesaid, and the undersigned agrees to remain liable for any deficiency then remaining. In case of depreciation in market value of said security at any time pledged for this loan a payment shall be made on account, or additional security added, as required by said Bank.

.....

.....

WRITTEN EXERCISE.

1. Does this record of a call loan show sufficient security, if the margin must be 10%?

The .....National Bank.

Loan No. 86739. New York, February 17, 190—

Folio No. 472.

Demand loan for $300,000.

To Robert Harlenan.

	200	Ill. Central	127	25	4
	400	B. & O.	106		
400	500	D. L. & W.	388		
	200	Pennsylvania	127		
1000	800	Reading	167		
	100	S. Pac.	108		

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NOTE. In reckoning value, fractions of quotations are omitted, and final value written to thousands and hundreds of dollars. The first item of value is $25,400.

Alterations of collateral are shown by quantities in italics.

2. Could any line of above stock be withdrawn without destroying margin?

3. In lots of 100 sh., how much N. Y. C., at 106, might be substituted for the D. L. & W.?

4. Compute the interest on this demand loan at 3%, Feb. 17–24; 3½%, Feb. 25–26; 4%, to Mar. 3 when "called."

5. On Mar. 11, $70,000 is borrowed at 3%, on 400 sh. N. Y. N. H. & H., at 202, as collateral. During a violent break in the market stock falls to 154. How much additional collateral should be demanded in order to preserve a 10% margin?

6. A call loan of $200,000 is secured by collateral of $80,000 U. S. bonds (103); 100 sh. Ill. C. (127); 200 sh. Penn. (128); 500 sh. U. P. (160), 200 sh. N. Y. C. (106) and 300 sh. C. P. (265). If the borrower pays $50,000 of the loan, what collateral may be released? Allow 10% margin except on Government bonds.

Incomes and Investments. The investor in stocks and bonds, as distinct from the speculator, is looking for a steady and safe return from his money. He must be familiar with factors that affect quotations, but he does not look at them in the same light. As a rule, he desires a fair interest on his money, security against loss, and a market for disposal in case of necessity. In all cases, it is well to have the advice of a reliable banker or broker.

Government securities, in the form of national or state bonds, are usually safe investments and are easily marketable, but they yield relatively low returns. Many municipal and county bonds are also reliable, but before buying it is wise to see what security is back of them and how regular has been the payment of past interest. Railway and industrial bonds should be examined even more strictly along the same lines.

In buying, one should note the date of maturity of the issue, especially if he desires a long term investment. If the quo-

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tation is at a premium, he should remember that the premium represents a portion of his investment which will not be repaid him when, at maturity, the bonds are settled. The nearer the maturity of the bonds, the greater is the effect of this premium.

In buying stocks, one should invest either in local organizations, whose policy, standing and resources he can examine, or in organizations of national reputation, whose stock has shown steadiness through a long period, and which is readily marketable. Before buying, one should look up quotations for months past to be sure that the current quotation is not far above the ruling price because of some temporary market influence. Times of depression are excellent periods for the purchase of sound stocks. One should be careful not to be misled by high rates of dividends. The safest investments on the market seldom pay over 5% or 6%. It is well to investigate and determine the average dividend for years past, to discover whether the rate is steady, steadily increasing, or decreasing, etc.

ORAL EXERCISE.

1. I can buy one stock, paying 6% dividends, at 120, and another, paying the same rate, for 150. How many dollars per share income in each case? Based on these rates, which is the better investment? Suggest a method of comparing the value of stock investments.

2. I have $20,000 to invest. A good 8% stock can be purchased at 200 and a 5% stock at 80. The money will buy how many shares of either kind? What is the income in each case? How may one compare investments, knowing the sum to be invested?

3. Investments in bonds are to be made to secure a yearly income of $720. What par value of 6% bonds will produce it? Of 8% bonds? The 6% bonds can be bought at par; the 8% bonds at 120. Security being equal, which is the better investment? How is the comparison made in this case?

ILLUSTRATION. Compare the value of C. B. & M. 4s at 101⅛, and P. R. R. 4½s at 107⅛, for an investment of $20,000.

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Solution.

C. B. & M. costs $101\frac{3}{8} + 1/8$, or $101\frac{1}{2}$.
A \$1000 bond will cost $10 \times \$101\frac{1}{2}$, or \$1015
P. R. R. costs $107\frac{7}{8} + \frac{1}{8}$, or \$108.
A \$1000 bond costs $10 \times \$108$, or \$1080.
$\$20{,}000 \div \$1015 = 19$ and \$715 remaining.
Therefore 19 C. B. & M. bonds could be purchased.
$\$20{,}000 \div \$1080 = 18$ and \$560 remaining.
Therefore 18 P. R. R. bonds could be purchased.
$19 \times \$40 = \$760$, income from C. B. & M. 4s.
$18 \times \$45 = \$810$, income from P. R. R. $4\frac{1}{2}$s.
$\$4 \div \$101.5 = .0394$ (3.94%), rate income on C. B. & M. bonds.
$\$4.5 \div 108 = .0416$ (4.16%), rate of income on P. R. R. bonds.
The P. R. R. bonds are evidently the better investment if *long term*.

EXERCISE.

1. What income will be received from the investment of so much as is possible of \$60,000 in U. S. 4s at $104\frac{1}{2}$, brok. $\frac{1}{8}$?

2. Erie 4s, at 108, brok. 1/8, earn ? % on investment.

3. Compare the investment values of L. & N. stock at 145, yielding 6%, and Pullman Palace Car Co. stock, at 181, earning 8%.

4. Brown has \$60,000 to invest. His choice narrows down to T. R. R. at 141 (7%); B. & O. 4% pfd. at 80, and D. L. & W. (10%) at 485. Compare investment values.

5. In the latter part of 1907 a serious financial stringency caused heavy breaks in stock quotations. This break was finally stayed on Oct. 26. From the following quotations, find the rates of income on investments based on each of the quotations (high, low and Oct. 26). Show that times of depression are good periods for investment in sound stocks.

1907.

High.	Low.	Stock.	Oct. 26.	Reg. Dividend.
$145\frac{1}{2}$	$92\frac{3}{4}$	L. & N.	95	6%.
$75\frac{1}{4}$	40	Mackay Co.	48	4
146	$100\frac{7}{8}$	Manhattan Elev.	$103\frac{1}{2}$	7
$92\frac{3}{4}$	48	Mo. Pac.	$51\frac{7}{8}$	5
147	108	Nash. & Chat.	115	6
$117\frac{3}{8}$	103	Nat. Bis. pfd.	103	7
$134\frac{7}{8}$	$96\frac{3}{4}$	N. Y. C.	$99\frac{3}{4}$	6
$92\frac{1}{4}$	56	Nor. & Western	$61\frac{5}{8}$	5
$141\frac{3}{8}$	$113\frac{7}{8}$	Penn. R. R.	115	7
$98\frac{5}{8}$	$72\frac{1}{2}$	People's Gas & Coal	$73\frac{5}{8}$	6

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Convertible Bonds. Some railroad corporations, and a few other organizations, have issued, of late, series of bonds that are convertible, at a fixed ratio, into shares of stock. Thus a certain company issues 4% bonds that may be exchanged for stock at a quotation of 150 (par 100). The bonds offer a safe but low rate of income. If the business gains in prosperity the dividends on the stock may increase to such a point that a greater rate of income, though less secure, may be obtained by conversion. Questions of control, speculation, etc., may lead, also, to conversion. The market price of both stocks and bonds also has its effect.

ILLUSTRATION. Mansfield R. R. 5% convertible bonds are exchangeable for stock at 120. The bonds were bought at 108, the stock is quoted at 136 and yields 8% dividends. Study the effect of converting $6000 par of the bonds.

Solution.

$6000 ÷ 120 = 50, number of shares by conversion.
5% of $6000 = $300, income from bonds.
8% of $5000 = $400, income from stock.

Increase of income by conversion, $100, or 33⅓%.
60 × $108 = $6480, original cost of bond investment.
50 × $136 = $6800, present selling value of equivalent stock.

Direct gain by converting and selling, $6800 − $6480 = $320.

NOTE. This does not take into account the effect of the time bonds have to run before maturing. Bankers have "bond tables" to show the real investment value of bonds.

EXERCISE.

1. $12,000 (par) of 5% bonds may be converted into ? shares of stock, if exchange quotation of stock is 150.

2. $30,000 of 4% bonds, bought at 92, are converted into stock at a quotation of 120. The stock is sold at the market quotation of 146½. Compute net gain or loss.

3. M. T. R. R. Conv. Gold 3½'s, due 1925, are convertible into stock at 75 (par 50). Bought at 90, the bonds yield ? % on investment. Compare market value by conversion, if stock quotation is 122⅝.

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4. N. P. Conv. 4s, of 1927, at 86, yield ? % on investment. $80,000 (par) of bonds are convertible into ? shares at 175. Compare investment values if 9% dividend is earned by stock.

5. R. M. Tr. Conv. 5s at 98 are convertible at par. The last stock quotation is 85⅜ and the last dividend 4%. Discuss advisability of exchange.

Building Loan Associations. Building loan associations are a form of bank organized under state laws to encourage the accumulation of money among people of limited means, with the object, primarily, of their making real estate investments and building homes. They originated in England in 1789. The first association of the kind in the United States was organized in 1831, at Frankford, Pa. Owing to many abuses in the past, laws are now in force in many states, restricting the methods and operations of associations. One state, for example, limits operating expenses to 2½% of indebtedness. It is well to study the laws of the incorporating state, before entering any association.

The capital of loan associations, consisting of the accumulated savings of members, is divided into shares of $50 to $250, usually $200. Each share runs until the sum of installments paid and dividends earned equals its *maturing value.* The shares are issued in series, at periodic intervals, say six months or a year. A member may buy any number of shares up to a fixed limit paying by small weekly installments. He receives the face value of the shares at maturity.

In most organizations, borrowers must be shareholders. They may borrow, on their shares as security, up to 90% of the sum paid in. The intending builder, who borrows, usually has a piece of land, which is pledged with his shares for his building loan. Here the maturing value of his shares is commonly taken. Usually a shareholder may withdraw his money at any time, although he is penalized by having his share of earnings reduced.

Often there are demands from members for more funds than

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are available for lending. It has become common, therefore, to auction off the money in the treasury. There may be a fixed minimum of interest, say 5%, and the one who bids the highest rate over 5% gets the money. In other cases the rate is fixed, the highest bidder being the one who offers the largest number of months interest in advance. This advance interest is subtracted from the face of the loan when the money is paid over. Rates are often higher than 5%, but since the stockholder has a share in the profits, the net rate is reasonable.

The repayment of loans, as in the case of other features, varies in different associations. Often the payments are monthly, and for equal amounts, but the proportion going to reduce the principal steadily increases and the interest decreases. Fully paid shares are accepted as cancelling the loan.

EXERCISE.

1. Recently, there were 2,029,927 persons said to be sharing in the benefits of 5737 associations in the United States, and the total assets were $618,795,414. Find the average membership and assets per association.

2. The paid in value of Arnold's ten shares is $746.80. He borrows 90% of this value on his personal note, receiving $——.

3. Norton subscribes for fifteen one-hundred-dollar shares in an association where the rate is 25c per week per share. What is his weekly payment? Ignoring dividends, how long will it take him to pay for his stock? To what sum is he entitled at maturity?

4. Norton borrows, on real estate security, the face of his shares, at 6½%. What is the quarterly interest on his loan?

5. Mason borrows $1,000, to be repaid at the rate of $10 per week, for 138 weeks, and a final payment of $7.79. What sum does he repay in all? What is the average rate of simple interest on his loan?

6. Randolph borrows $2,000 at 6%, on a bid of 32 weeks' advance interest. What proceeds of his loan are paid him?

7. Anderson wishes to obtain $4,000. He applies for twenty shares (par, $200), agreeing to pay the usual dues of $1.00 per month per share,

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and 1½% of dues additional, as premium. If the rate of loan is 6%, what is his total monthly payment?

8. An association issues one thousand $200 shares on payments of $1.00 per month. What sum has it to lend the first month? Including 8% interest on the first month's loans, what has it to lend the second month? Construct a table showing compounded funds to the end of the sixth month, and the book value and profits of each share in the company.

Distribution of Profits. When only one series of shares is issued, the distribution of profits for any one year consists simply in dividing the profits by the number of shares to determine the profits per share. If, however, one series follows another, at short intervals, the problem is complicated. Many methods are followed, the most common being that of averaging.

ILLUSTRATION. An association, at the end of its third year, has in force three annual series of 1,000, 500 and 1,000 shares respectively. The payments are $1.00 per month per share. Its net profits to the time of issue of the fourth series are $4860. What profits are due each share?

Solution.

The first dollar on a share, in series (1) has run 36 months, the second 35 months, and the last 1 month, an average time of 18½ months. In a similar way, the average dollar in series (2) has run 12½ months, and in series (3), 6½ months. $36, $24, and $12 have been paid in respectively, on shares of the three issues.

Series (1) 1000 shares, @ $36 yield $36,000, on an average of 18½ mo. $= 18\frac{1}{2} \times 36{,}000 =$ $666,000 for 1 mo.

(2) 500 shares, @ 24 yield 72,000, on an average of 12½ mo. $12\frac{1}{2} \times 24{,}000 =$ $150,000 for 1 mo.

(3) 1000 shares, @ 12 yield $12,000, on an average of 6½ mo. $= 6\frac{1}{2} \times 12{,}000 =$ $ 78,000 for 1 mo.

Equivalent for 1 mo. $894,000.

Series (1) evidently shares $\frac{666{,}000}{894{,}000}$ in profits, or $\frac{666}{894{,}000}$ per share.

(2) evidently shares $\frac{150{,}000}{894{,}000}$ in profits, or $\frac{300}{894{,}000}$ per share.

(3) evidently shares $\frac{78{,}000}{894{,}000}$ in profits, or $\frac{78}{894{,}000}$ per share.

$\frac{1}{894{,}000}$ of $4860 = $.0054.

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Profits per share:

Series (1) 666 × $.0054, or $3.5964.
Series (2) 300 × .0054, or 1.6200.
Series (3), 78 × .0054, or .4212.

EXERCISE.

1. At the end of one year, the book value of one series of an association is $12.39, and of a second series, $6.10. Series (1) has 2,000 shares and series (2) 1200 shares. The year's profits are $1860. Apportion these profits according to book values.

2. The Martinsburg Building and Loan Asso. has issued five annual series as follows: (1) 2,000 shares; (2) 5,000 shares; (3) 3500 shares; (4) 1600 shares; (5), just out, 1800 shares. Dues $1.00 per month. Apportion profits to date of $11,456.

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CHAPTER XL.

DOMESTIC AND FOREIGN EXCHANGE.

Domestic Exchange.

INTRODUCTORY EXERCISE.

1. Name common reasons for sending money to persons in other localities.

2. In what different ways may a person send the actual money?

3. Name objections to sending the actual money.

4. To what extent are the postal authorities liable for money sent by registered mail?

5. Who is liable for losses of money sent by express or messenger?

6. In what ways, to your knowledge, may payment be made without sending the actual money? Describe such as you are familiar with, and state advantages and disadvantages of each.

7. Is it cheaper to send the actual money? Is it safer?

The process of making payments, or of settling accounts due at other localities, without sending the actual money, is termed *exchange*. Exchange within a country is termed *domestic* or *inland exchange*.

The credit papers commonly used in effecting exchange are money orders, checks and drafts.

Money Orders. Money orders are issued by the National Government and by express companies. They are really drafts or orders, issued at one office, drawn on the office in the locality in which it is desired to make payment, and payable, as a rule, on presentation. Identification before payment may be required.

The funds paid by the purchaser of the money order are not transferred by government or express company to the office of payment, but payment at that office is made from any

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funds on hand. Proper bookkeeping records of the transfer are kept. Both express and postal orders may be deposited like checks for collection. Many hotels accept and cash them. They may be transferred by endorsement—a single endorsement being allowed on postal orders, while express orders are freely negotiable.

If more than one hundred dollars is to be sent by postal order, additional orders must be taken out. The individual express money order is limited to fifty dollars.

EXPRESS MONEY ORDER.

NOTE. The cost of an order equals the face plus the fee.

FEES FOR MONEY ORDERS DRAWN ON DOMESTIC FORM.

Payable in the United States and its possessions; also for orders payable in Bermuda, British Guiana, British Honduras, Canada, Cuba, Mexico, Newfoundland, the United States Postal Agency at Shanghai (China), the Bahama Islands, and certain other Islands in the West Indies.

For Orders From	$ 0.01 to $ 2.50	..... 3 cents.
From	$ 2.51 to $ 5.00	..... 5 cents.
From	$ 5.01 to $ 10.00	..... 8 cents.
From	$10.01 to $ 20.00	.....10 cents.
From	$20.01 to $ 30.00	.....12 cents.
From	$30.01 to $ 40.00	.....15 cents.
From	$40.01 to $ 50.00	.....18 cents.
From	$50.01 to $ 60.00	.....20 cents.
From	$60.01 to $ 75.00	.....25 cents.
From	$75.01 to $100.00	.....30 cents.

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Telegraph Orders.

Telegraph orders are obtainable in the larger cities. The money is paid in at the sending office, and a telegram is sent to the telegraph company's agent at the place of payment, directing him to pay the sum, from any funds on hand, to the person specified, *on identification.* The rates are usually one per cent. of the face of the order plus twice the cost of a ten word message. The message is frequently sent in the Company's private cypher or code, each word standing for a phrase.

EXERCISE.

1. Let some pupil describe, from personal experience, the purchase of a money order, giving the steps to the process.

2. Let some pupil describe, from personal experience, the cashing of a money order.

3. Compute the total cost of money orders to cover each of the following amounts. Give the denominations of money orders where more than one required to make a payment. Amounts: $1.64, $5.00, $7.00, $1.50, $25.00, $48.00, $65.50, $200.00, $165.00, $78.50, $2.50, $37.28, $96.84, $299.00.

4. An express agent issues money orders on a certain day for $34.16, $345., $23.69, $50., $76.15, $20.25 and $107.10. He started the day with a balance in his money order drawer of $124.00. During the day he cashed orders for $23.90, $45.50, $120.00, $13.00, $112.00, $3.26 and $1.56. What should be his balance in the drawer at the close of the day?

5. Determine the individual and total cost of these telegraph orders from Washington, the rate being 1%: (*a*) To Philadelphia $600.00, plus message cost of 50 cents; (*b*) To New York, $850, message 50 cents; (*c*) To Newark, Del., $386.50, message 40 cents; (*d*) To Indianapolis, $590; message 80 cents and to New Orleans, $2500, message 75 cents.

Checks. It is customary for business men and others to keep the major portion of their cash funds on deposit in banks or trust companies. They then make practically all payments by *checks*, or orders on the bank to pay to the person named in the check the sum specified.

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FORM OF CHECK.

No. 826 *Philadelphia, October 17, 1912*

GIRARD TRUST COMPANY

Pay to the order of —— *Henry Newton* ——

Four Hundred $\frac{00}{100}$ —— *Dollars*

$\$400\frac{00}{100}$ *John Doe*

NOTE. In this instance, John Doe desires to make a payment of $400 to Henry Newton. Doe draws a check on the bank, in which he has money deposited, ordering it to make the payment to Newton. Newton may either take the check to the bank and cash it, after being identified, or he may deposit it with his own bank, after endorsing it, and the bank will collect payment from the drawer's bank. The latter bank will charge the amount against Doe's deposit.

The checks of others, received in the course of business are deposited to the credit of the depositor's account, as in the case of actual cash. It is estimated that ninety per cent. of all business transactions involving money settlements are settled by checks.

In commercial banks, checks may be drawn against a deposit at any time, but they must not exceed, at any time, the depositor's balance. Checks are drawn payable (1) "To ————, or order"—in this case, being negotiable; (2) "To Cash"—as a rule payable to the depositor himself; (3) "To Bearer"—payable to anyone presenting the check; or (4) "To ————," payable only to the person named. Depositors desiring to draw money for a specific use, as for example, to pay wages, may draw checks to the order of the particular expense, thus "Pay to the order of Pay Roll." Such checks are cashed by the depositor, or after endorsement, by his special representative. In some localities this form of check is now forbidden.

As in the case of notes (pp. 307, 308) checks may be transferred by endorsement to others before being finally cashed or deposited.

Business men deposit in their own banks, practically in the

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same manner as cash, the checks received by them from others and drawn on local or out-of-town banks. The banks receiving these checks on deposit must collect payment from the banks on which they are drawn. To avoid the necessity of sending messengers daily to all other local banks on which they hold checks, to make collections, banks form local *clearing houses* at which representatives of each bank meet daily at a specified hour. Here, in a systematic way, all checks drawn against a given bank, and brought by representatives of other banks are charged against it, and all checks it presents against other banks are credited to it. Balances are then struck. In each case where the debits exceed the credits, the bank concerned pays the debit balance to the clearing house officials before a certain time. Immediately after the receipt of this money from the banks that are said to owe the clearing house, it is paid out to the banks that have a credit balance. The clearing house thus begins and ends the day without money. By means of this institution balances of millions are sometimes settled by the actual payment of only a few thousand dollars.

ILLUSTRATION. Rosslyn has three banks. On a certain day their representatives meet. The First National Bank presents $12,000 in checks against the Second National Bank, and $18,756.42 against the Third National Bank. The Second National Bank presents $19,540 in checks against the First National Bank and $15,126.10 against the Third National Bank. The Third National Bank presents $14,118 in checks against the First National Bank and $11,121.70 against the Second National Bank. The Clearing House credits each bank with the checks it presents and charges it with the checks drawn on it and presented by other banks, thus:

Bank.	Charges.	Credits.	Dr. Bal.	Cr. Bal.
First Nat. Bank	$33,658.00	$30,756.42	$2,901.58	
Second Nat. Bank	23,121.70	34,666.10		$11,544.40
Third Nat. Bank	33,882.52	25,239.70	8,642.82	
			$11,544.40	$11,544.40

The Clearing House collects the amounts of the debit balances against the First and Third National and pays them over to the Second National Bank.

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The checks that each bank receives from the clearing house are taken back by its messenger and charged against the accounts of the persons drawing them.

Each bank receives, with other deposits, many checks drawn on out-of-town banks. It may collect these by mail, direct from the banks on which they are drawn, or, as is the common custom, it may collect them through its *correspondent* bank or banks in other localities, commonly in the larger cities, with which it keeps accounts. In the same way, banks in the villages and towns of its neighborhood, and banks of other cities, may keep accounts with it.

ILLUSTRATION. Peter Morton draws a check on his bank, The Idaho National Bank of Washington, in favor of James Rawson of Newark, N. J., and sends it to the latter in settlement for a purchase of merchandise. James Rawson deposits the check to the credit of his account in his home bank, the First National. The First National Bank credits Rawson and sends the check to its correspondent bank nearest Washington, the Fortieth National Bank of Baltimore, charging that bank. The Fortieth National Bank credits the Newark bank, and sends on the check, properly endorsed, to its correspondent bank in Washington, say the Twentieth National Bank. The Twentieth National Bank credits the Fortieth National Bank of Baltimore, and exchanges the check with the Idaho National Bank through the Clearing House. The Idaho National Bank charges the check against Morton's account.

Thus it is evident that all the checks a depositor draws against his account, and gives out in payment, either locally or at other points, find their way back to his own bank and are charged against his account. Sometimes they take very roundabout paths back, but the whole history is shown by the endorsements on the back. It will be found interesting to study the endorsements of cancelled checks.

Of course, any two correspondent banks are continually exchanging checks for collection, and their debit or credit balances with each other are constantly changing. At more or less periodic intervals, however, these balances are settled in full or in part by transfers of actual currency.

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Collection Fees. Some banks charge for out-of-town collections made for other than their depositors, and sometimes for collections for their depositors. The fee (rate of exchange) for such collections, while still more or less variable, is becoming steadily more uniform, especially in the East, being frequently only 1/10%. Sometimes there is a minimum charge of 10 cents or 25 cents.

Bank Drafts and Cashiers' Checks. It frequently happens, owing to the risk of loss, that objection is made to receiving the check of a private depositor. In such cases the payer may have his check certified by an official of his bank on which the draft is drawn. The certification is stamped across the face of the check and signed by the cashier or other bank officer. The bank thus guarantees the check and protects itself by immediately charging the check against the depositor's account. Such checks are more readily accepted. In case, however, a certified check is not used, it should not be destroyed, but should be deposited to the credit of the account against which it was originally drawn.

The payer may also make settlement by means of a *bank draft*, purchased at his own bank, or at any other that will accommodate him, and drawn on some correspondent bank either in the neighborhood of the payee, or in some large central city. Exchange on New York or Chicago is acceptable anywhere. These drafts are often drawn to the order of their purchaser in order that he may endorse them over to the intended payee and thus secure proof of payment. The payee may deposit them or transfer them by endorsement, the same as checks. The charge (rate of exchange) is usually the face plus a slight premium, generally 1/10%, although a bank will often issue drafts to its regular customers at a cost equal to the face.

If the correspondent or paying bank has funds to the credit or issuing bank, the drafts are charged against these. If not,

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the issuing bank will send bank checks to its correspondent for collection, or will give the bank credit and balance up later. As a rule, no charge is made for the collection of bank drafts when deposited for that purpose, if they are drawn on banks at the large money centers.

FORM OF BANK DRAFT.

No. 6129 *Seattle, Wash., Mar. 2, 1912.*

JOHNSON, MORTON AND CO.

Bankers

Pay to the order of.....*James F. Porter*..... .....826\frac{45}{100}$

Eight hundred, twenty-six $\frac{45}{100}$ Dollars.

To Fifteenth National Bank, *Andrew Norton,*

St. Paul, Minn. Cashier.

FORM OF BANK DRAFT.

In place of a bank draft, an ordinary cashier's check on his own bank may be obtained by a depositor, its cost being charged against his account. Such checks, being official in character, and having the weight of a bank back of them, pass more readily among strangers than do private checks.

Certificate of Deposit. Instead of opening a regular checking account, one may deposit money in a bank and receive therefor a *certificate of deposit*, transferable by endorsement. This form is often used when some fixed sum of money is to be deposited for safe-keeping. Some savings banks and trust companies will issue certificates for fixed periods, such as one year, bearing interest at a certain rate, usually higher than that allowed on deposits subject to check.

A CERTIFICATE OF DEPOSIT.

6000\frac{00}{100}$ CHICAGO, ILLINOIS, June 16, 191—

NATIONAL BANK

This certifies that *Thomas Medway* has deposited

Six Thousand $\frac{00}{100}$ Dollars,

payable to the order of.....*C. T. Robinson*..... . ...

on the return of this certificate properly indorsed.

Martin Kent,

Cashier.

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Commercial Drafts. Ordinary commercial drafts are frequently used for settlements and collections.

825\frac{67}{100}$ COLUMBUS, OHIO, Sept. 19, 191—

.....*At sight*.....Pay to the order of

.....*Ourselves*.....

....*Eight hundred twenty-five* $\frac{67}{100}$_____Dollars..

Value received and charge to the account of

To *Thompson Bros.* *Lewis Mark & Son.*

No. 67 *Baltimore, Md.*

NOTE. Thompson Bros. owe Lewis Mark & Son $825.67. When the account is due Lewis Mark & Son draw a sight draft on Thompson Bros. and give it to their home bank to collect. The Columbus bank sends the draft to its correspondent in Baltimore. The bank in Baltimore presents the draft to Thompson Bros. for settlement. When payment is received the Columbus bank is notified and it credits Lewis Mark & Son's account for the amount less a small fee (collection and exchange).

Time or sight drafts are used frequently to collect for merchandise at time of delivery. Goods are shipped to a customer and a through bill of lading is taken from the transportation company. This is a receipt for the goods and an agreement to deliver them to the consignee, or to his order. The selling firm also makes out a sight draft on the customer, if the terms are cash on delivery, or a time draft, if credit for a certain period is to be allowed. The draft and bill of lading are fastened together and given to the seller's bank. The bank endorses the draft and sends it to its correspondent bank in the neighborhood of the purchaser. This bank presents the draft to the customer for payment, if a sight draft, or for acceptance if a time draft. When payment or acceptance has been made, the bank surrenders the bill of lading and the customer may secure his goods. Time drafts frequently are left in the bank for presentation again when payment becomes due.

Rates of Exchange. Postal and express money orders cost more than their face value. Bank drafts frequently sell at, or above, their face value. Bank and commercial drafts

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may sell for less than their face value. Exchange is *at par* if the cost of a draft equals its face; it is *above par,* or at *a premium,* if the cost exceeds the face; it is *below par,* or at *a discount,* if it sells for less. Exchange on small sums is usually at a premium to repay cost of handling. The rate of exchange varies with the demand for money at certain centers of trade and with the cost of shipment of actual currency.

ILLUSTRATION. In the harvest season for grain, eastern purchasers are sending checks west in settlement for purchases. Suppose, for example, that large orders are being received in Chicago from New York customers. Checks and drafts drawn on New York banks and presented or deposited in Chicago banks will tend to seriously decrease, or to more than exhaust, the balances to the credit of the New York banks. At such times exchange on Chicago will be selling at a premium in New York, because the New York banks selling such exchange will be under the necessity of shipping actual currency to Chicago to meet the drafts.

At the same time, however, Chicago purchasers will find exchange on New York below par, because Chicago banks have so much due them from New York, that drafts on the latter city may be met easily without the transfer between the cities of the actual currency. The sale of such drafts really benefits Chicago by lessening the balance due from New York to Chicago.

In spring, when manufactured and imported goods are going out from New York all over the country, and merchants all over the country owe New York dealers and manufacturers, conditions are reversed. There is such a demand for exchange on New York that the rate of exchange is almost sure to go above par.

The rate of exchange seldom if ever goes above the rate for shipping the actual money. If the cost of shipping money between two given cities is \$10 per \$10,000 the rate of premium will not go above 1/10%.

Collection rates for drafts are somewhat arbitrary, being governed by general trade conditions. As a rule, however, bank drafts on the large money centers are collected without charge.

EXERCISE.

1. Secure a bank deposit slip. Find out how to make a deposit. Explain the process of opening an account and making the original deposit.

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2. If you owed a man in Albany a payment of $760, which would probably be the cheapest safe way of making settlement?

3. C. Y. Portner collects through his bank, at a charge of 1/10% for collection, checks on New York for $456.89 and $887.50 and on Philadelphia for $1,456. What is the net sum to be credited to his bank account?

4. Extend the following form showing collections made by one bank for its correspondent bank. Collection 1/10%.

FORTIETH NATIONAL BANK.

————, N. Y., September 15, 191—.

MR. ROBERT T. BROWN, Cashier.

Washington, D. C., Ninth National Bank,

Dear Sir:

We have credited your account this day for the proceeds of the collections stated below.

JAMES FADELEY, *Cashier.*

Your No.	Payer.	Amount	Collection.	Proceeds.
456	Robert Chase	480 00	....	....
461	A. B. Nortins	1239 80	....	....
511	Bruce & Co.	2311 45	....	....
512	Henry Ames	477 62	....	....
589	Besley Bros.	5465 00	....	....

5. Banks A, B, C and D form a Clearing House. On Sept. 16, Bank A presents checks against Bank B for $12,178.80; against Bank C for $4566.75; against Bank D for $56,124.59. Bank B presents checks against Bank A for $45,000; against Bank C for $34,123; against Bank D for $19,234.59. Bank C presents checks against Bank A for $13,543; against Bank B for $24,118.25; and against Bank D for $7,188.55. Bank D presents checks against Bank A for $12,325; against Bank B for $35,455.60 and against Bank C for $41,157.20. Find the debit or credit balance of each bank with the Clearing House and determine what payments must be made to settle all balances.

6. What is the net sum placed to the credit of John Grandfield on deposits of checks for $435, $350, $527.50 and $56.88, on which a collection fee of 1/8% is charged?

7. Compute the cost of a bank draft on Richmond, Va., for $723.90, at 1/10%.

8. How large a draft may be purchased for $1127.13 at 1/10% premium?

9. Determine the proceeds from the collection through an express company of an account of $2,356, at a rate of $1.75 per $1,000.

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10. Draw up a check on an assumed bank to pay for a bank draft of $2264 at 1/5% premium.

11. Robert Gaines sold H. M. Chilton a bill of goods amounting to $1764.80. Gaines drew a draft in settlement and sent it, with the bill of lading for the goods, through his bank. What sum was added to the credit of his account when the bank had collected the draft, its charge being 1/5%?

12. A. C. Crandall sold a bill of goods for $7500 to George Grimes, the terms being "30 day acceptance." Crandall drew the draft and sent it through his bank for acceptance. After acceptance, the bank discounted the draft for Crandall at 1/10% and 6% interest. What proceeds did Crandall receive?

13. A commercial draft for $475 cost $474.05. Was exchange at a premium or at a discount? At what rate?

14. A. M. Beardsley drew a draft on John Evans, for $467.56 and collected it through his bank at a rate of collection of 1/20%. What sum should be credited to his account?

15. Was exchange at a premium or at a discount, and at what rate, if a draft for $23,450 cost $23,567.25?

16. Boston is selling drafts on New Orleans at 3/4% premium. Which way is the balance of trade? What is the cost of a draft on New Orleans face value of $1245?

17. Suppose the balance of trade between Indianapolis and Pittsburg is largely in favor of the latter. What does this mean? In which city is it likely that exchange will be at a premium? Why? At a discount? Why?

18. Name several cities that might have a balance of trade in their favor in the spring. In the fall. Give your reasons.

FOREIGN EXCHANGE.

All forms of payment possible under domestic exchange are available for payments abroad. Theoretically, foreign exchange is based on the same general principles as domestic exchange. In practice, however, all computations are affected by differences in currency denominations and standards, and by methods of quoting exchange. From the banker's standpoint, foreign exchange is, in many ways, the most intricate of all forms of banking.

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Rates of Exchange. Foreign rates are variously expressed, but may be broadly classified under three heads.

1. Rates per foreign coin. Thus a London exchange rate of "4.875" means that one pound sterling can be bought for $4.875. A rate of "40⅜" on Amsterdam means that one guilder exchange on that city costs $.40⅜.

2. Rates on a set number of coin. Exchange on German cities is usually quoted "per four marks." Thus a quotation of "96," on Berlin, means that a four marks bill of exchange costs $.96.

3. Rates in "coin per dollar." This form of exchange is often quoted for France and other Latin countries, as Spain, Italy, Switzerland, etc. Thus a rate of 5.15⅞ on Paris means that a draft on Paris for 5.15⅞ francs may be purchased for one dollar.

NOTE. Form (1) is sometimes used for exchange on any country especially in connection with some forms of money orders.

ILLUSTRATIONS.

(1) Compute the cost of a £450 draft on London, purchased at 4.875.

Solution.

$4.875
450
243750
19500
$2193.75, Ans.

Since £1 cost $4.875, £450 will cost 450 × $4.875.

(2) Compute the cost of a draft on Berlin for 1200 M., purchased at 96.

Solution.

4)1200
300, no. sets 4 M.
$0.96
300
$288.00 cost.

In 1200 M. there are 300 sets of 4 M.
If one set costs 96c, 300 sets cost 300 × 96c.

(3) Compute the cost of a 12,000 fr. draft on Paris, purchased at 5.15.

Solution.

$ 2330.09
5.15)12000.
1030
1700
1545
1550
1545
5000
4635

Since 5.15 francs cost $1.00, 12,000 francs will cost as many dollars as 5.15 is contained times in 12,000.

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Foreign Money Orders. The International Postal Money Orders and the express orders, which are similar to domestic money orders, are issued for amounts not exceeding one hundred dollars, and are usually payable in the money of the country on which they are drawn. Fees for this service are small.

1. What will an express money order for £5 6s. cost at $4.87?

2. How large a draft on London will $50 buy, at the same quotation?

3. Robbins wishes to send a friend in Rome an order for 290.5 lira. At 19.5c per lira, the order costs $?

4. For $40 one may buy an order on Berlin for ? M., if the rate is 24c per M.

NOTE. Money payments may be made by cable in the same way in which they are made by telegraph. In cable orders, a charge is made for each word of the address.

5. At 20c per word, and 1% of the amount, find the cost of a 24-word cable order on London for £600, at a rate of exchange of $4.8725.

Travelers' Checks. Travelers may secure from express companies and banks travelers' checks, which can be cashed

FORM OF TRAVELER'S CHECK.

at hotels and banks abroad. They are issued for fixed amounts and state on their face their value in the coinage of the different countries in which they may be presented for "cashing."

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The purchaser signs his name on the face at the time of purchase, and again on face or back, for purposes of identification, at time of cashing. The checks are sometimes issued in books or series.

Letters of Credit. A traveler's letter of credit is an instrument issued by a bank to its representatives abroad, authorizing them to honor drafts of the person named in the body of the letter, to total not exceeding an amount stated.

The person desiring a letter of credit deposits with the issuing bank money or bonds as security. On receiving the letter of credit, he places his signature on the face, and also gives the issuing banker other specimens of his signature, which are forwarded to the correspondent banks with the formal notice of the issuance of the letter. Once abroad, whenever the holder of the letter desires money, he stops at any correspondent bank named in the letter and presents his personal draft for the amount desired. After the signature has been verified, he is paid the amount in the coinage of the country. This amount is indorsed, also, on the letter of credit. When the "credit" called for has been exhausted, the letter is taken up by the bank making the last payment, cancelled, and returned to the issuing bank.

EXERCISE.

1. Determine the cost of a 12,000 franc letter of credit, at 5.18 and 1% commission.

2. On a £600 letter of credit, are endorsed payments of £120, Nov. 10; £50, Dec. 5; £180, Jan. 16; £125, Feb. 12. Purchased at 4.86 and 1% commission, and unused portion redeemed at same rate, what repayment should be made to the holder?

Commercial Letters of Credit. These letters are issued to mercantile houses desiring to import goods to enable them

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to get credit from exporters for the usual periods of 30, 60 or 90 days. On proper evidence of the shipment of goods, a foreign bank at point of shipment, will buy the shipper's draft against the person named in the letter, at the prevailing rate of commercial exchange, reimbursing itself by sending the draft to party drawn on for collection, or by selling the draft in the open market. Credits are issued usually in pounds sterling because such exchange is easily negotiated in any part of the world. They are sometimes issued payable in the money of the country to which they are sent. Commercial credits are issued in four parts, one being sent to the exporter, one to the bank named in the credit, and one to the importer, or purchaser, the fourth being retained. Prices vary with credit term, financial responsibility of applicant, and many other conditions. The problems of computation are the same as those involved in other draft transactions.

Bills of Exchange. Drafts drawn by a person or bank in one country on a person in another country are termed *bills of exchange.* They include bankers', commercial and documentary bills. Bankers' bills are a bank's checks, drawn against its deposits in foreign banks. Commercial bills are those drawn by one merchant against another. Documentary bills are commercial bills, secured by bills of lading, insurance policies, or other documents that control the merchandise in transit. Bills of exchange were formerly issued in triplicate. They are now issued, as a rule, in duplicate, so worded that the payment of either one makes void the other. Sometimes the ordinary single draft is used. Owing to delays and dangers of transit, the duplicate drafts were formerly sent by different routes or mails. Now the tendency is to keep the duplicate on file.

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A DUPLICATE BILL.

PHILADELPHIA, PA., April 19, 19—

2450 M.

2 *At sight* of this SECOND of Exchange (first unpaid), pay to the order of.....*Ourselves*.....
Twenty-four hundred and fifty Marks,.....
Value received and charge to the account of
To *K. R. Schmidt*
Berlin

S. T. COLES & Co.

SIMPLE FOREIGN BILL.

No protest tear this off before presenting.

£1000— Philadelphia, *March 12th*, 190—
.....*On demand*.....Pay to the order of
John J. Doe.....
.....*One thousand*_____Pounds sterling
.....
Value received and charge to account of
To *J. S. Morgan & Co.*
No. 16 *London, England.*

John Smith.

Fluctuation of Exchange. The rate of exchange is affected by numerous causes. If exported goods greatly exceed imports in value, the rate of exchange tends to lower because there is less demand for remittances abroad. If more money is due abroad than is due the home country, the rate tends to rise. When money is in demand here, and rates rise, rates of exchange may fall because there is less demand for bills. If money rates are high abroad, exchange rates may rise because of the demand for bills in order to send money to a place of good investment. In general, the question of supply and demand seriously affects the quotations.

The discount rates on commercial bills, at foreign money centers, also affect the rate of exchange. If rates of discount are low, commercial drafts may "cash" for a greater sum than could be realized by selling them as exchange. The question of the parity of units of foreign coinage, as compared with the units of our own coinage, also has its effect. The exchange rate will seldom go beyond the limits of the cost of buying

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and shipping the actual gold—these costs including commission, freight, insurance, and often interest lost while money is in transit. That is, the rate of exchange does not go beyond the point where it is cheaper to send the actual money.

Newspaper quotations are given, usually, for both sight and time paper. *Par of exchange* is either (1) *mint par*—the equality of actual value of pure metal in units of money in the countries concerned; or (2) *commercial par*—a rarely used term referring to a practical equality of debt between countries that reduces the rate of exchange to the face value plus the cost of shipping.

ILLUSTRATIONS OF EXCHANGE.

1. *A*, in New York, sells *B* in London merchandise, $600. *A* draws his draft on *B*, in pounds sterling. He may then discount the draft at a bank; or he may sell it in open market to some party who wishes to make a payment in England. In the former case, the bank either sends the draft to England for collection—in which case the money is probably held by the correspondent bank in England, and the home bank draws and sells its bank exchange against the sum, and like sums, to any purchaser—or it may sell the original draft to a customer. In case *A* sells the original draft direct, or through a broker, the party buying it, *C*, who may owe *D* in England, sends the draft, properly endorsed, to the latter, who presents it to *B* for payment. By such a general process, it is evident that *A* gets his payment at once, and that sooner or later *B* pays the draft on presentation and thus settles for the goods that he bought. Incidentally, the bill has served the further purpose of making settlements between other parties.

2. In case *A* had been the buyer and *B* the seller, *B* might carry through the same process shown above. The settlement might start with *A*, however, who might go to a bank or broker, and buy a draft on England for the amount due, sending it to *B* who would cash it. In this case *A* corresponds to *C*, in ex. (1).

Notice, in each case, that this process permits each person to make or to receive settlement in the money of his own country.

ORAL EXERCISE.

1. In what coinage would an American importer order of a French exporter? If he bought exchange, with which to make settlement, in what coinage would it read? With what money would he make settlement for the draft?

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2. An English dealer orders goods from the catalogue of an American manufacturer. He finds prices quoted in what coinage? If he pays by bank draft, the draft is made out in what money? If he is drawn upon, in settlement, the draft is made out in what money? If the manufacturer sells the draft in New York he will receive money of what country?

How would you determine:

3. The cost of a draft on Paris, at 5.19?
4. The cost of a draft on London at 4.88?
5. The cost of a draft on Berlin at .958?
6. The proceeds from the sale of a draft on Lyons at 5.185?
7. The proceeds from the sale of a draft on Amsterdam at 40¼%?
8. The proceeds of a draft on Berlin at 96¼?
9. Which are the higher rates, 5.19 or 5.20? 96 or 95¾?

EXERCISE.

(For Mint exchange rates, see page 281.)

1. What is the face of a draft that an English exporter should draw on a New York customer, at mint par, to cover the purchase of 240 yd. broadcloth at 0/14/6 per yd.? If the exporter sells this draft in the market for 4.87, what proceeds does it yield him?

Compute the cost of drafts on:

2. Paris, for 9700 francs at 5.18
3. Berlin, for 12,500 M. at .962.
4. Glasgow, for £161 4s., at 4.87¼.
5. Amsterdam, for 752 g., at .405.
6. Rome, for 768 L. at .238.
7. Geneva, for 1284.6 fr., at 5.18¾, brok. 1/8%.
8. Is the balance of trade probably for or against this country if exchange is quoted at 5.2, 4.88, .412?

EXCHANGE RATES.

(From a daily newspaper.)

	Sight.	60 days
Cable transfers	4.87¼	.....
Bankers' sterling	4.87	4.85½
Commercial bills	.....	4.84½
Francs	5.16¼	5.17½
Reichsmarks	.95¾	.95¼
Guilders	.40½	.40¼

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NOTE. Use the newspaper quotations on page 373 in the following:

9. James Bros. & Co. purchase an invoice of £2000 worth of woolens, paying by 60-day bank draft. Cost of draft?

10. How large drafts would $500 buy at each of the above quotations?

11. A. B. Norton drew on a Paris customer a draft for 12,500 fr. and sold it at 1/8% commission, receiving what amount?

12. What does it cost to cable an agent £120?

13. B. C Baker shipped S. Tournier, Paris, 5000 bu. wheat, invoiced at 4.8 fr. drawing a 30 day draft on the purchaser. This draft, accompanied by bill of lading and insurance certificate, he sold to Mason and Dean, private bankers, at 5.19, and should receive what sum?

14. Mason & Dean sold the draft just mentioned to Robert Bros., at 5.18, receiving $—, and making a profit of $—

15. Robert Bros. mailed the same draft, in settlement of account, to a Paris customer, who discounted it at a bank at 1%, receiving — fr. The bank presented the draft to Tournier (ex. 13) who was allowed to settle for it instead of accepting it, at 1/2% discount. He then received the bill of lading that entitled him to the wheat.

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CHAPTER XLI.

DEPRECIATION.

Depreciation is the loss of value of any property due to wear, new inventions, exhaustion of product, or other causes. It is the difference between original value and marketable value at time of disuse.

ILLUSTRATIONS. (1) In spite of repairs, a factory or piece of machinery deteriorates to the point where it must be discarded or rebuilt. This deterioration is *depreciation*. (2) New inventions or improvements may make old machinery valueless. (3) A lessening of demand for a product may cause the disuse of machinery, and a consequent lessening in value. (4) A mine deteriorates from exhaustion of its ore, etc.

To avoid the necessity of meeting the entire cost or replacement in any one year, it is customary in large concerns, to allow for depreciation, setting aside each year during the probable useful life of the property, sufficient funds to equal the cost of replacement when necessary. The sum set aside is largely a matter of scientific guess-work based on past experience. It is considered a running expense and may amount to 30% of such expenses. Allowing for individual variation, the following table shows common rates.

ANNUAL RATES OF DEPRECIATION.

Modern fireproof steel and tile factories	4% to 5%.
Steel construction, partly fireproof	6%.
Warehouses, modern, fireproof	2½% to 4%.
General freehold buildings, office and general use	1½% to 3%.
Cheaply constructed building, according to use	8% to 15%.
Fixtures of power and lighting plants	12% to 15%.
General fixtures of buildings, other than machinery	6% to 15%.
Machinery (average 11½%)	8% to 15%.
Horses, work	15% to 25%.

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Annual amounts of depreciation are computed (1) as a fixed proportion of the original value; (2) as a fixed per cent. of reduced values; or (3) by the sinking fund method.

1. Based on Original Value.

Illustration. A machine in a cotton mill, costing $560, has an average working life of 10 years, when it is disposed of for $120. Determine the annual allowance for depreciation.

Solution. Depreciation = Original cost − disuse value = $560 − $120 = $440. Since this must be covered in 10 years the annual depreciation is 1/10 of $440, or $44.

EXERCISE.

1. Compute the annual depreciation at 2¾%, on a fireproof warehouse costing originally $364,200.

2. Compute the annual depreciation on $340,200 worth of fixed machinery, at 12%, and on tools originally worth $3640, at 18%.

3. A department store owner has 40 delivery horses originally costing $5680. He reckons the depreciation, at 22½%, as $——.

4. A boiler, costing $520, is estimated to have a life of 12 years, and then to be worth $60. Estimate annual depreciation and rate of depreciation.

5. A contractor employs a $14,500 dredge in contract work. In computing cost of work, as a basis for bids, he includes interest on the cost of the dredge and depreciation. He reckons interest at 6% per annum and depreciation 1¾% per month. What should he allow for interest and depreciation on a contract that he estimates will take 7½ months?

II. Based on Reduced Values.

With machinery, for example, repairs grow heavier with age. Hence, with uniform depreciation and increasing repairs, the working cost is heaviest when the machine is least useful. To equalize the yearly cost therefore, depreciation is often heavier when repairs are light, decreasing as repairs grow heavy. Depreciation of this type is often expressed as a fixed per cent. of the net value of the previous year.

Illustration. Compute the third year's depreciation, at 10% on reduced values, of machinery costing $900.

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Solution.

$900	Orig. value
90	Depreciation at 10%.
810	Net value, end 1st year.
81	Dep. 10%.
729	Net value 2d year.
72.90	Dep. 10%, for 3d year. Ans.

NOTE. If term and rate of depreciation are known, tables similar to the following may be computed for shop use.

TABLE OF DEPRECIATION PER $100 OF ORIGINAL VALUE.

Term Year.	2%	3%	4%	5%	10%	15%	20%
1 yr.	2.00	3.00	4.00	5.00	10.00	15.00	20.00
2	1.96	2.91	3.84	4.75	9.00	12.75	16.00
3	1.92	2.82	3.69	4.51	8.10	10.84	12.80
4	1.88	2.74	3.54	4.29	7.29	9.21	10.24
5	1.85	2.66	3.40	4.07	6.56	7.83	8.19
6	1.81	2.58	3.26	3.87	5.90	6.66	6.55
7	1.77	2.50	3.14	3.68	5.31	5.66	5.24
8	1.73	2.42	3.00	3.49	4.78	4.81	4.19

EXERCISE.

1. Compute the depreciation for the sixth year, at 8% on reduced balances, on fixtures originally worth $8240.

2. A factory costing $82,600.00 is five years old. What is its inventory value, if depreciation is reckoned on reduced balances at 4½%?

By Table.

3. Reckon the depreciation for the eighth year, on machinery worth originally $6525, if the depreciation rate is 10%.

4. What is the total loss of value in seven years, of a boiler costing $1800, and depreciated 15% annually?

5. Compute the inventory value at the end of 8 years, of an equipment of cotton mill machinery, costing originally $38,250, and depreciated at 20% annually.

6. Determine a rate of depreciation for machinery that is expected to depreciate from $900 to $630 in 7 years.

III. SINKING FUNDS.

Depreciation is sometimes provided for by establishing a sinking fund, that is a fund which is invested at compound interest at the rate necessary

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to provide for renewals. Frequently this fund is based on annual investments of equal amounts. Business establishments seldom establish a sinking fund, since, when prosperous, funds earn a higher rate by being continued in the business, than by outside investment.

SINKING FUND TABLE.

Showing Accumulations at Compound Interest of Annual Investments of One Dollar.

Year	At 4%	At 5%	At 6%	Year	At 4%	At 5%	At 6%
1	1.040000	1.050000	1.060000	11	14.025805	14.917127	15.869941
2	2.121600	2.152500	2.183600	12	15.626838	16.712983	17.882138
3	3.246464	3.310125	3.374616	13	17.291911	18.598632	20.015066
4	4.416323	4.525631	4.637093	14	19.023588	20.578564	22.275970
5	5.632975	5.801931	5.975319	15	20.824531	22.657492	24.672528
6	6.898294	7.142008	7.393838	16	22.697512	24.840366	27.212880
7	8.214226	8.549109	8.897468	17	24.645413	27.132385	29.905653
8	9.582795	10.026564	10.491316	18	26.671229	29.539004	32.759992
9	11.006107	11.577893	12.180795	19	28.778079	32.065954	35.785591
10	12.486351	13.206787	13.971643	20	30.969202	34.719252	38.992727

EXERCISE.

1. What is the sum that must be invested annually in a 4% sinking fund to provide for replacing a $12,000 equipment of machinery in 10 years, if the estimated value of the equipment at the end of the period is $1950?

2. What allowance for depreciation should be made annually for machinery that is expected to deteriorate in value $6750 in 5 years, when it must be replaced? The sinking fund is invested at 6%. What at 5%?

3. A certain factory building is estimated to have a life of 20 years. It is built at a cost of $46,000. Estimate uniform annual depreciation. Compute annual allowance for sinking fund at 6%.

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CHAPTER XLII.

COST-KEEPING.

Knowledge of the exact cost of the articles that he makes is of vital importance to a manufacturer. Owing to slight margins of profit, carelessness in estimating materials and labor, or needless waste, may lead to large losses.

The determining of the exact cost of an article, or piece of work, is termed *cost-keeping*. It necessitates (1) the exact measurement of the material used and wasted, (2) the computing of the time spent on the work by each workman, and (3) the computation of the exact cost of material and labor. The labor cost is often greater and more variable than the material cost, and is considered the vital element in cost-keeping.

NOTE. The computation of the cost of production is often exceedingly involved, and varies with each class of work, but the fundamental principles are very simple and are worth considering, since the subject is becoming a matter of general discussion, and of frequent mention in current publications.

ILLUSTRATION. A printing office receives an order for 2000 booklets of a specified character. At once a duplicate docket (record sheet) is prepared (page 380) to show full particulars of cost. The original docket accompanies the work from start to finish, quantities and costs being entered at each stage, as determined, the form being completed when the work is finished. Each workman keeps his separate ticket, and marks on it the hours that he spends on each job. These tickets are turned into the office at night, and the time and labor costs transferred to the duplicate docket here on file. Labor costs are also summed up and entered on the original docket as it passes from hand to hand. The duplicates thus serve as checks. Each job, however, must bear its share of the expense of the office force, foremen, etc. (the non-productive labor), and its share of rent, wear on machinery, lighting, etc. (burdens). This expense, after long experience, is expressed as a per cent. of direct cost, or of labor cost only. To this, if no advance price has been quoted, is added the per cent. of profit. The completed cost cards are filed to serve as a basis for estimating on future work. Errors, carelessness, and excessive cost of labor, may also

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be traced from them. Many improvements are suggested by a study of cost cards.

JOB WORK DOCKET

B 4/30 — James C. Norton — Nov. 19, 1912

DUE 11/26 — 726 F St., N. W.

DESCRIPTION

2000 booklets, 6 x 9, 16 pp.
L. P. Type — Black Ink
Cover two colors, wire stitched

CHARGED Nov. 26, 1912 — AMOUNT $75.00

			ITEMS		TOTALS	
STOCK SUPPLIED WITH DOCKET						
2 R 2 Q S. No. 3 25×37 per Rm.		$ 4.80	10	08		
1 R 0 Q 12 S R. 16 Green		$10.00	?		?	
R Q S		$				
R Q S		$				
COVERS		$				
TICKETS FROM WORK — PRINTING COST	COMPOSITION		19	08		
	PRESSWORK		7	21		
	INK		1	60	?	
BINDING COST	LABOR		4	18		
	MAIL AND POSTAGE		5			
	STOCK		2	60	?	
OUTSIDE WORK COST	2 Electros 1.40				2	80
			TOTAL COST		?	

FOREMAN PRINTERY — FOREMAN BINDERY

EXERCISE.

1. Extend the job work docket, shown above.

2. Compute the cost of a box requiring 2 pc. wood, 1×6×8; 2 pc. 1/2×6×12, and 2 pc. 1/2×8×12; waste 4%. Cost of material, 2½c per ft.; labor, sawing, etc., $.006 per ft.; assembling $.025 per bx.; indirect expense, 20% of labor. (Reckon values to four decimal places.)

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3. Complete the following card.

NO. PIECES	NAME OF ARTICLE	SHOP ORDER
600	B — # 678	7306

ORDERED	BEGUN	FINISHED
Jan. 27, 1912	Jan. 29	Feb. 6

DEPARTMENT	OPER.	HOURS	RATE	COST		MATERIAL	COST	
Machine	Sawing	9	.15	?		1450' 1/4 Oak @ 2.00	13	05
"	Shaping	12	.12	?		Filler	4	38
"	Sanding	16	.09	?		Oil & Varnish	5	99
Finishing	Filling	12 1/2	.20	?		Waste		87
	Varnish'g	13	.25	?				
	Polish'g	15 1/2	.30	?				
						MATERIAL COST	?	
						*SHOP BURDEN	?	
						COST PER PC. MAT.	?	
						COST PER PC. LABOR	?	
LABOR COST — 600 PC.		?		?		COST PER PIECE	?	

*12 1/2% of labor

4. Complete the following card.

FORSBERG STEEL CO.

DATE Feb. 17, 1912

ARTICLE 3000 Sprockets NO. C — 6731

QUANTITY	OPERATION	TIME	RATE PER 100	NO. OF OPERATOR	
3000	Block	26h	.45	721	?
3000	Blank	32h	.52	483	?
3000	Finish	65h	.90	521	?
	Coal, 18¢ per hr.	? hr.			?
	7000# steel			21/4¢	?
	Hammer, 123 hr.			1.18	?
	Punch 7 hr.			.80	?
	Press 7 hr.			.25	?
				Total	?
				+ 35%	?
				Gross	?
					?

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NOTE. In the form just shown, operations are paid for on the "piece" plan, and coal and use of machinery are charged by the hour, previous experiments having determined what is a fair charge.

The construction engineer, or contractor, estimates costs *per unit of finished work*, or *per day*, and holds these unit costs as a basis for future estimating.

EXERCISE.

1. A gang of seventeen men, working on a concrete construction, averaged 46 cu. yd. of concrete per day. Estimate labor costs *per day* and *per cubic yard*, based on the following organization:

		Per day.	Per cu. yd.
5 men loading barrows,	at $1.75 per day,	$——.—	$——.—
9 men mixing and placing,	at $1.75 per day,	——.—	——.—
2 men tamping,	at $1.75 per day,	——.—	——.—
1 foreman,		3.00	——.—

2. A gang of men averaged 42 lin. ft. of concrete conduit (3/4 cu. yd. per lin. ft.) per day. Estimate the labor cost per day and per cu. yd.

12 men mixing, at	$1.80
4 helpers	1.50
6 loading stone and sand	1.50
6 wheeling	1.50
8 pouring concrete	1.80
2 supplying water	1.50
2 placing metal	2.35
1 sub-foreman	2.75
1 foreman	3.50

3. Estimate the unit cost per cubic yard for 260 cu. yd. of concrete, laid in seven 8-hour days.

MATERIALS AND LABOR.

0.85 bbl. cement, per cu. yd., at $2.90 per bbl.
.25 cu. yd. sand, per cu. yd., at $1.12.
.50 cu. yd. broken stone, per cu. yd., at $.80.
.40 cu. yd. rubble, at $.55.
Water, for the entire quantity, $8.00.
Labor, 9 men, at 18c per hour, each.
Foreman, $3.60 per day.

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CHAPTER XLIII.

BIDS AND ESTIMATES.

Those desiring work done, or material supplied, often advertise for sealed proposals, or bids, from such contractors as care to meet their requirements. These bids are based on *specifications* supplied by the advertiser, which state in detail the work to be accomplished, *how* it is to be done, the time of completion, the character of material to be used, the conditions as to the acceptance of the completed work, payments, etc.

The prospective bidder makes an estimate of the *cost* of the work, allows for his profit and prepares his bid in due form, usually on a blank supplied with the specifications. The sealed proposals are opened, frequently in the presence of all bidders who care to attend, on a fixed date, and the contract awarded. While the lowest bidder, if reliable, frequently secures the contract for the work, parts of the work *may* be given to different contractors. The proposal often names prices for parts of the work, or for the work as a whole. Usually the bid must be accompanied by a certified check for a certain percentage of the amount bid, as security for the acceptance and performance of contract, if tendered; or a bond must be given as security.

Model Proposal for a Gravel Road.

PROPOSAL.

To the Board of Chosen Freeholders, County of.....
and State of New Jersey:

Gentlemen—The undersigned hereby declare...that....he....ha.... carefully examined the annexed specifications and the drawings therein referred to, and will provide all necessary machinery, tools, apparatus and other means of construction, and do all the work and furnish all the material called for by said specifications in the manner prescribed by the

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specifications and the requirements of the Engineer and Supervisor under them, for the following prices:

(1) Price per cubic yard for earth excavation, without classification, as per plans and cross-sections, throughout the length and width of the road.....per cubic yard.

(2) Price per acre for grubbing and removing objectionable material from sidewalks.....per acre.

(3) Price per lineal foot for completed tile drain.....per lineal foot.

(4) Price per cubic yard for compacted gravel as specified.....per cubic yard.

(5) Price per cubic yard for carting gravel more than one half mile and each additional half-mile or fraction thereof.....per cubic yard.

(6) Price per cubic yard for stripping or removing earth from top of gravel bed.....per cubic yard.

(7) Price per square yard for each ordered inch in depth in excess of thickness named.....per square yard.

(8) Price (lump) for the whole road complete, according to the specifications and plans prepared by the Engineer.....

Accompanying this proposal is a certified check for the sum of one thousand ($1,000) dollars, payable to the order of the Director of the Board of Chosen Freeholders of.....County; which check is to be forfeited as liquidated damages, if, in case this proposal is accepted, the undersigned shall fail to execute a contract with said Board of Chosen Freeholders, under the conditions of this proposal, within the time provided for by the foregoing advertisement for proposals; otherwise said check is to be returned to the undersigned.

Signed.....

Address.....

.....N. J.,.....

In preparing his bid the contractor must allow for such things as:

1. Preparatory expense. Cost of preparing for the real work, such as setting up working machinery, clearing ground, etc.

2. Plant expense and supplies. Cost of maintaining tools, machinery, worksheds, etc. Supplies for equipment. Depreciation allowance.

3. Materials. Those used in the real construction. Freight and cartage.

4. Labor. Skilled and unskilled workmen.

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5. Superintendence and general expense. Wages of firemen, timekeepers, superintendent, clerical force. Supplies for their use. This expense varies from 5% to 15% of labor.

6. Contingencies. Allowance for unforeseen expense, such as delays from bad weather, labor troubles, accidents, rise in price of materials. Often in excess of 10% of cost.

7. Profit allowance. At times a fixed per cent. of cost. At times, one rate on cost of materials, say 10% to 15%, and a second rate, say 15% to 25% on labor. These rates are often made high enough to cover probable contingencies.

NOTE. Estimates of cost are based on current prices of labor, freight, materials, etc.: estimates of quantities, largely on past experience. Often the unit cost, as obtained by cost-keeping methods, is the basis for an estimate. The exercises that follow illustrate a few principles of the subject.

ORAL EXERCISE.

1. A bidder for a contract for office supplies believes that he can secure the material for $3800. He reckons freight at $87.50; labor at $32.50. Estimate his bid on a profit allowance of 20%.

2. Compute a bidder's profit of 15% on labor estimated to cost $380.

3. Compute a bidder's allowance for 12½% profits on materials costing him $1840.

4. A contractor estimates the gross cost of a prospective structure at $28,500. If he allows 20% for profits and contingencies, what should his bid be?

EXERCISE.

1. A contractor estimates the cost of materials, plant, etc., for a certain work, at $46,758, and of labor at $39,200. He adds 10% to labor for contingencies. He then reckons a profit of 20% on labor and 12½% on materials. What is the amount of his bid?

2. Gross estimated cost of materials, $15,286.50; plant expense, $3495; labor, $7296.40. Profit on materials, 15%; on labor, 25%. Amount of bid?

3. Reckon quantities of material, cost of labor and amount of bid, for a contract of 260 squares of slate roofing, basing values on these figures: (1) Unit cost per square, 214 pc. slate, $5.60; freight, $1.40; loading and hauling, 35c; slate waste, 2%; 16 lb. sheathing paper, 50c; 1 lb. nails, 5c; 2¼ lb. nails, galvanized 9c; labor, $1.70. (2) Superintendence, 10%. (3) Labor contingency, 10%. (4) Profit, on net cost of labor, 20%; on materials, 15%.

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4. Compute the face of bid on 14,365 cu. yd. concrete foundation, based on a unit cost per cu. yd., as follows: Stone, $1.15; gravel $.14; sand, $.08; cement, $1.36; labor, $1.05; superintendence, $.05. Profits on labor, 25%; on materials, 15%.

5. Compute bid (ex. 4), allowing a straight profit of 20% on cost per unit.

6. A contractor agrees to cut and supply 60,000 ft., B. M., of 3″ plank, basing this bid on the following cost per unit M ft.: stumpage, $1.75; cutting, $1.40; skidding, $.75; sawing, $2.40. Compute his bid, allowing 25% for profit and contingencies.

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CHAPTER XLIV.

PARTITIVE PROPORTION AND PARTNERSHIP.

Partitive proportion, one phase of ratio, is the process of dividing a number into parts proportional to two or more given numbers.

ORAL EXERCISE.

1. Compare 4 and 6.

2. A's 12 shares compare how with B's 4 shares? Together they have how many shares?

3. Divide \$320 into parts proportional to 2 and 3.

Suggestion. $2 + 3 = 5$ parts.
\$320 = 5 parts.
1 part = \$64.
2 " = ——?
3 " = ——?

4. Divide \$7200 in the ratio of 5 to 7.

5. Divide 1200 yd. into two lots having the ratio 3/5.

6. Divide 6300 lb. into parts proportional to 1, 2 and 3.

7. Divide \$1800 into parts proportional to 1/4, 1/3, and 1/6.

8. Divide \$600 between A and B, so that A receives 3 times as much as B.

9. Two dealers unite in ordering a carload of 40 tons of soft coal. The freight bill is \$24. If the first dealer, who bought 15 tons, settles the bill, what should the second dealer pay him?

10. A piece of freight is carried 200 miles on one railway, and 150 miles on a second, for a total charge of \$35.00. What should each railway receive?

A *partnership* is an association of two or more persons for the purpose of conducting a business in common, and of sharing in its profits or losses. Partnerships are formed under verbal or written agreements. The latter usually include a

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statement of the conditions concerning investments, withdrawals, and the distribution of profits.

Collectively, the persons forming a partnership are termed a *firm*, a *house*, or a *company*. Individually, they are known as partners. Partnerships may be conducted under fictitious names, under the names of the partners, or under the name of one or more partners "and Co." Partners are *real* or *ostensible*, if they are really partners and are known to the public as such. They are *silent* partners if their connection with the firm is not known to the public; *nominal*, if they make no investment and have no share in the proceeds of the business; or *limited* if they have only a partial legal responsibility for the debts of the firm. Real and nominal partners are responsible for all liabilities of their firm. Silent partners when their connection becomes known must assume their share of responsibility for the liabilities of the firm. Limited and silent partnerships are not allowed in some states; in others it is often required that at least one partner shall be *real*.

The *capital* of the partnership is the property invested in the business. It includes money, any tangible form of real or personal property, labor, good will, etc.

NOTE. *Good will* is the term applied to the value of past custom of an old established place of business, in bringing future trade, or of confidence in a partner who "lends his name," etc. Such a partner is often credited with good will as an investment, at some money value, on which his share of profits may be based.

The *net capital or present worth* of a firm at any time is the excess of resources or assets (property owned and debts due) over liabilities (debts owed). If liabilities are in excess there is *net insolvency*.

FOR DISCUSSION.

Compare partnerships and stock companies as to:

1. Method of organization.
2. Method of investing.
3. Withdrawal from business.
4. Sale of interest in business.
5. Effect of death of partner or stockholder.
6. Government and management.
7. Individual responsibility for debts.

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ORAL EXERCISE.

1. Total gains − total losses = ?
2. Total losses − total gains = ?
3. Resources ± ? = net capital.
4. Original capital + net gain = ?
5. Original capital − net loss = ?
6. Resources − liabilities = ?. How check?
7. May a firm gain during a period and be insolvent at its close?

EXERCISE.

Supply missing words or values. Solve mentally if possible.

1. Gains $2400; losses $560; net ——, $——.
2. Gains, $2150; losses $2680; net ——, $——.
3. Capital at commencing $16000; at close, $18500; net —— = $——.
4. Original capital, $5000; gains, $1250; losses $640; net capital at close, $——.
5. Net insolvency, Jan. 1, 1907, $2500; Jan. 1, 1908—Resources, $5260; liabilities $5000. Result of business for the year a ——of $——.
6. A & Co. invested $12,000, and a year later had cash, $2000; mdse., $10,500; real estate $2500, but owed debts of $2000. A & Co.'s net capital then was $——.
7. Jan. 1, 1912. Net capital $5000. Jan. 1, 1913, gross gains during year $3000; gross losses, $1200. Present resources $8000; liabilities $900. Prove the existence of an error in these figures.

Distribution of Profits and Assessment of Losses.

In partnerships, profits are distributed and losses met in proportions agreed upon. These may be arbitrary or graded for special service, or based on amounts of investments; or a combination of these factors.

I. EQUAL DIVISION OF PROFITS.

ORAL EXERCISE.

1. Divide $726 profits equally between Adams and Brown.
2. A's investment was $4000 and D's $6000. They share equally a profit of $800, but D withdraws $100 of his share. The net capital of the firm is then $?, and that of D, $?

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3. A invests $2000, B $4000, and C $3000. Determine their present worths, after they have shared equally a loss of $960.

EXERCISE.

1. July 1, 1908. Investments: A, $12,600; B, $14,800; C, $9,650.
Jan. 1, 1909. Withdrawals; A, $1200; B, $400.
Jan. 1, 1909. Gains: On mdse., $7260.50; on real estate $1260.
Losses: Expense, $3126.50; fuel and light $568.40; taxes $165.
Find the present worth, if profits are shared equally.

2. On Jan. 1, Evans invested $12,000, Brown $8000 and Thompson $15,000 in the real estate business. A year later their resources were: Cash $7290; accounts and notes receivable, $11,360; real estate $26,750, while they owed notes for $3296.50 and taxes of $561.30. Find the present worth of each member.

II. PROFITS SHARED IN ARBITRARY PROPORTIONS.

ORAL EXERCISE.

1. Divide $792 between A and B in the ratio of 3 to 1.

2. A receives 2/3 of the profits of a business in which he invested 1/4 of the capital of $12,000. Find his present worth if the profits are $900.

EXERCISE.

1. Crown and Briggs entered into partnership on Jan. 1, each investing $8000. Owing to Crown's greater experience, he receives 2/3 of the profits. On July 1, their profit and loss account stands as follows.

PROFIT AND LOSS.

Expense	$429.60	Merchandise	$3129.65
Rent	600.	Discounts	128.46
Discounts	86.	Stocks and Bonds.	82.50

Find the net gain and present worth of each partner.

2. A invests $9000 and B his services, under an agreement that the latter shall receive $900 yearly salary, to be paid from any profits, and 1/4 of the profits remaining. The profits at the end of a year amount to $3126.50. Find each partner's present worth, B having withdrawn only one-half of his salary.

3. A, B and C enter into partnership to manufacture advertising novelties. A invests $6000, and the other partners $4500 each. The partners agree: (1) To share losses equally; (2) To pay A $1200 salary

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out of any profits, for giving all his time to managing the business; (3) To divide any remaining profits in parts the proportion of 3 (A), 2 (B), and 2 (C). At the end of the year the profits are found to be; on merchandise, $9720; discounts, $112.50; and the expenses of the business exclusive of A's salary, $4326.50. Find the present worth of each partner.

III. PROFITS SHARED ACCORDING TO INVESTMENTS.

ORAL EXERCISE.

1. A invests $5000 and B $7000. What is the partnership capital?

2. Each partner invests what part of the capital? Each should receive what part of any profits?

3. What should each receive of $1400 profits? What shall each pay of $350 loss?

What is each partner's share, according to the following investments?

(*a*) A, $6000; B, $10,000. (*b*) A, $5000; B, $4000; C, $3000.

Determine individual gain, loss and present worth in the following:

	Investment.	Gain.		Investment.	Loss.
4.	A, $2000; B, $10,000	$1500.	6.	E, $3000; H, $8000	$3300
5.	C, $6000; D, $8000	$1860.	7.	G, $6000; K, $1500	$4200

EXERCISE.

1. The net profits of the firm of Thompson and Harris for the last year are $3584.64. This is to be apportioned to the partners' investments. Make the apportionments if Thompson's investment was $6000 and that of Harris, $1700.

2. Brown, Kann and Eastman engaged in a retail business, investing respectively $12,000, $8000 and $19,000. Brown, who was the only active partner, was permitted to take $1500 from the profits, annually, for his services, the balance of the profits being divided according to investment. Net profits for the past year were $7843.12. Make the distribution.

At the time of apportioning profits or losses a business statement is often prepared, showing a summary of the resources and liabilities, the investments and present worth, and, at times, the losses and gains.

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EXERCISE.

1. Complete the following business statement.

PARTNERSHIP STATEMENT.

Harris and Davidson, December 31, 19—

Resources.				
Cash	5280	45		
Merchandise on hand and unsold	15212	20		
Notes Receivable	5000	—		
Accounts Receivable	8240	60		
Liabilities.				
Accounts Payable			3204	50
Notes Payable			2500	—
Present worth, Harris & Davidson			?	
	?		?	
Gains.				
Merchandise			12760	20
Interest and Discount			521	60
Losses.				
Labor	3640	—		
Rentals	1815	60		
Expense	2116	45		
Harris, Net Gain	?			
Davidson, Net Gain	?			
Check.				
Harris, Investments	10000	—		
Harris, Net Gain	?			
Harris, Present Worth			?	
Davidson Investment	13000	—		
Davidson Withdrawal	681	—		
Davidson Net Investment	?			
Davidson Net Gain	?			
Davidson Present Worth			?	
Harris and Davidson Present Worth			?	

NOTE. Share profits according to investment.

IV. INTEREST ON INVESTMENTS.

Sometimes interest on investments is paid from the profits, the balance of profits being apportioned otherwise.

ILLUSTRATION. *Example.* A invests $6000 and B $9000 in a business, realizing $3450 profits the first year. Six per cent. interest is allowed on investments, the balance of profit being apportioned in the ratio of 1 : 2. Find present worth of each partner.

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Solution.

Interest on A's investment, 1 yr. on \$6000 =	\$360
Interest on B's investment, 1 yr. on \$9000 =	540
	900
\$3450 − \$900 = \$2550, net profits.	
A's share, 1/3 of \$2550	\$ 850
B's share, 2/3	1700
A's present worth, \$6000+\$360+\$850 =	7210
B's present worth, 9000+ 540+1700 =	\$11240

EXERCISE.

1. Andrews entered into partnership with Gates on Jan. 1, 1913, investing \$6000, while Gates invested \$11,000. They agreed to allow each other 6% interest on investments, and to share the balance of profits in the ratio of 2 to 3. Find the present worth of each member on Jan. 1, 1914, after the apportionment of profits of \$6585.

2. Jan. 1, 1913, Carleton invested \$12,000; Johnson, \$8500. Oct. 1, 1913, Carleton withdrew \$3000. Jan. 1, 1914, profits for past year \$4216.50. Interest allowed at 6%, and balance of profits shared equally. Apportionment?

The average interest per partner is sometimes computed, and each partner is credited with the excess of his investment interest over the average, or charged with any deficit. The entire profits are then divided by agreement.

ILLUSTRATION. Last illustrative example.

Solution.

Interest on A's investment	\$360
Interest on B's investment	540
	2)900
Average interest.	450

\$450 − \$360 = \$90 A's deficit (Debit).
540 − 450 = 90 B's excess (Credit).
A's 1/3 of profits = 1/3 of 3450 = \$1150
B's 2/3 = 2300
A's present worth = \$6000 + \$1150 − \$90 (deficit) = \$7,060
B's present worth = \$9000 + \$2300 + \$90 = \$11,390

EXERCISE.

1. April 1, 1912, A. M. Gates and J. B. Farquhar entered into partnership for the purpose of carrying on a manufacturing business. Gates invested \$16,000 and Farquhar \$12,500. It was agreed that interest should be allowed and charged at 6%, and that gains and losses should be divided

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equally. On Aug. 1, 1912, Gates withdrew $1000, and on Oct. 1, Farquhar withdrew $500. On Apr. 1, 1913, the books were closed and showed the following:

Gains.		Losses.	
Mdse.	$15,289	Labor	$7240
Stocks and Bonds	986.40	Equipment	1645.20
Mdse. discounts	560	Discounts	300
		Expense	1298.75

Find the present worth of each partner after apportionment of net gain.

2. Oct. 1, 1912, M. Andrews, B. T. Chase, and T. C. Lewis formed a partnership. Lewis invested $8500 and the other partners $7500 each. They agreed to allow and charge interest at 6% and to divide profits equally. On Dec. 1, Lewis withdrew $1000 and Chase $500. At the close of the year, assets and liabilities were as follows:

Assets.		Liabilities.	
Cash	$5486.50	Bills pay	$1250
Mdse.	6798.15	Accts. pay.	1670.60
Real estate	9725.		
Bills Rec.	3000.		
Accts. Rec.	4520.		

Make statement showing condition of business.

VI. APPORTIONMENT BASED ON AVERAGE INVESTMENT.

The sum that has the same earning power for a stated period as two or more sums invested for different periods, is termed an average investment. Profits and losses are often apportioned by average investment. The apportionment may be made by ratio or by interest.

ILLUSTRATION. *Example.* At organization, A invests $12,000, but withdraws $2000 at the end of 8 months; B invests $6000 withdrawing $3000 after 6 months. How should the first year's gains of $4750 be apportioned?

Solution (1).

A's investment of	$12,000	for 8 mo. =	$96,000 for 1 mo.
A's investment of	$10,000	for 4 mo. =	40,000 for 1 mo.
A's average investment of			$136,000 for 1 mo.
B's investment of	6,000	for 6 mo. =	36,000 for 1 mo.
B's investment of	3,000	for 6 mo. =	18,000 for 1 mo.
B's average investment		=	54,000 for 1 mo.
A's average + B's average		=	190,000 for 1 mo.

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A's share = 136/190 = 68/95.
B's share = 27/95.
1/95 of profit of \$4750 = \$50 68/95 = 68 × 50 or \$3400 = A's profit.
27/95 = 27 × 50 or 1350 = B's profit.

Solution (2). Using any rate, say 6%.

Interest on \$12,000 for 8 mo.	= \$480
Interest on 10,000 for 4 mo.	= 200
A's invest. earning power	= \$680
Interest on \$6000 for 6 mo.	= \$180
Interest on 3000 for 6 mo.	= 90
	\$270

Total earning power \$680 + \$270 = \$950, of which A's investment represents 68/95 and B's 27/95.

Final step—as in previous solution.

EXERCISE.

1. M. C. Barton and James Thompson form a partnership to carry on an agricultural supply store. Barton invests \$6000 for 9 mo., and then adds \$2000. Thompson invests \$14,000, but withdraws \$4000 at the end of 4 months. At the end of a year their summary accounts stand as follows:

Farm Implements.			Fertilizers.		
Cost \$6240	Sales	\$5268.50	Cost \$5692	Sales	\$7218.20
	On hand	3650.		On hand	965.
Seeds.			General Expense.		
Cost \$2132	Sales	\$3586.40	Cost \$1296.18		
	On hand	1125.			

Apportion the profits in accordance with average investment.

2. Chas. Parsons and T. P. Newton form a partnership to carry on the grocery business. Complete the proprietors' accounts, apportioning profits by average investment.

Chas. Parsons.

Mar. 1, Mdse. \$2000	Jan. 1, Invest \$8000.
Jan. 1, Pres. worth?	Jan. 1, Net gain?

T. P. Newton.

July 1, Withdraw \$6000	Jan. 1, 1908, Investment \$18,000
Jan. 1, Pres. worth?	Jan. 1, 1909, Net gain ?

PROFITS AND LOSSES.

Losses.		Profits.	
General Expense	\$1503.14	Merchandise	\$9290.68
Rentals	900	Discounts	587.40
Labor	1650		
Delivery	2160		
Parson's net gain	?		
Newton's net gain	?		

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3, 4. Solve the problems of the last exercise by average investment.

GENERAL EXERCISE.

1. Frank T. Morris, R. P. Norton and Henry Collins form a partnership on April 1, to carry on a furniture business, investing respectively, $12,000, $6000 and $20,000. On June 15, Morris withdraws $500 and on Dec. 1, Collins withdraws $2500. At the close of the first business year, the books showed the following facts: Cost of mdse. purchased, $21,690; sales of mdse, $32,169.40; mdse. on hand unsold, $6,742.10. Cost of real estate, $12,000; alterations and repairs, $3167.50; estimated value at close of year, $14,500; notes in favor of firm, $5640; accounts due the firm, $5680; accounts owed by the firm, $1520; profits on discounts, $750; notes owed by the firm, $1200, mortgage on real estate, $6000; general expense for the year, $9650 45; cash, $18,811.45.

Prepare partnership statements at close of year under following conditions:

1. Share respectively in proportions of 2, 1 and 3.

2. Interest allowed and charged at 5% and profits divided equally.

3. $1200 salary allowed Morris, balance of profits being divided according to original investment.

4. Profits shared according to average investment.

FOR DISCUSSION.

1. Which system of apportionment is most fair, considering simply investments?

2. Which permits recognition of special service?

3. When is it impossible to use the method of "average investment"?

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CHAPTER XLV.

EQUATION OF PAYMENTS AND OF ACCOUNTS.

INTRODUCTORY EXERCISE.

1. Which party to a debt loses by its delayed payment?

2. Who gains by payment before due?

3. How may one measure, arithmetically, this gain or loss?

4. $600, which I owe, is overdue 1 mo. Money being worth 6%, what ought I to pay the creditor for the delay? Why?

5. $200 worth of mdse. bought Jan. 10, on 2 mo. credit, should be paid for on what date? The creditor is entitled to how many dollars extra, if payment is made May 10? If paid Jan. 10 would the creditor gain or lose? What might he allow off?

Debts draw interest from the time they are due until paid. Discounts, sometimes equal to interest on the debt, may be allowed on payments before due. If several payments are due the same party on different dates, a single payment equal to their total may be made on such a date that the overdue interest may equal the discount on advance payments. This date is termed the *average* or *equated* date. The process of determining it is termed *averaging* or *equating*. (Why is the settlement on the equated date fair to both parties?)

EQUATION OF PAYMENTS.

ORAL EXERCISE.

1. The interest on $400 for 6 mo. = interest on $200 for ? mo.

2. I owe $400 due in 2 mo. By paying $200 to-day for what extra time might I fairly retain the balance?

3. I owe $600 in 3 mo., and $600 in 5 mo. When might the total debt be paid without gain or loss to either party?

ILLUSTRATION (1). On Jan. 10, A. C. Brown bought $900 worth of mdse., payable, $200, Jan. 20; $400, Feb. 9; the balance, Mar. 10. (1) He

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settles in full Mar. 20. What does he owe? (2) On what date would the face of debt be a fair settlement? Interest 6%.

Solution (1). Debt paid Mar. 20.

Date Due.	Payment.	Overdue.	Overdue Interest.
Jan. 20	$200	59 da.	$1.97
Feb. 9	400	39 "	2.60
Mar. 10	300	10 "	.50
Debt	$900	.	$5.07
Int. due	5.07		
Due	$905.07		

NOTE. Use exact time.

(2) By paying Mar. 20, Brown pays $5.07 extra. The earlier he pays the less interest he owes. The interest on $900 for 1 da. is 15c. ∴ 5.07 represents 507/15 days loss of interest, or 34 days. ∴ The equated date for payment of $900, is 34 days before Mar. 20, or Feb. 14.

It is evident that *interest is found on each payment from due date to date of assumed settlement.* The total interest, divided by the interest on the total debt, gives the number of days of delayed payment. If there is discount in place of interest, the real payment should be made later.

ILLUSTRATION. Find the equated date for payments of $600 due Mar. 10; $300, April 8; $600, May 12.

Solution. Assume a date of settlement, called a *focal* date.

Select as focal date, May 12, date of last payment.

Date.	Payment.	Overdue.	Interest.
Mar. 10.	$600	63 da. (to May 12)	$6.30
Apr. 8.	300	34	1.70
May 12	600	0	0
Due	$1500		.25)$8.00 due
Interest on $1500 for 1 da. = 25c.			32, no. of days.

∴ By paying May 12, the debt is overdue 32 days. 32 days back from May 12 = Apr. 10, the equated date.

Check Solution. If Apr. 10 is correct, there should be no gain or loss by paying face on that date. Take Apr. 10 as focal date.

Date	Payment	Overdue	Before due	Interest	Discount
Mar. 10	$600	(to Apr. 10) 31 da.	——	3.10	——
Apr. 8	300	2	——	.10	——
May 12	600	——	32 da.	——	3.20
			check	3.20 =	3.20

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NOTE. This means that $1500 may be paid in full settlement on Apr. 10. On any date thereafter the amount due is $1500 + interest from Apr. 10.

Remember: 1. Any date may be used as focal date or trial date.

2. By using first or last date, one interest computation is saved. Why?

3. The *face* of debt is always due on the equated date.

EXERCISE.

Find the equated date, and check by using equated date or another focal date.

1. A. B. Chase, Dr.

1912		
Jan. 16	To mdse.	$300.
Jan. 29	To mdse.	900.
Mar. 12	To mdse.	400.

What is due Dec. 1?

2. S. P. Randall, Dr.

1912		
Mar. 12	To mdse.	$420.
May 16	To mdse.	500.
Aug. 15	To mdse.	320.

3. James Field, Dr.

1912		
Oct. 16	To mdse.	$520.
Oct. 20	To cash	120.
Dec. 15	To mdse.	800.
1913		
Jan. 3	To mdse.	320.

In the following, re-write the accounts by substituting the true dates when due, obtained from the credit terms.

4. Robert Everhart, Dr.

1912		
Feb. 21	30 da. credit	$300
Feb. 25	2 mo. credit	600
Mar. 16	1 mo. credit	900

5. James Donnelly, Dr.

1912		
Mar. 3	To mdse.	$240.
Mar. 18	To mdse., 30 da.	380.
Mar. 30	To mdse., 2 mo.	456.40
Apr. 4	To mdse., 60 da.	820.60
Apr. 16	To mdse.	900.

6. Jones & Co., Dr.

1912			
Apr. 16	Mdse.	60 da.	$456.10
Apr. 29	Mdse.	30 da.	829.30
May 5	Mdse.	3 mo.	420.00
June 16	Mdse.	——	36.50

EQUATION OF ACCOUNTS.

Accounts having debits and credits are equated to determine the *due date* of the *balance*. The most convenient focal date is the latest named in the account.

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ILLUSTRATION.

James C. Westcott.

1909								
Mar.	12	Mdse.	$360		Mar.	30	By cash	$300
	29		300		Apr.	10	By note	120
May	4		240				(Balance	$480

Solution. Here we have both debit and credit interest, the former being charged on debts overdue, and the latter being credited on payments made to focal date.

Focal date, May 4.

Date.	Debit Amounts.	Period (to May 4).	Dr. Interest.
Mar. 16	$360	49 da.	$2.94
Mar. 29	300	36 da.	1.80
May 4	240		
	$900		$4.74
	Credit Amounts.		Cr. Interest.
Mar. 30	$300	35 da.	$1.75
Apr. 10	120	24 da.	.48
	$420		$2.23

Balance = $900 − $420 = $480.

Net interest = Dr. int. − cr. int. = $4.74 − $2.23 = $2.51, dr. int. Since the interest is debit, payment is overdue.

Interest on $480 for 1 day = 8c.

2.51 ÷ .08 = 31 +, the number of days overdue.

May 4 − 31 days = Apr. 3, equated date.

Check. Assume Apr. 3 as focal date.

Date.	Amount.	Period.	Dr. Int.		Cr. Int.
Mar. 16	$360	18 da.	$1.08		
Mar. 29	300	5 da.	.25		
May 4	240	31 da. (before due)			$1.24
Mar. 30	300	4 da. (before due)			.20
Apr. 10	120	7 da. (delayed)	1.14		
			$1.47	check	$1.44

NOTE. The difference between dr. and cr. interest = the remainder in first solution, when dividing total interest by interest for one day.

EXERCISE.

Find and check the equated date.

1. C. P. NEWTON.

1912			1912		
Mar. 16	To mdse.	$1400	Apr. 4	By cash	$ 200
May 28	"	2200	June 6	"	1000

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2. THE NEWBOLD CO.

1912			1912		
Nov. 19	To mdse.	$26,580	Nov. 30	By cash	$200
30	"	41,590	Dec. 15	"	300
Dec. 12	"	16,250	Dec. 31	Note	500
30	"	38,845			
1913					
Jan. 20	"	268			

3. CHAS. P. RANDALL.

1912				1912		
July 15	To mdse.,	30 da.	$360	Aug. 5	By cash	$400
31	"	15 da.	180	Sept. 10	Note	300
Aug. 12	"	30 da.	420	10	Cash	300
Sept. 6	"	2 mo.	620			

4. JAMES BROWNELL & BRO.

1912			1912			
Feb. 28	By note	$ 300	Feb. 16	By mdse.	60 da.	$325
Mar. 16	Mdse.	120	20	"	30 da.	244.60
Apr. 2	Cash	400	Mar. 18	"	3 mo.	840
May 10	"	600	29	"	2 mo.	628

NOTE. Account sales are often *averaged* in the same manner as general accounts.

Washington, D. C., May 16, 1913.

ACCOUNT SALES OF POTATOES.

5. Sold for account of James Parker,
Hyattsville, Md.

By Robert Harper,
Commission Merchant.

1913			
	Sales.		
Apr. 30	20 bbl. potatoes.	$2.40	
May 6	50 " "	3.00	
8	120 " "	2.60	
12	10 " "	2.00	
	Charges.		
Apr. 28	Freight and cartage		.46
May 12	Storage		.30
12	Commission, 5% of sales		?
—— ?	Net proceeds, due by equation		?

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CASH BALANCE.

The *cash balance* of an account on any particular date is the amount due on that date. If interest and discount are not allowed, the *cash* balance is the same as the account balance; if allowed, it equals the regular balance plus or minus the net interest or discount; or it equals the balance of the account plus interest for the time between equated date and average date, if later than equated date; or minus interest, if given date is earlier. The first illustrative example, page 398, shows the cash balance on March 20.

EXERCISE.

Find the cash balance directly for each of the accounts in the preceding exercise, checking by using equated date.

Ac. 1. What is due July 1?

2. What is due Jan. 8?

3. What is due Oct. 31?

4. What is due Jan. 1, 1913?

5. What is due July 1?

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CHAPTER XLVI.

BILLING.

INTRODUCTORY EXERCISE.

1. Bring to class several bills for goods purchased. State the information that each heading gives.

2. What information is given concerning the articles bought?

3. Who makes out a bill for goods? What becomes of it? Trace its life.

4. Of what value is the bill to the purchaser? What might happen in case no bill was given?

The *bill of merchandise* is the statement in detail of merchandise sold in the course of trade, and is prepared by the seller for the buyer. Formerly the term *invoice* was applied to statements of merchandise sold wholesale, and of incoming goods from abroad. Now the terms are frequently used interchangeably.

I. SIMPLE BILL.

Washington, January 11, 1913.

C. M. BROWN,
Groceries and Provisions.

Sold to Henry Eastman,
1426 13th Street.

1	1	sk. Flour			85
2	6	lb. Lard	@ .17	1	02
3	1½	pk. Apples	.72		?
4	10	lb. Gran. Sugar.	.06½		?
5	2	qt. Molasses	.18		?
6	3	lb. Rice	.09		?
7	2	lb. Starch	.05		?
8		Yeast			02
9					?
10					
11		Paid,			
12		C. M. Brown			
13					
14					

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EXERCISE.

1. State in your own words just what took place according to the bill on page 403.

2. Are there any non-essentials in this bill? Add two.

3. If settlement is not made, how is the bill affected? How, if sent C. O. D.?

4. What is the advantage of numbering the items on the bill?

5. Rule up three bill forms and prepare three original bills for a Saturday's marketing. Secure reliable information as to quantities and prices.

Receipting for Payment.

When payment is made, bills are receipted by the creditor or by the party representing him, who writes the word "Paid," or some equivalent expression, across the face of the bill over his signature.

EXERCISE.

1. Why is the creditor's signature necessary?

2. Examine the forms of receipt of the model bills in this section, and throughout the book, and determine, in each case:

(*a*) What facts are stated in the receipt.

(*b*) By whom receipted and under what probable conditions.

(*c*) What part of the receipt may be printed.

(*d*) Under which of the following classes the receipt belongs:

(1) Receipt by the direct creditor.

(2) Receipt in the firm name by a partner.

(3) Receipt of a corporation by an officer.

(4) Receipt for the direct creditor by a third party.

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II. SIMPLE TWO COLUMN BILL.

January 31, 191–

CHESTNUT GROVE DIARY,
1421 Oak Street.
JOHN C. NORTON, JR.
Sanitary Cream and Milk.

Sold to *James Fielding*
546 Pine Street.

Acc't Rendered					
132 Qts. Milk		.09	———		
6 Pts. "		.05	———		
Pts. Gilt-Edge Cream					
7½ Pts. " "		.19	———		
Gills " "					
Pts. Choice "					
Hf. Pts. Choice "					
Gills " "					
14 Qts. Buttermilk		.08	———		———

Received payment
JOHN C. NORTON, JR.

EXERCISE.

1. Extend the above bill. How should the form be changed if $3.00 is paid on account? How, if nothing is paid?

2. You are directed by James Kent to deliver daily 2 qt. 1 pt. of milk and 1 pt. of cream. In addition, you deliver: Jan. 5, 3 qt. milk; Jan. 8, 2 qt. milk; 1 pt. cream; Jan. 21, 2 qt. 1 pt. milk; 1 pt. cream. Prices: 9c per qt. for milk; 16c per pint for cream. Render a bill and have your clerk receipt for payment.

3. Design an original bill for similar use for an ice company.

4. In what other businesses could similar bills be used?

TERMS OF SALE.

As is evident from the problems and illustrations given, the terms of payment on bills of goods may vary markedly with the character and quantity of sales. The list of terms that follow are selected from actual bills.

LIST OF TERMS OF SALE.

1. Cash.
2. Net cash.

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3. Cash or exchange on New York.
4. Account.
5. Interest after 30 days.
6. 30 days; 2/10.
7. Net 30 days; note to your own order, payable at a Philadelphia bank.
8. Net 90 days; 5% discount for exchange on New York in 30 days.
9. 30 days. If paid in 10 days, 2% discount.
10. 30 days.
11. 30 days. If not promptly paid interest will be charged from date of sale.
12. Cash, 5%; 30 days, 2%; 60 days, net.
13. Ten day note.
14. 60 days from Aug. 10; 2% 10 days.
15. 5% 30 days, or 6% 10 days. Payable in New York City funds.
16. Due, October 2. Less 2% if paid in 10 days.
17. C. O. D.
18. 5/60; 6/30; 7/10.
19. Best discount allowed for unexpired time is at the rate of 6%.
20. Net 30 days, or 1% discount in 30 days, or 2% discount in 10 days. New York funds.
21. Net after 60 days.
22. Subject to sight draft after 60 days. Payable in gold or its equivalent.
23. Net 30 days; 2% discount 10 days. No discount allowed after ————, 191—.
24. Accounts due the first of the month.
25. 2% discount if paid ——, 191—; Due net, ——, 191—.

EXERCISE.

1. Give your understanding of the meaning of each of the terms stated above.

2. Examine any bills that you can secure and see whether you can add to the above list.

3. State the effect of each set of terms on the settlement of a bill of merchandise amounting to $1600, purchased August 1.

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III. FORM OF MANUFACTURER'S INVOICE AND LETTER OF TRANSMISSION.

INVOICE.

Name *John Smithson* Address *Albany, N. Y.*
Order No. *17296*

Bought of WASHINGTON FLOUR CO.
Manufacturers
Fancy Patent Flours
Buffalo, N. Y.

Ship to *Order Washington Flour Co.* Date 12/30/191—
Destination *Albany, N. Y.*
Via. N. Y. C. R. R.

Bbls.	H. Bbls.	No. Sks.	Sz. Sks.	Kind.	Brand.	Marks.	Price Per Cwt.	Amount.
180					Ames		5.65	?
	50				Buffalo Prime		4.90	?
		200	24	Paper	Conrad		4.60	?
		80	49	Cott'n	Golden Queen		4.80	?
		40	98	Grain	Harvest		4.40	?
		260	140	Jute	Bakers Pride		4.45	?
								?

Car	Initials	Date
47329	L. V. R. R.	12/30/191—

The above is an invoice of goods shipped this date, in accordance with your order of 12/17/191—

Washington Flour Co.
per *Temple.*

EXERCISE.

1. Extend the bill and compare with previous bill, as to columns, entry of items, etc.

2. Explain the advantages and disadvantages of the use of many columns.

3. From the standpoint of the customer, what portion of the heading might be dispensed with?

4. Prepare a similar original invoice for a wholesale order, using newspaper market quotations. Add terms. Receipt the bill, signing your name as officer of the selling company.

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IV. CLOTH MANUFACTURER'S INVOICE.

Goods Purchased will not be Taken Back Except for Damage or Imperfection.
No Claims allowed unless reported within 10 days after receipt of goods.

MORRIS & NORTON

DRESS GOODS

NEW YORK, May 21, 191—

JOHN HARRIS & Co.
N. Y. City.

Terms
5 per cent. 30 das., or
6 per cent. 10 das.

Register 1337
Book 45 Folio 14

THIS INVOICE PAYABLE IN NEW YORK CITY FUNDS

19 Pcs NORFOLK LT PERCALE 45^{1} 45^{2} 45^{2} 45^{1} 45^{3} 45^{2} 46^{3} 46^{2} 45^{1} 46^{3} 46^{3} 45^{2} 45^{3} 45^{3} 46^{1} 45^{3} 46^{1} 46^{3} 45^{2}		$10\frac{1}{2}$	
16 Pcs NORFOLK DK PERCALE 45 45^{1} 45^{3} 46^{2} 45^{3} 45^{2} 45^{2} 45^{2} 46^{1} 45^{1} 46^{3} 45^{2} 45^{2} 46^{1} 45^{3} 45^{2}		10	
17 Pcs $\frac{10}{4}$ PEPPERELL BROWN 45 45^{2} 46^{3} 45^{2} 45^{1} 45^{1} 45^{2} 45^{3} 45^{1} 45^{1} 46^{3} 45^{2} 45^{1} 45^{3} 45^{2} 45^{1} 45^{3}		09	

EXERCISE.

1. Extend the bill. What amount will settle it June 19? June 24? August 23? October 16? On what date is the face value due?

2. On October 19, the Fall River Manufacturing Co. sells you, on terms of 4/60; 5/30; 6/10; 36 pc. C. K. Checks, 45, 46, 47^{1}, 48, 45^{2}, 46^{1}, 42^{2}, 41^{2}, 45^{2}, 46, 44, 45^{3}, 44^{2}, 46, 47, 48^{1}, 45^{3}, 42^{2}, 43^{2}, 44^{1}, 42^{2}, 46, 47, 48^{2}, 45, 41^{2}, 42, 44, 47^{3}, 45^{2}, 43^{2}, 44^{1}, 45^{2}, 43^{2}, 42^{2}, 44 @ 23c.

Also 12 pc. Ginghams: 45^{1}, 47, 46^{2}, 43, 45, 49^{2}, 50, 48^{2}, 47, 49, 51^{2}, 49^{2} @ $9\frac{3}{4}$.

Draw up the bill and receipt for the company for settlement on November 16.

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3. Ascertain by inquiry the names, common piece lengths and prices of four standard cloths and draw up an original bill. Name your own terms and receipt for payment.

V. DISCOUNT BILLS.

See form on page 242.

Claims for errors and overcharges must be made within ten days of delivery.
TERMS, 30 days net; 5% cash.

Louisville, *October 25, 191–*

THE KENAWHA SUPPLY COMPANY.

GENERAL CONSTRUCTION AND RAILWAY SUPPLIES.

1468 River Street.

Your Order No. *5657*
Your Req. No.....
Shipped *L. L. Frt. 10/25/1—*
Our No. D–*3456*
Salesman *Roberts*.

Sold to *John P. Carey,*
Evanston, Ill.

Quantity Furnished.	Description.	List.	Disc. %	Extension.	Total.
16	sq. 10×14 Gal. Sheet Roofing	6 87			
164	ft. Gal. Valley	11			
48½	ft. Ga. Climax	14			
		Less	20%		
160	lb. W. P. R. Wire	12			
			30%		
2	C. Machine Bolts, ½×3½,	4 72			
1½	C. Machine Bolts, 5/8×4	7 15			
1½	C. Machine Bolts, ¾×8	13 85			
			50, 10		
		Trade	5%		

Received payment, October 17, 191—
THE KENAWHA SUPPLY CO.
J. Towne, Treas.

1. Extend and explain bill.

2. How does it differ in arrangement from the bill on page 242?

3. Prepare discount bills for examples on pages 237, 239 and 242. Supply names of customers and dealers where necessary.

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VI. MONTHLY STATEMENT.

Retail Statement.

Mrs. Samuel Henderson, Chicago, Ill., Sept. 30, 19—
The Morrison.

IN ACCOUNT WITH
HENRY NORTON & COMPANY.

		Items.	Charges.	Credits.
Sept. 1	To Balance (Bill rendered)		68 40	
2	12 yd. Lawn .18	— —		
	8 Lawn Mull .30	— —	—	
5	3 bt. Ribbon 1.15	— —		
	6 yd. Embroidery .28	— —	—	
6	2 yd. Elastic .12	— —		
	5 Binding .15	— —		
	4 yd. Linen .65	— —	—	
12	1 set Furs,	165 00		
	1 Ermine Cape	240 00		
	12 yd. Silk 1.25	— —	—	
*Cr.	1 Set Furs			165 00
24	1 rem. Gingham	65		
	9 yd. Calico .10	— —		
	5 C. Thread .10	— —	—	
Cr.	1 rem. Gingham			65
27	2 Shirtwaists	9 60		
	3 pr. Blankets 4.00	— —		
	2 pr. Pillows 2.25	— —		
	2 Bolsters 2.50	— —	—	
			— —	— —
		Less	— —	

*Note 1. Credit items are usually entered in red.

Note 2. Where individual purchases have covered many items on any one date, and a bill has been rendered at the time, details are not entered on the statement, but the date of each purchase, the words "to Bill Rendered" and the amount of individual bill are entered. This is common in wholesale statements.

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EXERCISE.

1. How does this statement differ from an ordinary bill of goods?

2. Extend the statement.

3. Explain just what has taken place according to the statement. How does it happen that credits occur?

4. Should the statement be receipted in case of partial settlement?

5. Prepare an original statement of a month's purchase of groceries. Receipt for payment.

EXERCISE.

1. Draw up a "to bill rendered" statement from the following account:

ROBERT H. LAWRENCE,
1356 18th Street, N. W.

191—				191—			
Mar. 3	Mdse.	42	16	Mar. 8	Cash	12	00
18	"	121	15	21	Mdse. ret.	5	40
23	Debit memo.	4	19	23	Overcharge		48
27	Mdse.	16	35	30	10 da. dft.	150	00
28	"	39	12	31	Cash	50	00
30	"	125	04				
31	"	11	85				

2. Draw up an original statement to cover one quarter of a year, containing at least fifteen debit items and six credit items of different character.

VIII. REQUISITION.

The requisition is a combined order and voucher or bill, used in ordering supplies for offices and workshops, when the approval of a higher official is necessary. Sometimes it does not involve prices, but is simply a request from an official or foreman, drawn on the stock clerk, for supplies needed, which the latter keeps in stock ready for use as required. In the illustration (page 412), however, the articles required must be purchased. The superintendent of a certain department asks for the supplies, and the general manager approves the request and directs the purchase. Requisitions vary greatly in different businesses and government offices.

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REQUISITION BOOK.

March 10, 191—

THE RANKIN MANUFACTURING COMPANY.

Charge to the *Repair* Shop; **Model Department.**

Please furnish for use in this shop the materials specified below:

Henry C. Carter,
Superintendent.

Items.	Quantity.	Material.		Estimated Cost.
1	400 b. f.	W. Pine, 2×3—6′, per M.	$42.00	—— —
2	2000 b. f.	Oak, 1×9, per M.	50.00	—— —
3	600 b. f.	Chestnut, per M.	36.00	—— —
4	5 qt.	M. Y. Varnish	.21	—— —
5	9 qt.	H. O. Finish	.24	—— —
6	2 lb.	Indian Red	.10	—— —
7	10 lb.	Vermilion Red	.24	—— —
8	18 lb.	Yellow Murdock	.26	—— —
				?

March 11, 191—
Approved and ordered purchased.
Jas. M. Dean,
Gen'l Man.

Received the above mentioned material on *March 16*, 191—
Henry C Carter,
Superintendent.

EXERCISE.

1. Extend the form.

2. State exactly what has taken place.

3. Prepare an original requisition for furniture and material required for the equipment of some business office. Sign as subordinate and have some other pupil sign as a superior officer.

FOR INVESTIGATION AND REPORT.

EXERCISE.

1. Collect specimen bills and statements from at least thirty distinct businesses. Classify these (*a*) in order of complexity; (*b*) according to the difficulty of the arithmetical computations involved.

2. Write a *brief* on the information furnished by the bill headings. Compare headings of the bills of retail, wholesale and manufacturing houses.

3. From your study of the bills discover how the "terms of payment" vary with the character of the business.

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4. What knowledge of arithmetic should a billing clerk have?

5. Study and report on the use and value of special columns. Give illustrations.

6. Collect, and submit an illustrated report on forms which, while not bills, are worked out on the same principle.

7. Show how each bill form is designed to meet the needs of the business in which it is used.

8. Examine bills and see how many different ways of entering items you can discover.

9. Report on the different forms of receipt for payment that you can discover. Explain the exact meaning of each

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CHAPTER XLVII.

STORAGE.

INTRODUCTORY EXERCISE.

1. What circumstances may lead a housekeeper to store her furniture?

2. Why does a commission merchant place certain merchandise in "cold storage"? Why does he not own a storage plant?

3. Why is grain "stored" in grain elevators?

4. If I rent a room in a furniture warehouse from July 15 to October 25, at $9.00 per 30 day month, or fraction thereof, the storage bill is $——.

5. The cold storage of 40 cases of eggs for 50 days, at 40c per month, or fraction, costs what amount?

Storage is the charge for storing goods in an elevator or warehouse. It is computed on quantity rather than on value. The *term of storage* is the time for which the storage rate is quoted. The *rate* may be for the day, week, month, or period of 30 days, etc. Fractions of terms are usually considered as full terms.

Storage warehouses and grain elevators are closely regulated by law. Often rates are fixed by law, by Chambers of Commerce, associations of warehousemen, etc. In the storage of grain, and of certain other produce, there are strict regulations for *grading* receipts of merchandise, since the person storing cannot expect to withdraw the identical property he deposited.

SIMPLE STORAGE.

Simple storage is storage computed at the time of withdrawal of goods.

EXERCISE.

(Solve mentally if possible.)

Compute the storage on:

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1. 200 cs. eggs, for 3 mo. at 10c per mo.

2. 500 cs. eggs, for 4 mo., at 10c per mo. for 2 mo., and 8c per mo. for succeeding months.

3. 800 lb. of cheese from August 17 to October 26, at 8c per 100 lb. per month of 30 days.

4. The storage rate on the following bill is 9c per 100 lb. per 30 da. Extend the bill, explaining each item.

March 31, 191—

Messrs. James Brown & Son,
To The Morton Storage Co., Dr.

Lot No.	Article.	Quantity.	In.	Out.	Quantity.	Time. Days.	Time. Terms.	Rate.	Amt.
6746	Cheese	20000 lbs.	Jan. 16	Feb. 15	5000 lbs.	30	1	9c	4.50
				Feb. 28	8000	43	2	18c	?
				Mar. 6	2000	49	2	?	?
				Mar. 28	5000	?	?	?	?
									?

5. Prepare a bill for the storage of 160 cs. eggs on Jan. 15, by James Quick, at the rate of 10c per month of 30 days. Deliveries: Jan. 28, 40 cs.; Feb. 19, 50 cs.; the balance, Mar. 13.

6. Compute the storage on 5000 lb. poultry, stored Mar. 15 at 1/4c per lb. per 30 da. Deliveries: Mar. 30, 1500 lb.; Apr. 4, 800 lb.; May 12, 2000 lb.; June 16, the balance.

7. Extend this bill. Rates: 10c per bale, 15c per cs., per 30 da.

October 31, 19—

Messrs. Cates & Co.,
To The Commercial Storage Co., Dr.

Quantity.	Marks and Nos.	Rec'd.		Deliv.		Rate.	Amount.
8	bales No. 675–83 △	June	8	Oct.	20	?	?
4	Cs. c □ No. 721–24		16	Aug.	19	?	?
12	bales A. C. ○ No. 829–41		27	Sept.	30	?	?
6	bales BC ◈ No. 726–31	July	5	Oct.	4	?	?
							?

In cases where goods are being constantly received and delivered, and simple storage is charged, it is assumed that all deliveries are made from goods longest in storage.

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ILLUSTRATION. Robert Connor stores 500 bbl. Apr. 16; 300 bbl. May 1; 200 bbl. May 19. He withdraws 400 bbl., Apr. 30; 200 bbl. May 18; 400 bbl., May 27. Compute his storage at the rate of 6c per month, or fraction.

Solution.

Date	Rec'd	Deliv.		Rate.	Cost.
Apr. 16	500 bbl.				
30		400 bbl.	Stored, Apr. 16–30, 14 da.	6c	$24.00
May 1	300				
18		200	100 Stored, Apr. 16–May 18, 32 da.	12c	12.00
			100 Stored, May 1–18, 17 da.	6c	6.00
19	200				
27		400	200 Stored, May 1–27, 26 da.	6c	12.00
			200 Stored, May 19–27, 8 da.	6c	12.00
	1000	1000	Total storage,		$66.00

EXERCISE.

1. A. C. Bronson's storage memorandum is as follows: Stored, Nov. 12, 2000 lb. poultry; Dec. 5, 2000 lb.; Jan. 18, 400 lb. Withdrew, Dec. 24, 2200 lb.; Jan. 10, 1500 lb.; Mar. 17, 700 lb. Compute the storage at the rate of 1/4c per lb. per month.

2. John T. Bartlett's memorandum is: Stored, Aug. 18, 50 bbl. potatoes; Sept. 15, 200 bbl.; Oct. 17, 300 bbl.; Withdrawn, Nov. 5, 200 bbl.; Nov. 20, 100 bbl.; Dec. 5, 100 bbl.; Dec. 20, the balance. Compute his storage bill at the rate of 6c per bbl. per month.

3. On Oct. 5, Brown stored 100 bbl. apples; Oct. 20, 50 bbl.; Nov. 16, 180 bbl. Nov. 1 he withdrew 80 bbl.; Dec. 1, 50 bbl.; Jan. 5, 100 bbl.; Feb. 9, the balance. Compute the storage at the rate of 12c per bbl. per month.

2. AVERAGE STORAGE.

When receipts and deliveries are frequent, it is a custom to average the time, and to charge *average storage*. Usually exact time, with 30 day periods, is used.

1. The storage of 200 bbl. for one month equals the storage of 1 bbl. for ? months.

2. The storage of 100 bbl. for 2 months equals the storage of 1 bbl. for ? months.

3. The storage of 500 bu. for 20 days equals the storage of 1 bu. for ? days.

ILLUSTRATIVE EXAMPLE. The following is a memorandum of flour stored by C. P. Dean with the Commercial Storage Co., at a rate of 4c

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average storage. Receipts: Feb. 6, 200 bbl.; Feb. 21, 150 bbl.; Mar. 8, 400 bbl.; Mar. 29, 200 bbl. Deliveries: Feb. 12, 100 bbl.; Mar. 9, 150 bbl.; Mar. 21, 300 bbl.; Apr. 4, 400 bbl.

Solution. Arrange entries in order of dates:

Date.	Rec'd.	Deliv.	Balance	Time.	Equiv. to 1 bbl. Stored for
Feb. 6	200 bbl.		200 bbl.	6 da.	1200 da.
12		100 bbl.	100	9	900
21	150		250	15	3750
Mar. 8	400		650	1	650
9		150	500	12	6000
21		300	200	8	1600
29	200		400	6	2400
Apr. 4		400	0		

The storage items are equivalent to the storage of one barrel for.....16,500 days.

16,500 days = 550 terms of 30 days each

At 4c per term, the storage = 550 × 4c = $22.00. Ans.

NOTE. Notice that each "one bbl." equivalent is the product of the balance on hand by the number of days that balance remains unchanged.

EXERCISE.

1. Solve the examples in the last written exercises by average storage.

2. Are charges lower by average, or by simple storage? Why?

3. Compute average storage at the rate of 1½c per bu. per 30 days, from the following memorandum: Received: Oct. 10, 12,000 bu.; Oct. 25, 14,000 bu.; Nov. 1, 3500 bu. Deliveries: Dec. 5, 2500 bu.; Dec. 8, 3000 bu.; Dec. 21, 5,000 bu.; Dec. 30, 8,000 bu.; Jan. 5, 11,000 bu.

4. On Jan. 16, Robert Osborn bought 2,000 bbl. of flour at $4.25 and placed it in storage at an average rate of 5½c per barrel. On Jan. 29, he bought and stored 200 bbl. at $4.30; on Feb. 16, he withdrew 300 bbl. for sale at $4.65; on Mar. 11, 500 bbl. for sale at $4.80, and disposed of the balance, on Apr. 3, at $5.10. Drayage charges amounted to 15c per bbl. What was his per cent. of profit?

FOR INDIVIDUAL REPORT.

1. Prepare a brief on the knowledge of arithmetic required by the average retail clerk.

2. Prepare a brief on the knowledge of arithmetic required by the average mechanic.

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3. Report on "What Knowledge of Arithmetic is Most Necessary." Secure the opinion of business men and manufacturers.

4. Report on "Arithmetic Computation Tables in General Use." Make clear their value and range of use. Give sections of typical tables illustrating their use and the principles of their constructions.

5. Write a brief on "Machines for Arithmetical Computation." Cover range, usefulness, speed, accuracy, cost and extent of use.

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APPENDIX I.

SIGNS AND SYMBOLS.

a/c ....account.
a/s ....account sales.
+.....addition.
O.K. ..all correct.
&.....and.
@.....at, each, to.
.....and so on.
B/L ...bill of lading.
c/o ....care of.
√.....check mark, correct.
⊙.....circle.
c.....cent.
°.....degree.
÷.....division.
$.....dollar.
=.....equal, equals.
∽.....equivalent.
′.....foot, minutes.
$^{1, 2, 3}$...fourths (written, as exponents: $2^1 = 2\frac{1}{4}$).
>.....greater than.
C.....hundred.
″.....inches, seconds.
<.....less than.
×.....multiplication; incorrect.
#.....number (written before a figure).
o/d....on demand.
%.....per centum.
#.....pounds (written after a figure).
£.....pound sterling.
:.....ratio.
∵.....since.
$\sqrt{\ }$....square root of.
−.....subtraction.
∴.....therefore.
M.....thousand.
△.....triangle.

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APPENDIX II.

STANDARD ABBREVIATIONS.

NOTE. The singular form is now commonly used for singular and plural, unless otherwise noted.

Ac. . . . . Account.
A. . . . . acre.
agt. . . . . agent.
ans. . . . . answer.
B.O. . . . . back order.
bg. . . . . bag.
bal. . . . . balance.
bl. . . . . bale.
bk. . . . . bank, book.
bbl. . . . . barrel.
bkt. . . . . basket.
bot. . . . . bought.
bx. . . . . box.
bu. . . . . bushel.
cn. . . . . can.
cd. . . . . card, cord.
car. . . . . carton.
cs. . . . . case.
C., C.B. . . cash, cash book.
Cash. . . . . cashier.
csk. . . . . cask.
c, ct. . . . . cent.
cg. . . . . centigram.
cm. . . . . centimeter.
ck. . . . . check.
ch. . . . . chest.
c.o.d. . . . . collect on delivery.
cml. . . . . commercial.
com. . . . . commission.
Co. . . . . company.
consgt. . . . consignment.
cr. . . . . crate, credit, creditor.
cu. . . . . cubic.
cwt. . . . . hundredweight.
da. . . . . day.
dr. . . . . debt, debtor, debit.
dept. . . . . department.
do. . . . . *ditto* (the same).
dol. . . . . dollar.
dz., doz. . . dozen.
dft. . . . . draft.
ea. . . . . each.
E.O.E. . . . errors and omission excepted.
etc. . . . . *et cetera* (and so forth).
ex. . . . . example, express.
exch. . . . . exchange.
e. g. . . . . *exempli gratia* (for example).
exp. . . . . expense.

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far.....farthings.
fir. .....firkins.
fol.....folio, page.
ft.....foot.
for.....foreign.
fw., fwd. . .forward.
fr.....franc.
f.o.b. .....free on board.
gal.....gallon.
gi.....gill.
g. .....gram.
gr. .....gross, grain.
guar. .....guaranty, guarantee.
hf. .....half.
hhd.....hogshead.
hr. .....hour.
cwt.....hundredweight.
in. .....inch.
int.....interest.
I, inv. ....invoice.
invt.....inventory.
ins.....insurance.
inst.....*instant* (the present month).
kg.....keg.
km. .....kilometer.
L.F.....ledger folio.
l.p.....list price.
lf., lv. ....loaf, loaves.
lb. .....pound.
m.p.....marked price.
mdse.....merchandise.
m. .....meter.
Messrs. ...*Messieurs* (gentlemen, Sirs).
mi.....mile.
min.....minute.
Mr. .....Mister.
Mrs.....Mistress.
mo. .....month.
no.....number.
O.B. .....order book.
oz. .....ounce.
p., pp.....page, pages.
pkg.....package.
pd.....paid.
pl. .....pail.
pr. .....pair.
pay.....payment.
d.....pence.
per.....*per* (by the).
p.c. .....*per centum.*
pk.....peck.
pwt.....pennyweight.
pc. .....piece.
pt. .....pint.
poc., pkt. .pocket.
lb. .....pound.
Pres. .....President.
prox. .....*proximo* (the following month).
qt. .....quart.
qr. .....quire.
R.R. .....railroad.
rm.....ream.
rec'd.....received.
M.....Reischsmark.

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rd. . . . . rod.
sk. . . . . sack.
S. . . . . sales.
sec. . . . . second.
sec'y. . . . . secretary
s. . . . . shilling.
set. . . . . settlement.
ship. . . . . shipment.
shipt. . . . . shipped.
sig. . . . . signed, signature.
st. . . . . street.
stk. . . . . stock.
sund. . . . . sundries.
trc. . . . . tierce.
T. . . . . ton.
tr., trans. . transfer.
treas. . . . . treasurer, treasury.
tb. . . . . tub.
ult . . . . . *ultimate* (last month).
via. . . . . *via* (by way of).
viz. . . . . *videlicet* (namely, to wit).
wk. . . . . week.
wt. . . . . weight.
yd. . . . . yard.
yr. . . . . year.

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